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Mass formulae for Λ-hypernuclei

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Abstract. Simple considerations regarding the Hamiltonian and the ground state wavefunctions of Λ-hypernuclei are used to derive several mass formulae. The parameters that occur in the mass formulae are determined by fitting the experimental binding energies. Information regarding the various interactions in hypernuclal is deduced from the values of these parameters. The 'best' mass formula is further used to predict energies of other light hypernuclei. Relationships between binding energies are also suggested and checked with observed data.

Keywords. Λ-hypernuclei; mass formulae; mass relationships.

1. Introduction

The binding energies of 21 Λ-hypernuclei (Povh 1978) and 2 ΛΛ-hypernuclei (Danysz *et al* 1963; Prowse 1966) are to date experimentally known. They provide vital information regarding the effective interaction of a Λ-particle with nucleons and of Λ-Λ interaction. A large amount of theoretical work (Gal *et al* 1971, 1972, 1978) has been done to understand the nature of these interactions. The approach for the most part has been to carry out detailed model calculations for different hypernuclei.

In this paper, the question of Λ-*N* and Λ-Λ effective interaction is examined from a different point of view involving mass formula and mass relationships for hypernuclear binding energies. More precisely, mass formulae for hypernuclei are derived based on a few basic assumptions regarding the nature of the Hamiltonian of the system and the ground state wave function. Such an approach has already been employed in the study of nuclei (Parikh 1978).

A brief description of this approach and its extension to Λ-hypernuclei is described in § 2. The various expressions for the binding energy of Λ-hypernuclei contain a few parameters. These are determined by fitting the mass formulae to the observed binding energies. The method of least squares was used for this purpose. The results of the fit are given in § 3 together with a discussion of the information they provide regarding the various interactions in hypernuclei. Binding energies of many as yet unobserved hypernuclei are also predicted. It has to be understood that in such an approach the fit to the observed energies cannot be as good as that for the model calculations (Gal *et al* 1971, 1972).

In § 4 it is shown that the mass formulae that have been derived, imply the existence of relationships between the binding energies that are generalizations of the Garvey-Kelson (Garvey & Kelson 1966; Garvey *et al* 1969) mass relationship. Experimental binding energies are used to test these relationships. Also predictions (of binding energies) based on mass relationships are made and compared with those resulting from the mass formula.

2. Derivation of the mass formula

In the nuclear case (Parikh 1978) the mass formula was derived by making the following assumptions:

(a) The nuclear hamiltonian H_N is a (1+2)-body operator.
(b) A single Slater determinant approximation to the ground state wave function gives a very good estimate of the ground state (g.s.) energy.
(c) The (average) potential in which each nucleon moves is taken to be axially-symmetric and time reversal invariant.

The mass formula was derived (Parikh 1978) by decomposing the unitary groups $U(N_p)$ and $U(N_n)$, where N_p (and N_n) are the number of single proton (and neutron) states, into the direct sum subgroups $U(2) \oplus U(2) \oplus \ldots\ldots \oplus U(2)$, where each U(2) stands for the group of transformations in the space of one specific time-reversed pair $|k\rangle$ and $|-k\rangle$. The energy of the ground state determinant is the average of the states belonging to a particular irreducible representation of the direct sum subgroup. As a result one can express the energy of that state in terms of the scalars of this direct sum subgroup (French 1967), which are p and n. The expression for the g.s. energy $E(A, T_3)$ of a nucleus with n neutrons and p protons (or $A = n + p$ and $T_3 = \frac{1}{2}(n-p)$) is then

$$E(A, T_3) = a_1 A + a_2 A^2 + a_3 T_3 + a_4 T_3^2 + a_5 AT_3 + a_6 s_p (s_p+1) + a_7 s_n (s_n+1).$$

Here s_p is a quasi-spin quantum number for the protons, having the value $\frac{1}{2}$ when p is odd and 0 when p is even. Similarly s_n is the quasi-spin quantum number for the neutrons. The seven unknown parameters $a_1, a_2, \ldots, a_7$ were determined (Parikh 1978) by making a least squares fit of the equation to the experimental energies. If one requires further that the mass formula be charge-independent one can modify the above equation to obtain

$$E(A, T=|T_3|) = a_1 A + a_2 A^2 + a_3 |T_3| + a_4 T_3^2 + a_5 A |T_3| + a_6 s_p (s_p+1) + a_7 s_n (s_n+1). \quad (1)$$

