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Masses and some electromagnetic properties of charm and *b*-quark hadrons in a bag model with a variable bag pressure

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Abstract. We investigate how a new bag phenomenology with a variable bag pressure as its main ingredient is successful in reproducing various properties of heavy hadrons. Our results for masses, magnetic moments and M1 radiative decay widths indicate that these properties arise in a natural and consistent manner from the theory and that no additional *ad hoc* assumption is required to explain them.

Keywords. MIT bag model; hadron masses; magnetic moments of hadrons; radiative transitions; charm; hadrons; *b*-quark hadron.

1. Introduction

The MIT bag model (Chodos *et al* 1974 a, b; De Grand *et al* 1975) provides a satisfactory dynamical framework for treating hadrons as systems of confined quarks. In this model, the quarks are confined to some region of space (the bag) by a pressure term B. The origin of B is not explained in the theory, and in the phenomenological applications of the model, it is treated as a universal parameter. However, the usual bag parametrisation has limited applications (De Grand *et al* 1975; Jaffe and Kiskis 1976). For example, it leads to poor agreement between the predicted masses for the charmed hadrons and the corresponding experimental numbers, and also yields a very small value for the proton magnetic moment. These discrepancies have been attributed (Ponce 1979), among other things, to the possible non-sphericity of the bag containing the quarks. Ponce (1979) has put forward an ansatz that the bag non-sphericity corrections can be incorporated if the zero point energy parameter Z_0 is taken to be mass- dependent. Hackman *et al* (1978), on the other hand, have added to the energy of the bag, a new term $c\,|N_q - N_{\bar{q}}|$, where c is a constant and N_q ($N_{\bar{q}}$) is the number of quarks (antiquarks) and have adjusted its value to correctly reproduce the magnetic moment of proton. Such attempts, though good working principles to lessen the gap between theory and experiment, are, however, *ad hoc* in character and lack a firm theoretical basis. The original MIT bag model, therefore, needs to be supplemented with new ideas and its basic parameters such as B and Z_0 require a deeper understanding and re-examination.

Recently, Joseph and Nair (1981) have proposed a phenomenological bag model, with a variable bag pressure as its main ingredient. In their model, the pressure term B, which has dimensions of energy-density, is not a universal constant as in the MIT bag model, but its value varies with density of hadronic matter, *i.e.*, it is different for each hadron. Also, the theory of hadronic structure, proposed by Callan *et al* (1979), which using QCD principles leads to the bag-like picture of hadrons, lends credence to the idea of B as an energy-density dependent factor. Joseph and Nair (1981) have calculated the masses of the light hadrons and the magnetic moments of SU(3) baryons in their approach, and the results obtained are in far closer agreement with the experiment than the MIT bag model. In the present paper, we argue in favour of the model by Joseph and Nair (1981) and investigate how the idea of a variable bag pressure modifies the MIT results concerning the various hadronic properties. For this, we recalculate masses of charm and b-quark hadrons, their magnetic moments, and the M1 transition rates of $3/2^+ \rightarrow 1/2^+ + \gamma$ and $1^- \rightarrow 0^- + \gamma$ decays in the new framework.

Interestingly, our analysis yields results comparable with those obtained by Ponce (1979) and Hackman *et al* (1978) for the charm and the bottom hadron masses. Our values of the charmed and b-flavoured baryon and vector meson magnetic moments compare favourably with those obtained by Bose and Singh (1980), who have used the method of Ponce (1979), *viz.*, a mass-dependent Z_0. For M1 transitions, our numbers are in fair agreement with those of Hackman *et al* (1978). These studies indicate that the new bag phenomenology, with a variable bag pressure as its main attribute, is quite successful in reproducing several features of hadrons, such as their masses, magnetic moments, radiative decays, etc. It has the added advantage that now these properties arise in a natural and logical manner from the theory and that no additional *ad hoc* assumption is necessary in the phenomenological implementation of the model.