For protons, neutrons and Λ particles we do the subgroup decomposition for all three of them and write the energy in terms of p, n, and n_Λ. The Λ particle has isospin zero and does not change the isospin structure. So we can convert n and p to A and T_3. Finally we want the expression to be charge-independent in this case also and arrive at

$$B_\Lambda (A, T=|T_3|, n_\Lambda) = a_1 n_\Lambda + b_1 n_\Lambda A + c_1 n_\Lambda |T_3| + d_1 n_\Lambda (n_\Lambda - 1)/2 + e_1 s_\Lambda (s_\Lambda + 1). \quad (2)$$

Here n_Λ is the number of Λ-particles and s_Λ the quasi-spin quantum number for Λ-particles which takes the value $\frac{1}{2}$ when n_Λ is odd and 0 when n_Λ is even. If one recalls that the experimental information is available only for $n_\Lambda = 1$ and $n_\Lambda = 2$, then it is easy to prove that one has actually four independent parameters instead of the five shown in (2). We therefore consider the four parameter formula,

$$B_\Lambda (A, T = |T_3|, n_\Lambda) = a_1 n_\Lambda + b_1 n_\Lambda A + c_1 n_\Lambda |T_3| + \\ + d_1 n_\Lambda (n_\Lambda - 1)/2. \tag{3}$$

The formula of (2) would be useful if one has data for multi Λ-hypernuclei. In (2) and (3) one may, instead of the term $n_\Lambda |T_3|$, introduce a term proportional to $n_\Lambda T(T+1)/A$ with $T = |T_3|$. The eigenvalue of the total isospin operator is then seen to occur in the mass formula.

Several features of this type (equation (3)) of mass formula have been discussed earlier (Parikh 1978). The crucial point to note in the present context is that at best the expression in (2) or (3) will be valid for a limited region of the periodic table. This arises from the fact that the nature of occupied single particle states changes appreciably from one region of the periodic table to another. Since experimental information is available only for those hypernuclei that have nucleons in the lowest s and p shells this is not a serious limitation.

The interpretation of the four parameters is very direct. The first two terms together represent the value of average field which each Λ-particle experiences. It is the sum of its kinetic energy and (average) Λ-nucleus potential energy. c_1 and d_1 represent respectively the residual Λ-nucleon and the residual Λ-Λ interactions.

The mass formula of (3) can be refined by having different parameters for the interactions of the Λ particle with nucleons in the s-shell and the p-shell. One then obtains for ($n_\Lambda = 1$) hypernuclei a five-parameter mass formula,

$$B (A, T = |T_3|, n_\Lambda = 1) = a_2 + b_2 A_s + c_2 A_p + d_2 |T_{3s}| + e_2 |T_{2p}|, \tag{4}$$

where $A_s = n_s + p_s, A_p = n_p + p_p, |T_{3s}| = \frac{1}{2} |n_s - p_s|, |T_{3p}| = \frac{1}{2} |n_p - p_p|,$

and n_s (p_s), n_p (p_p) are respectively the number of neutrons (protons) in the s-shell and the p-shell. In (4) we do not retain the term proportional to $n_\Lambda (n_\Lambda - 1)/2$ as we restrict ourselves to single Λ-hypernuclei. Furthermore as there are good reasons (for example see Gal *et al* 1972) to include three-body ΛNN interaction terms in the Hamiltonian, the mass formula should also reflect this feature. So we construct the three-body terms with the operators n_Λ, A and T_3 and add them to our earlier expression given in (3). Then we get the following mass formula for single Λ-hypernuclei ($n_\Lambda = 1$)

$$B_\Lambda (A, T = |T_3|, n_\Lambda = 1) = a_3 + b_3 A + c_3 |T_3| + d_3 \binom{A}{2} + e_3 T_3^2. \tag{5}$$

In addition to (5) in which there are independent coefficients for the terms proportional to $|T_3|$ and T_3^2 we have also considered the four parameter mass formula of (6) in which the dependence is proportional to $T(T + 1)$

$$B_\Lambda (A, T = |T_3|, n_\Lambda = 1) = a_3 + b_3 A + c_3 \binom{A}{2} + d_3 T(T + 1). \tag{6}$$

Equation (6) can, as before, be modified to differentiate between the nucleons in the s-shell and the p-shell. As we shall see in the next section, the dominant contribution to the energy comes from the two-body term proportional to A (equation (6)) and hence we modify (6) only to the extent shown below.

$$B_\Lambda (A, T = |T_3|, n_\Lambda = 1) = a_4 + b_4 A_s + c_4 A_p + d_4 \binom{A}{2} + e_4 T(T+1). \tag{7}$$

As we shall see in the next section, for single Λ-hypernuclei, (6) gives us the " best " mass formula in the sense that it is quite simple and has optimal predictive power that one can achieve in this formalism.

3. Determination of the parameters in the mass formula

Results of the least squares fit of the various formulae (3) to (7) to the experimental binding energies are described in this section.

Table 1 shows the values of the four parameters with the standard errors (Δa_1, etc) when (3) is fitted to all the 23 binding energies (21 single Λ-hypernuclei and 2 double Λ-hypernuclei) known experimentally.

On comparing the calculated and the observed energies of these hypernuclei it is found that for all the 23 hypernuclei the absolute deviation between the calculated and the experimental value is less than 1 MeV. Mean deviation between the two sets is zero and the r.m.s. deviation is 0·503 MeV. The largest absolute deviation of 0·93 MeV is found for the two $\Lambda\Lambda$-hypernuclei $_{\Lambda\Lambda}He^6$ and $_{\Lambda\Lambda}Be^{10}$.

In order to make a comparison between the different mass formulae (3) to (7) we use in the fit only the 21 single Λ-hypernuclear energies. This is because there are just two double Λ-hypernuclei known, and it is not worthwhile to include them in the fitting procedure as that would increase the number of parameters by one. Table 2 gives the r.m.s. deviation and the maximum deviation between the calculated and the observed (21) energies for the five different mass formulae.

Table 1. The value of the parameters with their standard errors in the mass formula of equation (3) fitted to 23 hypernuclei.

$a_1 \pm \Delta a_1$	$b_1 \pm \Delta b_1$	$c_1 \pm \Delta c_1$	$d_1 \pm \Delta d_1$
$-1·135 \pm 0·334$	$1·057 \pm 0.035$	$0·617 \pm 0·299$	$3·785 \pm 0·552$

Table 2. The rms deviation (MeV) and the maximum deviation (MeV) between the 21 calculated and observed energies of hypernuclei for different mass formulae.

	Mass formula of equation (3)*	Mass formula of equation (4)	Mass formula of equation (5)	Mass formula of equation (6)	Mass formula of equation (7)
RMS deviation	0·503	0·392	0·370	0·374	0·374
Maximum deviation	0·93	0·80	0·92	0·99	0·99

*This is a fit to 23 hypernuclei as opposed to the others which are for 21 hypernuclei.

It is seen that as the mass formula is made more complex the fit improves but the maximum deviation does not decrease by any significant amount. Note also that (6) and (7) give identical fits indicating that having different parameters for s and p shell nucleons does not improve the results. On closer examination it is found that the hypernucleus $_\Lambda Be^9$ always gives the largest deviation. Therefore we next exclude from the fit $_\Lambda Be^9$ and two other hypernuclei having large deviations. We find that the mass formula (equation(6)) with 4 parameters fits the 18 energies quite well with r.m.s. deviation of 0·235 MeV. Table 3 shows the values of the four parameters and their standard errors when (6) is fitted to the energies of 21 as well as 18 hypernuclei. In table 4 the calculated (E_c) (equation (16) with the values of the parameters shown

Table 3. The value of the parameters in the mass formula of equation (5) fitted to 18 hypernuclei with their standard errors.