The outline of the paper is as follows. In § 2, we write down the preliminaries of the original MIT bag and the new bag phenomenlogy. In § 3, we calculate the baryon and meson masses in the charm and the b-quark sectors, using the new approach. Section 4 is devoted to the determination of the magnetic moments of these heavy hadrons. The M1 transition rates for $3/2^+ \rightarrow 1/2^+ \gamma$ and $1^- \rightarrow 0^- \gamma$ are calculated in § 5.

2. Preliminaries

2.1 *The* MIT *bag hamiltonian*

The expression for the original MIT bag hamiltonian has been discussed in detail at several places. We simply summarize here various terms contributing to it. The total energy of the bag system and hence the mass of a hadron is written as

$$M(R) = E_Q + E_V + E_0 + E_M, \tag{1}$$

where

(i) $$E_Q = \sum_i N_i w_i, \tag{2}$$

is the kinetic energy term. N_i is the number of quarks and antiquarks of the ith flavour. A quark of mass m moving in a spherical bag of radius R has a kinetic energy w given by

$$w\,(mR) = \frac{1}{R}\,[x^2 + (mR)^2]^{1/2}. \tag{3}$$

x obeys the eigenvalue equation

$$\tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{1/2}}. \tag{4}$$

(ii) The bag volume energy

$$E_V = \frac{4}{3}\,\pi\,R^3\,B. \tag{5}$$

is the energy due to bag pressure B.

(iii) $$E_O = -\,Z_0/R, \tag{6}$$

is a term accounting for zero-point fluctuations.

(iv) The term E_M contains the residual gluonic interactions between the quarks and will be proportional to α_c, the strong coupling constant. This interaction is found to be magnetic in character.

$$E_M = \sum_{i>j} \lambda\,\sigma_i \cdot \sigma_j\,M_{ij}; \tag{7}$$

where $$M_{ij} = 8\,\alpha_c\,\frac{\mu'\,(m_i\,R)\,\mu'\,(m_j\,R)}{R^3}\,I\,(m_i\,R, m_j\,R), \tag{8}$$

$$\mu'\,(mR) = \frac{R}{6}\left[\frac{4wR + 2mR - 3}{2wR\,(wR - 1) + mR}\right], \tag{9}$$

λ is 1 for a baryon and 2 for a meson, σ_i, σ_j are spins of ith and jth quarks. $I\,(m_i\,R, m_j\,R)$ is a slowing varying function of $m_i\,R$ and $m_j\,R$ and ≈ 1.

The radius R of the bag is determined by demanding $\partial M/\partial R = 0$. The bag model is defined by the parameters B, Z_0, α_c, R, which are obtained by fitting (1) to the known hadron masses and to the value of proton magnetic moment.

2.2 *The new bag hamiltonian*

In order to relate the bag pressure term B to the density of the hadronic matter, it is noted that the energy density $\epsilon \equiv E/V$ of quark matter in the limit of zero quark masses is (Chaplin and Nauenberg 1977; Baym and Chin 1976)

$$\epsilon = B + An^{4/3}, \tag{10}$$

where A is a constant, and gets a contribution chiefly from the quarks' kinetic energy, and the lowest order colour interaction energy. Thus the pressure P of the quark matter is

$$P = -\frac{\partial E}{\partial V} = -B + 1/3\, A n^{4/3}, \tag{11}$$

leading to an equation of state

$$P = 1/3\,(\epsilon - 4B). \tag{12}$$

Requiring the system to be stable (*i.e.*, $P = 0$), and writing

$$\epsilon = B + \rho, \tag{13}$$

where ρ represents the contribution to the hadronic mass M from all the sources except the volume tension, it is found that (Joseph and Nair 1981)

$$B = \rho/3; \tag{14}$$

and

$$M = \frac{4}{3}\left[\sum_i N_i\, w_i + \sum_{i<j} \lambda\,(\sigma_i \cdot \sigma_j)\, M_{ij} - \frac{Z_0}{R}\right]. \tag{15}$$

Further, since the bag model results are not very sensitive to small variations in the bag size, the idea of average radii R_B and R_M for baryons and mesons, respectively, has been introduced, where (Bardeen *et al* 1975)

$$R_B \approx R_M\,(3/2)^{1/3} \tag{16}$$

Equation (15) can then be written as

$$M = 4/3\left(\sum_i N_i\, w_i + \sum_{i<j} \lambda\, \sigma_i \cdot \sigma_j\, M_{ij} - E_c\right),$$

where the zero-point energy E_c is now a constant, with one value for baryons and another for mesons.

In order to evaluate the model parameters, *viz.*, the quark masses, the bag radii R_B and R_M, the gluon coupling constant α_c, and the zero point energy E_c, the known values of the axial vector coupling constant

$$g_A = \frac{5}{9}\left(\frac{2w^2 R^2 + 4mR \cdot wR - 3mR}{2wR(wR - 1) + mR}\right), \tag{17}$$

the proton magnetic moment (in nuclear magneton), and the mass separation (Review Particle Properties 1980) $\Lambda - N$ are employed to yield $R_B = 8.88\ \text{GeV}^{-1}$, $m_n = 0{\cdot}114$ GeV, $w_n = 0{\cdot}294$ GeV, $m_s = 0{\cdot}302$ GeV, $w_s = 0{\cdot}427$ GeV. Using the ground-state baryon masses, one gets $\alpha_c = 0{\cdot}94$, and $E_c = 0{\cdot}068$ GeV. Finally, for mesons, assuming that the quark masses remain the same and that the meson radius is given by

$$R_M \approx R_B\,(2/3)^{1/3} = 7{\cdot}75\ \mathrm{GeV}^{-1}, \tag{18}$$

the quark kinetic energies have been found to increase to

$$w_n = 0{\cdot}321\ \mathrm{GeV}, \quad w_s = 0{\cdot}448\ \mathrm{GeV}.$$

Employing the same value of α_c as for baryons, and fitting the experimental mass of K-meson, E_c for mesons, comes to be 0·178 GeV.

3. Masses of the charm and the bottom hadrons

To estimate the masses of various ground-state baryons belonging to the charm and the b-quark sectors, we have first to determine the quark kinetic energies w_c and w_b, and the corresponding quark masses m_c and m_b. For this, we make use of the mass separations $(\Lambda_c - N)$ and $(\Lambda_b - N)$, and obtain

$$w_c = 1{\cdot}294\ \mathrm{GeV},$$

$$w_b = 3{\cdot}714\ \mathrm{GeV}.$$

For the mass of Λ_b, we are guided by the theoretical predictions of Singh *et al* (1981). We then solve the transcendental equation

$$\tan\,(w^2R^2 - m^2R^2)^{1/2} = -\frac{(w^2R^2 - m^2R^2)^{1/2}}{(wR + mR - 1)}, \tag{19}$$

numerically, yielding

$$m_c = 1{\cdot}279\ \mathrm{GeV},$$

$$m_b = 3{\cdot}443\ \mathrm{GeV}.$$

Using the values of the parameters as obtained above, we now estimate masses of $1/2^+$ and $3/2^+$ ground-state charm and b-quark baryons from (15). Our results are displayed in table 1, along with the experimental values (Review Particle Properties 1980), wherever available.

For mesons, assuming that the quark masses remain the same and that the meson radius is given by (18), we again solve (19) numerically to find that the quark kinetic energies increase to

$$w_c = 1{\cdot}307\ \mathrm{GeV}$$

$$w_b = 3{\cdot}749\ \mathrm{GeV}.$$

Table 1. Mass spectrum (in GeV) of c-quark and b-quark baryons in lowest cavity mode.