	$a_3 \pm \Delta a_3$	$b_3 \pm \Delta b_3$	$c_3 \pm \Delta c_3$	$d_3 \pm \Delta d_3$
Parameters of equation (6) as fitted to 21 single hypernuclei	$-2{\cdot}300 \pm 0{\cdot}468$	$1{\cdot}396 \pm 0{\cdot}127$	$-0{\cdot}042 \pm 0{\cdot}017$	$0{\cdot}159 \pm 0{\cdot}109$
Parameters of equation (6) as fitted to 18 single hypernuclei	$-2{\cdot}403 \pm 0{\cdot}311$	$1{\cdot}452 \pm 0{\cdot}090$	$-0{\cdot}048 \pm 0{\cdot}012$	$0{\cdot}135 \pm 0{\cdot}078$

Table 4. Comparison of the calculated (by equation (6) with the values of the parameters as shown in table 3) and the observed binding energies of the 21 single hypernuclei.

Hyper-nucleus	p	n	n_Λ	$E_c \pm \Delta E_c$ in MeV	$E_o \pm \Delta E_o$ in MeV	$E_c - E_o$ in MeV
$_\Lambda H^3$	1	1	1	0·45 ± 0·36	0·13 ± 0·05	0·32
$_\Lambda H^4$	1	2	1	1·91 ± 0·42	2·04 ± 0·04	− 0·13
$_\Lambda He^4$	2	1	1	1·91 ± 0·42	2·39 ± 0·03	− 0·48
$_\Lambda He^5$	2	2	1	3·12 ± 0·48	3·12 ± 0·02	0·00
$_\Lambda He^6$	2	3	1	4·48 ± 0·56	4·18 ± 0·10	0·30
$_\Lambda He^8$	2	5	1	7·26 ± 0·80	7·16 ± 0·70	0·10
$_\Lambda Li^7$	3	3	1	5·59 ± 0·65	5·58 ± 0·03	0·01
$_\Lambda Li^8$	3	4	1	6·86 ± 0·75	6·80 ± 0·03	0·06
$_\Lambda Li^9$	3	5	1	8·14 ± 0·87	8·53 ± 0·15	− 0·39
$_\Lambda Be^8$	4	3	1	6·86 ± 0·75	6·84 ± 0·05	0·02
$_\Lambda Be^{10}$	4	5	1	9·04 ± 0·97	9·11 ± 0·22	− 0·07
$_\Lambda B^9$	5	3	1	8·14 ± 0·87	7·88 ± 0·15	0·26
$_\Lambda B^{10}$	5	4	1	9·04 ± 0·97	8·89 ± 0·12	0·15
$_\Lambda B^{11}$	5	5	1	9·96 ± 1·10	10·24 ± 0·05	− 0·28
$_\Lambda B^{12}$	5	6	1	11·04 ± 1·24	11·37 ± 0·06	− 0·33
$_\Lambda C^{12}$	6	5	1	11·04 ± 1·24	10·76 ± 0·19	0·28
$_\Lambda C^{13}$	6	6	1	11·86 ± 1·38	11·69 ± 0·12	0·17
$_\Lambda B^{7*}$	4	2	1	5·77 ± 0·67 5·77	5·16 ± 0·08	0·61
$_\Lambda B^{9*}$	4	4	1	7·70 ± 0·85 7·70	6·71 ± 0·04	0·99
$_\Lambda C^{14*}$	6	7	1	12·71 ± 1·54 12·71	12·17 ± 0·33	0·54

*For these three cases we give both the values of E_c, first by fitting equation (6) to all 21 single hypernuclei and then fitting it to the other 18 of them and predicting these three from that mass formula. As we see, they turn out to be the same.

in table 3) and the observed (E_0) energies of the 21 hypernuclei are compared. ΔE_c is the most probable error in the calculated value which is due to the standard errors in the four parameters (see table 3). ΔE_0 is the experimental error. Except for the lightest hypernuclei the quantities ΔE_c are close to 15% and are substantially larger than the uncertainties in the experimentally determined energies. It is interesting to point out that while the r.m.s. deviation reduces substantially (fiom 0·37 MeV to 0·24 MeV) when the fit is made to 18 instead of 21 hypernuclei, the binding energy predictions for the 3 hypernuclei in the former case match almost exactly to the fitted values obtained in the latter case. This is shown in table 4. It seems difficult to obtain mass formulae based on the general considerations described in the last section that will significantly improve the quality of fit to the data.