Multiplet	Particle	Quark Content	MIT bag model: Jaffe and Kiskis (1976)	MIT bag model: Ponce (1979)	MIT bag model: Hackman *et al* (1978)	Present analysis	Experiment
$1/2^+$ baryons	Σ_c^{++}	*cuu*	2·357	2·380	2·495	2·421	2·430
	Λ_c^+	$c(ud)_A$	2·214	2·243	2·253	2·300	2·273
	Ξ_c^+	$c(su)_S$	2·507	2·530	2·564	2·580	—
	$\Xi_c^{+\prime}$	$c(su)_A$	2·396	2·425	2·450	2·520	—
	Ω_c^0	*css*	2·653	2·678	2·705	2·766	—
	Ξ_{cc}^{++}	*ccu*	3·538	3·511	3·636	3·720	—
	Ω_{cc}^+	*ccs*	3·690	3·664	3·781	3·897	—
	Σ_b^+	*buu*	—	5·735	—	5·670	—
	Λ_b^0	$b(ud)_A$	—	5·555	—	5·550	—
	Ξ_b^0	$b(su)_S$	—	5·880	—	5·836	—
	$\Xi_b^{0\prime}$	$b(su)_A$	—	5·736	—	5·753	—
	Ω_b^-	*bss*	—	6·022	—	6·008	—
	Ξ_{cb}^+	$b(cu)_S$	—	6·842	—	6·980	—
	$\Xi_{cb}^{+\prime}$	$b(cu)_A$	—	—	—	6·954	—
	Ω_{cb}^0	*bcs*	—	6·988	—	7·056	—
	Ξ_{bb}^0	*bbu*	—	10·003	—	10·194	—
	Ω_{bb}^-	*bbs*	—	10·142	—	10·370	—
$3/2^+$ baryons	Σ_c^{++*}	*cuu*	2·461	2·481	2·495	2·496	2·480
	Ξ_c^{+*}	$c(su)_S$	2·603	2·624	2·633	2·633	—
	Ω_c^{0*}	*css*	2·742	2·764	2·766	2·793	—
	Ξ_{cc}^{++*}	*ccu*	3·661	3·630	3·727	3·775	—
	Ω_{cc}^{+*}	*ccs*	3·795	3·764	3·854	3·930	—
	Ω_{ccc}^{++*}	*ccc*	4·827	4·747	—	5·200	—
	Σ_b^{+*}	*buu*	—	5·769	—	5·682	—
	Ξ_b^{0*}	$b(su)_S$	—	5·912	—	5·850	—
	Ω_b^{-*}	*bss*	—	6·051	—	6·021	—
	Ξ_{cb}^{+*}	*bcu*	—	6·919	—	6·909	—
	Ω_{cb}^{0*}	*bcs*	—	7·054	—	7·165	—
	Ξ_{bb}^{0*}	*bbu*	—	10·048	—	10·211	—
	Ω_{bb}^{-*}	*bbs*	—	10·148	—	10·347	—
	Ω_{bbc}^{0*}	*bbc*	—	11·152	—	11·540	—
	Ω_{bbb}^{-*}	*bbb*	—	14·248	—	14·766	—

Using these values and taking $\alpha_c = 0.94$, $E_c = 0{\cdot}178$, we now estimate the masses of 0^- and 1^- mesons in the charm and the bottom sector, and list them in table 2, along with the corresponding experimental numbers (Review Particle Properties 1980). The mass values obtained for η_c is disturbed off the experiment. The discrepancy may be due to additional contributions coming from gluon annihilation processes. (De Rujula *et al.* 1975)

Our numbers show quite an improvement over those obtained by Jaffe and Kiskis (1976) who for the charmed hadrons used the original version of the MIT bag without employing any additional consideration. Further, the results of the present analysis favourably match the values obtained by Ponce (1979) and Hackman *et al* (1978), who have to propose *ad hoc* ansatze to bring the desired agreement between theory and experiment.

4. Magnetic moments

The magnetic moment of a quark q of mass m is given by

$$\mu_q = 1/2 \int_{\text{bag}} d^3x \, \mathbf{r} \times \mathbf{j}, \tag{20}$$

with the current

$$\mathbf{j} = e_q \, q^+ (x) \, \sigma \, q \, (x), \tag{21}$$

Table 2. Mass spectrum (in GeV) of *c*-quark and *b*-quark mesons in lowest cavity mode.

Multiplet	Particle	Quark Content	MIT bag model: Jaffe and Kiskis (1976)	MIT bag model: Ponce (1979)	MIT bag model: Hackman *et al* (1978)	Present analysis	Experiment
0^- mesons	D_c^+	$c\bar{d}$	1·726	1·800	1·867	1·867	1·865
	F_c^+	$c\bar{s}$	1·885	1·957	2·015	2·040	2·030
	η_c	$c\bar{c}$	2·931	2·971	—	3·220	2·960
	D_b^-	$b\bar{u}$	—	5·232	—	5·170	—
	F_b^0	$b\bar{s}$	—	5·372	—	5·340	—
	G_b^-	$b\bar{c}$	—	6·347	—	6·490	—
1^- mesons	D_c^{+*}	$c\bar{d}$	1·969	2·009	2·015	1·992	2·007
	F_c^{+*}	$c\bar{s}$	2·099	2·141	2·139	2·124	2·140
	ψ	$c\bar{c}$	3·095	3·095	3·194	3·150	3·095
	D_b^{-*}	$b\bar{u}$	—	5·299	—	5·250	—
	F_b^{0*}	$b\bar{s}$	—	5·431	—	5·360	—
	G_b^-	$b\bar{c}$	—	6·388	—	6·510	—
	Υ	$b\bar{b}$	—	9·460	—	9·760	—

where e_q is the charge of the quark and $q(x)$ is the quark wave function. The value of the above integral in lowest quark eigenmode is

$$\mu_q = \frac{R}{6}\left(\frac{4wR + 2mR - 3}{2wR\,(wR - 1) + mR}\right) e_q. \tag{22}$$

Using the values of R, mR and wR for baryons as obtained in §§2 and 3, the magnetic moments of the individual quarks that make up the baryons come out to be

$$\mu_u = 1{\cdot}86 \text{ nm}, \qquad \mu_d = -\tfrac{1}{2}\mu_u = -0{\cdot}93 \text{ nm},$$

$$\mu_s = -0{\cdot}68 \text{ nm}, \qquad \mu_c = 0{\cdot}48 \text{ nm},$$

$$\mu_b = -0{\cdot}082 \text{ nm}.$$

For the magnetic moments of 1^- mesons, we find that the magnetic moments of the quarks that constitute the 1^- mesons have the values

$$\mu_u = 1{\cdot}70 \text{ nm}, \qquad \mu_d = -0{\cdot}85 \text{ nm},$$

$$\mu_s = -0{\cdot}645 \text{ nm}, \qquad \mu_c = 0{\cdot}47 \text{ nm},$$

$$\mu_b = -0{\cdot}081 \text{ nm}.$$

The magnetic moments of $1/2^+$ baryons, $3/2^+$ baryons, and 1^- mesons, both in the c-quark and b-quark sectors, can now be evaluated in terms of μ_q from the known flavour and spin wave functions of the baryons and the mesons. Here we make use of the principle of quark additivity and assume that the orbital angular momentum of quarks is zero. So we write

$$\mu_{B,M} = \sum_i \mu_q\,(m_i R). \tag{23}$$

The magnetic moments of both baryons and mesons thus calculated are listed in table 3. Our values of the baryons magnetic moments compare favourably with those determined by Bose and Singh (1980), who have used the method of Ponce (1979), *viz.*, a mass dependent Z_0. It may be mentioned here that the experimental numbers for the magnetic moments of c- and b-flavoured hadrons are still not available. Nevertheless, we present our results since there may be other studies accessible to experimental verification where the magnetic moments of heavy hadrons may be needed.

5. M1 radiative transitions

An expression for M1 transition width is given by (Deshpande *et al* 1977; Hackman *et al* 1978)

$$\Gamma = \frac{e^2}{4\pi} k^3\,(16/3) \sum_{\substack{\alpha,\,\beta \\ \alpha \geqslant \beta}} \mu_\alpha\,\mu_\beta\, C^{PQ}_{\alpha\beta}, \tag{24}$$

Table 3. Magnetic moments of charmed and b-quark baryons and mesons (in nm)