As mentioned before, the values of the parameters (table 3) provide information about the properties (on the average) of Λ-nucleus, Λ-nucleon and ΛNN interactions. The results of table 3 indicate that, in this region, the kinetic energy together with the (average) potential energy of a Λ-particle, is attractive for all $A > 1$. The ΛNN interaction has an attractive part and a repulsive part. For a fixed value of A the hypernucleus with higher isospin value has a greater binding energy.

From the experimental binding energies it is possible to estimate the strength of the Λ-nucleus potential. Assuming that the Λ-particle moves in a square well potential, one can evaluate the kinetic energy of the Λ-particle by the expression (Povh 1978)

$$T = \frac{3\,\hbar^2\,\pi^2}{10\,M_R\,R^2}. \tag{8}$$

Here M_R is the reduced mass of the Λ-particle in the nucleus and R is the r.m.s. radius of the residual nucleus. The values of R are known (experimentally) for many nuclei and are tabulated in (Barrett & Jackson 1977). Thus T can be evaluated for various nuclei and knowing the binding energy one obtains the potential energy. This is shown in table 5. The crucial point to note is that the well is shallower by about 20 MeV compared to the nuclear case.

It should also be obvious that the mass formula can be used to predict the binding energies of other hypernuclei in this region. In table 6 these are shown for several single Λ-hypernuclei having $1 \leqslant p \leqslant 8$, $1 \leqslant n \leqslant 8$ and $n_\Lambda = 1$. We have used the

Table 5. Estimates of the depth of the Λ-nucleus potential well.

Hypernucleus	Kinetic energy (in MeV)	Potential energy (in MeV)
$_\Lambda Li^7$	18·89	− 24·47
$_\Lambda Li^8$	20·81	− 27·61
$_\Lambda Be^{10}$	18·43	− 27·54
$_\Lambda B^{12}$	20·38	− 31·75
$_\Lambda C^{13}$	18·32	− 30·01
$_\Lambda C^{14}$	18·94	− 31·11
$_\Lambda N^{15}$	17·38	− 30·97

Table 6. Predictions of the binding energies of some light hypernuclei as yet not known.

Hypernucleus	p	n	n_Λ	Predicted binding energy (in MeV)
$_\Lambda He^7$	2	4	1	5·86 ± 0·67
$_\Lambda Li^5$	3	1	1	3·39 ± 0·50
$_\Lambda Li^6$	3	2	1	4·48 ± 0·56
$_\Lambda Li^{10}$	3	6	1	9·49 ± 1·01
$_\Lambda Li^{11}$	3	7	1	10·77 ± 1·19
$_\Lambda Li^{12}$	3	8	1	12·12 ± 1·41
$_\Lambda Be^7$	4	2	1	5·86 ± 0·67
$_\Lambda Be^{11}$	4	6	1	10·24 ± 1·10
$_\Lambda Be^{12}$	4	7	1	11·44 ± 1·27
$_\Lambda Be^{13}$	4	8	1	12·68 ± 1·46
$_\Lambda B^8$	5	2	1	7·20 ± 0·80
$_\Lambda B^{13}$	5	7	1	12·14 ± 1·39
$_\Lambda B^{14}$	5	8	1	13·25 ± 1·57
$_\Lambda C^9$	6	2	1	8·69 ± 0·97
$_\Lambda C^{10}$	6	3	1	9·45 ± 1·01
$_\Lambda C^{11}$	6	4	1	10·24 ± 1·11
$_\Lambda C^{15}$	6	8	1	13·84 ± 1·72
$_\Lambda N^{10}$	7	2	1	10·13 ± 1·18
$_\Lambda N^{11}$	7	3	1	10·78 ± 1·19
$_\Lambda N^{12}$	7	4	1	11·44 ± 1·27
$_\Lambda N^{13}$	7	5	1	12·14 ± 1·39
$_\Lambda N^{14}$	7	6	1	12·84 ± 1·54
$_\Lambda N^{16}$	7	8	1	14·46 ± 1·89
$_\Lambda O^{13}$	8	4	1	12·68 ± 1·46
$_\Lambda O^{14}$	8	5	1	13·26 ± 1·57
$_\Lambda O^{15}$	8	6	1	13·85 ± 1·72
$_\Lambda O^{16}$	8	7	1	14·46 ± 1·89
$_\Lambda O^{17}$	8	8	1	15·09 ± 2·08

mass formula as given by (6) for this purpose. The most probable errors are also shown.