Multiplet	Particle	Quark content	Bose and Singh (1980)	Present analysis
$1/2^+$ baryons	Σ_c^{++}	cuu	1·95	2·32
	Σ_c^+	$c(ud)_S$	0·36	0·42
	$\Lambda_c^{+'}$	$c(ud)_A$	0·50	0·48
	Σ_c^0	cdd	−1·23	−1·40
	Ξ_c^+	$c(su)_S$	0·47	0·63
	$\Xi_c^{+'}$	$c(su)_A$	0·50	0·48
	Ξ_c^0	$c(sd)_S$	−1·09	−1·24
	$\Xi_c^{0'}$	$c(sd)_A$	0·50	0·48
	Ω_c^0	css	−0·98	−1·07
	Ξ_{cc}^{++}	ccu	0·17	0·18
	Ξ_{cc}^+	ccd	0·86	0·95
	Ω_{cc}^+	ccs	0·84	0·87
	Σ_b^+	buu	2·32	2·45
	Σ_b^0	$b(ud)_S$	0·59	0·64
	$\Lambda_b^{0'}$	$b(ud)_A$	0·084	0·082
	Σ_b^-	bdd	−1·12	−1·27
	Ξ_b^0	$b(su)_S$	0·73	0·81
	$\Xi_b^{0'}$	$b(su)_A$	0·084	0·082
	Ξ_b^-	$b(sd)_S$	−0·98	−1·04
	$\Xi_b^{-'}$	$b(sd)_A$	0·084	0·082
	Ω_b^-	bss	−0·84	−0·88
	Ξ_{cb}^+	bcu	—	1·58
	Ξ_{cb}^0	bcd	—	−0·27
	Ω_{cb}^0	bcs	—	−0·11
	Ω_{ccb}^+	bcc	—	0·66
	Ξ_{bb}^0	bbu	−0·62	−0·73
	Ξ_{bb}^-	bbd	0·14	0·20
	Ω_{bb}^-	bbs	0·09	0·12
	Ω_{cbb}^0	bbc	—	−0·26

Table 3. (*continued*)

Multiplet	Particle	Quark content	Bose and Singh (1980)	Present analysis
3/2⁺ baryons	Σ_c^{++*}	*cuu*	3·91	4·10
	Σ_c^{+*}	*cud*	1·34	1·40
	Σ_c^{0*}	*cdd*	−1·20	−1·38
	Ξ_c^{+*}	*cus*	1·54	1·65
	Ξ_c^{0*}	*cds*	−1·01	−1·13
	Ω_c^{0*}	*css*	−0·78	−0·88
	Ξ_{cc}^{++*}	*ccu*	2·54	2·81
	Ξ_{cc}^{+*}	*ccd*	0·19	0·26
	Ω_{cc}^{+*}	*ccs*	0·39	0·28
	Ω_{ccc}^{++*}	*ccc*	1·45	1·43
	Σ_b^{+*}	*buu*	3·44	3·64
	Σ_b^{0*}	*bud*	0·78	0·84
	Σ_b^{-*}	*bdd*	−1·84	−1·94
	Ξ_b^{0*}	*bsu*	1·01	1·09
	Ξ_b^{-*}	*bsd*	−1·62	−1·69
	Ω_b^{-*}	*bss*	−1·40	−1·44
	Ξ_{cb}^{+*}	*bcu*	2·04	2·25
	Ξ_{cb}^{0*}	*bcd*	−0·40	−0·53
	Ω_{cb}^{0*}	*bcs*	−0·22	−0·28
	Ω_{ccb}^{+*}	*bcc*	0·89	0·88
	Ξ_{bb}^{0*}	*bbu*	1·37	2·02
	Ξ_{bb}^{-*}	*bbd*	−0·95	−1·09
	Ω_{bb}^{-*}	*bbs*	−1·28	−0·84
	Ω_{cbb}^{0*}	*bbc*	0·31	0·64
	Ω_{bbb}^{-*}	*bbb*	−0·08	−0·09
1⁻ mesons	D_c^{+*}	$c\bar{d}$	1·18	1·32
	D_c^{0*}	$c\bar{u}$	−0·9	−1·02
	F_c^{+*}	*cs*	1·03	1·11
	D_b^{-*}	$b\bar{u}$	−1·54	−1·78
	D_b^{0*}	$b\bar{d}$	0·64	0·76
	F_b^{0*}	$b\bar{s}$	0·48	0·56
	G_b^{-*}	$b\bar{c}$	−0·56	−0·55