It may be interesting when data is available to predict the binding energies of double and multi Λ-hypernuclei.

4. Mass relationships

It was shown (Parikh 1978) that the mass formula which was derived for nuclei exactly satisfied the Garvey-Kelson (Garvey & Kelson 1966) type mass relationships.

Therefore, in the first instance it should be clear that one can also obtain (following the arguments of Parikh 1978, Garvey *et al* 1969) new mass relationships in which one has a bound system of neutrons, protons and Λ-particles. The simplest of these extended relationships are obtained by adding a single particle to the 6 nuclei related by the usual Garvey-Kelson (Garvey & Kelson 1966) relations. More precisely one obtains,

$$M(p-2, n+2, \Lambda=1) - M(p, n, \Lambda=1) + M(p-1, n, \Lambda=1) - M(p-2, n+1, \Lambda=1) + M(p, n+1, \Lambda=1) - M(p-1, n+2, \Lambda=1) = 0, \tag{9}$$

and

$$M(p, n+2, \Lambda=1) - M(p-2, n, \Lambda=1) + M(p-2, n+1, \Lambda=1)$$
$$- M(p-1, n+2, \Lambda=1) + M(p-1, n, \Lambda=1) - M(p, n+1, \Lambda=1)=0. \tag{10}$$

In these equations, $M(p, n, \Lambda)$, etc denote the total binding energy of the hypernuclei. Although the experimental data is scant there are three cases in which the binding energies of all the six hypernuclei on the left side of the equations are known. These provide at present the only checks of the mass relations in (9) and (10). Although the relationships connect the total energies of six hypernuclei, we have assumed that the (residual) nuclear energies exactly satisfy the equations and hence attempted to verify the relationships for the Λ-nucleus energy (Povh 1978) $E_0(p, n, \Lambda)$ (see table 4).

By taking $p = 5$ and $n = 3$ in (9) one gets

$$E_0({}_{\Lambda}Li^9) - E_0({}_{\Lambda}B^9) + E_0({}_{\Lambda}Be^8) - E_0({}_{\Lambda}Li^8) + E_0({}_{\Lambda}B^{10}) - E_0({}_{\Lambda}Be^{10})=0. \tag{11}$$

All these energies are known (Povh 1978) experimentally and substituting them one finds for the left side the value 0·47 ± 0·72.

Secondly substituting ($p = 5$, $n = 3$) and ($p = 6$, $n = 4$) in (10) one obtains (12) and (13) respectively

$$E_0({}_{\Lambda}\mathrm{B}^{11}) - E_0({}_{\Lambda}\mathrm{Li}^7)+E_0({}_{\Lambda}\mathrm{Li}^8) - E_0({}_{\Lambda}\mathrm{Be}^{10}) + E_0({}_{\Lambda}\mathrm{Be}^8) - E_0({}_{\Lambda}\mathrm{B}^{10})=0, \tag{12}$$

$$E_0({}_{\Lambda}\mathrm{C}^{13}) - E_0({}_{\Lambda}\mathrm{Be}^9) + E_0({}_{\Lambda}\mathrm{Be}^{10}) - E_0({}_{\Lambda}\mathrm{B}^{12}) + E_0({}_{\Lambda}\mathrm{B}^{10})$$
$$- E_0({}_{\Lambda}\mathrm{C}^{12}) = 0. \tag{13}$$

On substituting the experimental energies the left side have the values 0.30 ± 0.50 (equation (12)) and 0·85 ± 0·75 (equation (13)). Although the errors are large and the number of checks very few, it seems reasonable to accept the validity of the mass relationships.

In addition to these simple extensions one can also derive more general relationships in which the binding energies of hypernuclei with different numbers of Λ-particles are related. For example (again excluding ΛNN interactions) these are

$$M(p-2, n+2, \Lambda-2) - M(p, n, \Lambda) + M(p-1, n, \Lambda-1)$$
$$- M(p-2, n+1, \Lambda-2) + M(p, n+1, \Lambda) - M(p-1, n+2, \Lambda-1) = 0, \tag{14}$$

and

$$M(p-2, n+2, \Lambda+2) - M(p, n, \Lambda) + M(p-1, n, \Lambda)$$
$$- M(p-2, n+1, \Lambda+1) + M(p, n+1, \Lambda+1)$$
$$- M(p-1, n+2, \Lambda+2) = 0. \tag{15}$$

It is as yet not possible to verify the extent of validity of these relations because of of insufficient data. It should be stressed again that all these mass relationships (equations (9), (10)) are exactly satisfied by the mass formulae (equations (3)-(7)).

If only five of the six energies related by either (9) or (10) are known experimentally one can use these and the appropriate equation to predict the unknown binding energy. A systematic search revealed ten such cases. These predictions are compared with the predictions based directly on the mass formula in table 7. For $_{\Lambda}He^7$ and $_{\Lambda}Li^6$ it turned out that there were more ways (independent) than one to obtain the energies. But as seems from table 7 they are not consistent with each other. Also the agreement with the predictions from the mass formula (table 6) is just fair. This remains a puzzle because at least in the three cases discussed earlier the mass relationships were reasonably well obeyed. One possible reason may be that in the light nuclei the Garvey-Kelson relationships are not well satisfied whereas in the present discussion it has been assumed that they are exactly obeyed.

After the manuscript got prepared we came to know of the observation of another single Λ hypernucleus $_{\Lambda}Li^6$ whose experimentally measured binding energy B_{Λ} is $4{\cdot}5 \pm 0{\cdot}5$ MeV (Bertini *et al* 1981). The prediction from our mass formula as given in table 6 is $4{\cdot}48 \pm 0{\cdot}56$ MeV.

Table 7. A comparison of the binding energies obtained from the mass relationships and the mass formula of equation (6).

	Prediction from mass relationship	Prediction from mass formula
$_{\Lambda}He^7$	$3{\cdot}08 \pm 0{\cdot}97$	$5{\cdot}86 \pm 0{\cdot}67$
$_{\Lambda}He^7$	$4{\cdot}73 \pm 0{\cdot}54$	$5{\cdot}86 \pm 0{\cdot}67$
$_{\Lambda}Li^5$	$3{\cdot}17 \pm 0{\cdot}21$	$3{\cdot}39 \pm 0{\cdot}50$
$_{\Lambda}Li^6$	$5{\cdot}70 \pm 0{\cdot}25$	$4{\cdot}48 \pm 0{\cdot}56$
$_{\Lambda}Li^6$	$5{\cdot}87 \pm 0{\cdot}24$	$4{\cdot}48 \pm 0{\cdot}56$
$_{\Lambda}Li^6$	$5{\cdot}04 \pm 0{\cdot}42$	$4{\cdot}48 \pm 0{\cdot}56$
$_{\Lambda}Be^{11}$	$9{\cdot}57 \pm 0{\cdot}33$	$10{\cdot}24 \pm 1{\cdot}10$
$_{\Lambda}B^8$	$7{\cdot}55 \pm 0{\cdot}33$	$7{\cdot}20 \pm 0{\cdot}80$
$_{\Lambda}C^{11}$	$8{\cdot}27 \pm 0{\cdot}48$	$10{\cdot}24 \pm 1{\cdot}11$
$_{\Lambda}N^{14}$	$13{\cdot}31 \pm 0{\cdot}78$	$12{\cdot}84 \pm 1{\cdot}54$

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Note added in proof

After we submitted this work for publication, we came across the work of Bhaduri and Nogami (Bhaduri RK and Nogami Y 1972 *Phys. Rev. Lett.* **28** 1397) which is similar to this as far as mass relationships are concerned.