where k is the photon momentum, α and β refer to the quark flavours, and μ_α is the quark transition moment defined by

$$\mu_\alpha = \frac{1}{2k} N^2 \int_0^R \mathrm{d}x\, x^2 j_1(kx) \times 2\left[j_0\left(\frac{x_\alpha x}{R}\right) j_1\left(\frac{x_\alpha x}{R}\right) \left(\frac{w_\alpha + m_\alpha}{w_\alpha}\right)^{1/2} \left(\frac{w_\alpha - m_\alpha}{w_\alpha}\right)^{1/2}\right]. \quad (25)$$

The quark normalisation N_α is given by

$$\frac{1}{N_\alpha^2} = R^3 j_0^2(x_\alpha) \left[\frac{2\, w_\alpha (w_\alpha - 1/R) + m_\alpha/R}{w_\alpha (w_\alpha - m_\alpha)}\right], \quad (26)$$

with the frequency of the lowest mode given by (3). In (25), j's are the spherical Bessel functions, and x_α satisfies the eigenvalue eq. (4). The magnetic coefficients $C_{\alpha\beta}^{PQ}$ are given by

$$C_{\alpha\beta}^{PQ} = \sum_{m,m'} \sum_{k,k'} \langle P \,|\, b_\alpha^{+}(m)\, Q_\alpha\, b_\alpha(m') \,|\, Q \rangle \langle Q \,|\, b_\beta^{+}(k)\, Q_\beta\, b_\beta(k') \,|\, P \rangle\, U_m^{+} \sigma_i U_{m'} U_{k'} \sigma_i U_k, \quad (27)$$

where $b_\alpha(m)$ is the destruction operator for a quark of type α with spin projection m, Q_α is the charge on the quark of flavour α, and Q denotes an intermediate state contributing to the magnetic energy of a particle P. The values of different $C_{\alpha\beta}^{PQ}$ which are of interest to the present work are given in appenda (A) and (B). Our values of the transition moments, which we evaluate using (25), are presented in tables 4 and 5 for mesons and baryons, respectively. We then employ (24) to calculate

Table 4. Quark transition moments, μ_α, for mesons.

Transition	q_1	q_2
$D_c^* \to D_c\, \Upsilon$	0·1810	0·7020
$F_c^* \to F_c\, \Upsilon$	0·1900	0·5046
$\psi \to \eta_c\, \Upsilon$	0·1562	0·1562

Table 5. Quark transition moments, μ_α, for baryons.

Transition	q_1	q_2	q_3
$\Sigma_c^* \to \Sigma_c\, \Upsilon$	0·1859	0·8403	0·8403
$\Sigma_c^* \to \Lambda_c\, \Upsilon$	0·1670	0·7318	0·7318
$\Xi_c^{+*} \to \Xi_c^{+}\, \Upsilon$	0·1863	0·6102	0·8361
$\Xi_c^{+*} \to \Xi_c^{+'}\, \Upsilon$	0·1752	0·5706	0·7290

Table 6. M1 radiative decay widths (in keV) for the charmed mesons and baryons.

Transition	MIT model Hackman *et al* (1978)	Present analysis
$D_c^{+*} \rightarrow D_c^{+} \Upsilon$	1·1	1·3
$D_c^{0*} \rightarrow D_c^{0} \Upsilon$	22·7	35·4
$F_c^{+*} \rightarrow F_c^{+} \Upsilon$	0·1	0·1
$\psi \rightarrow \eta_c \Upsilon$	21·0	27·9
$\Sigma_c^{++*} \rightarrow \Sigma_c^{++} \Upsilon$	3·3	6·3
$\Sigma_c^{+*} \rightarrow \Sigma_c^{+} \Upsilon$	1·5	2·9
$\Sigma_c^{0*} \rightarrow \Sigma_c^{0} \Upsilon$	2·7	5·4
$\Sigma_c^{+*} \rightarrow \Lambda_c^{+} \Upsilon$	176·7	125·7
$\Xi_c^{+*} \rightarrow \Xi_c^{+} \Upsilon$	1·5	2·3
$\Xi_c^{+*} \rightarrow \Xi_c^{+'} \Upsilon$	74·0	89·4

the M1 radiative decay widths and list our results in table 6 both for $3/2^+ \rightarrow 1/2^+$ and $1^- \rightarrow 0^-$ transitions of charmed particles. For simplicity, we have neglected the dynamical mixing induced by transitions to pure gluon states in our calculations involving isoscalar pseudoscalar mesons. It is interesting to note that our values compare reasonably well with the ones which Hackman *et al* (1978) obtained by postulating an additional *ad hoc* term to the bag energy.

6. Conclusion

The MIT bag model, with a universal bag pressure, though a very successful tool in reproducing some of the salient features of hadrons, has met with only limited success when applied to the heavy quark states and the nonspectroscopic calculations. Several authors have attempted to resolve the discrepancies observed in the implementation of the model. Such attempts are, however, *ad hoc* in character, and do not emerge in a natural manner from the theory. Recently, Joseph and Nair (1981) have introduced a new bag phenomenology, with a variable bag pressure as its chief attribute, and have satisfactorily reproduced the light hadron masses and magnetic moments of SU(3) octet baryons, in better agreement with the experimental values than the MIT bag model. Also, Callen *et al* (1979) have presented a theory of hadronic structure leading to a bag-like picture of hadrons in which the pressure B is implicitly an energy-density dependent factor. Keeping this aspect in view, and following Joseph and Nair (1981), we have investigated how the new bag phenomenology helps in reproducing various hadronic features, by recalculating the masses and magnetic moments of charmed and b-quark hadrons, and the M1 transition rates for $1^- \rightarrow 0^- \gamma$ and $3/2^+ \rightarrow 1/2^+ \gamma$ decays in the charm sector. Interestingly, our results favourably match the values obtained by Ponce (1979) and Heckman *et al* (1978), who, to lessen the gap between theory and experiment, proposed a mass dependent Z_0 and an additional term to the bag energy, respectively. The success of the new phenomenology lies in the fact that the hadronic properties hitherto explained by making *ad hoc* assumptions are now derivable from the theory.

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Appendix A. The magnetic transition coefficients for mesons as defined in Hackman *et al* (1978) are tabulated as follows.

Transition	q_1q_1	q_1q_2	q_2q_2
$D_c^{+*} \to D_c^+ \Upsilon$	12/27	−12/27	3/27
$D_c^{*0} \to D_c^0 \Upsilon$	12/27	24/27	12/27
$F_c^{*+} \to F_c^+ \Upsilon$	12/27	−12/27	3/27
$\psi \to \eta_c \Upsilon$	12/27	24/27	12/27

Appendix B. The magnetic transition coefficients for baryons as defined in Hackman *et al* (1978) are given below.

Transition	q_1q_1	q_1q_2	q_1q_3	q_2q_2	q_2q_3	q_{33}
$\Sigma_c^{++*} \to \Sigma_c^{++} \Upsilon$	16/27	−16/27	−16/27	4/27	8/27	4/27
$\Sigma_c^{+*} \to \Sigma_c^{+} \Upsilon$	16/27	8/27	−32/27	1/27	−8/27	16/27
$\Sigma_c^{0*} \to \Sigma_c^{0} \Upsilon$	16/27	8/27	8/27	1/27	2/27	1/27
$\Sigma_c^{+*} \to \Lambda_c^{+} \Upsilon$	0	0	0	3/27	12/27	12/27
$\Xi_c^{+*} \to \Xi_c^{+} \Upsilon$	16/27	8/27	−32/27	1/27	−8/27	16/27
$\Xi_c^{+*} \to \Xi_c^{+\prime} \Upsilon$	0	0	0	3/27	12/27	12/27

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