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Effect of electromagnetic perturbation on charge imbalance in a superconductor: A two-fluid description

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Abstract. Using a generalized two-fluid pictures for the charge of superconductor and ordinary Boltzmann equation for quasiparticle excitations, the effect of frequency and wave-vector dependent electromagnetic perturbation on charge imbalance near transition temperature T_C is studied. In a situatiod where both the effective charge and distribution function of quasiparticles deviate from their equilibrium values, the charge imbalance is shown to possess a propagating solution at frequencies greater than inelastic scattering rate. In situations where charge imbalance is created by injection of quasiparticles, the charge imbalance relaxation rate is shown to decrease. We also study the effect of applied perturbation on quasiparticle diffusion length and hence on superconductor—normal interface boundary resistance.

Keywords. Non-equilibrium superconductor; two-fluid model; charge imbalance; quasiparticle diffusion length; boundary resistance.

1. Introduction

During last few years, the dynamics of non-equilibrium superconductors have received considerable attention. One of the important aspects of non-equilibrium superconductivity has been centred around the concept of charge imbalance used to explain measurements of Clarke and colleagues (Clarke 1972; Clarke and Paterson 1974) of non-equilibrium potential induced by quasiparticle injection into a superconducting film. This and several other non-equilibrium effects in a superconductor have been treated by two methods. The first, developed by Schmid and Schon (1975a), is based on the equation of motion for the full Green's function matrix. Entin-Wohlman and Orbach (1978) linearised the Eliashberg theory (Eliashberg 1969; Gorkov and Eliashberg 1968) to study the response of a superconductor to frequency (ω) and wave-vector (κ) dependent perturbations. This method, though detailed and quantitative, involves mathematical complexities. The second method of treating non-equilibrium phenomena is through the framework of the Boltzmann equation for quasiparticle distributions introduced by Pethick and Smith (1978, 1979). The simplicity of the Boltzmann equation approach brings out clearly the physics involved in the non-equilibrium process. The quasiparticle formalism is valid provided the spatial variations are slow on the scale of the temperature-dependent coherence length $\xi(T) \sim v_F/\Delta$, and the temporal variations are slow on a scale $1/\Delta$, where v_F and Δ are, respectively, the Fermi velocity and energy gap in units of $\hbar$. Under these conditions, the quasiparticles respond instantaneously to the local gap, and the state of normal component may be specified in terms of a scalar quasiparticle distribution function rather than a matrix distribution involving anomalous correlations, as in more general cases.

In this paper the effects of frequency (ω) and wave-vector (κ) dependent electromagnetic perturbation on charge imbalance, using a generalised two-fluid picture for charge and ordinary Boltzmann equation for excitations in superconductor has been reported. The temperature has been kept close to the transition temperature T_C in order that the exact solutions of the Boltzmann equation are valid. The basic ingredients of the two-fluid model are given by Betbeder-Matibet and Nozieres (1969) who studied the transport equations in clean superconductors. However, it was only after the work of Leggett and Takegi (1977) on an analogous two-component model for the spin of ^{3}He that the physical contents of the two-fluid model were brought out. The salient features of this theory have been summarised in § 2. In §3, the effects of applied perturbation on charge imbalance created due to deviation in the effective charge as well as in distribution function of quasiparticles from their equilibrium values have been reported. At frequencies that are larger compared to the superfluid response factor γ, and also the rate of inelastic scattering processes $\tau_{\text{in}}^{-1}(0)$ the charge imbalance is found to possess a propagating solution. In § 4, the theory is extended to account for the effect of applied perturbation on charge imbalance created due to injection of quasiparticles. The effects of applied perturbation on quasiparticle diffusion length and hence on superconductor normal interface boundary resistance have further been studied. Calculations predict appreciable effects at microwave frequencies.

2. Charge imbalance (a two-fluid description)

In the BCS theory the total charge density (measured in units of electronic charge) may be written as:

$$Q_{\text{tot}} = \sum_{k\sigma} \{U_k^2 f_{k\sigma} + V_k^2 (1 - f_{k\sigma})\}, \tag{1}$$

where U_k and V_k are the usual coherence factors given by

$$U_k^2 (\xi_k) = V_k^2 (-\xi_k) = \tfrac{1}{2} (1 + \xi_k/E_k), \tag{2}$$

with $$E_k = (\xi^2 + \Delta^2)^{1/2} \text{ and } \xi_k = \epsilon_k - \mu_s. \tag{3}$$

Here ξ_k is the normal state quasiparticle energy ϵ_k measured with respect to chemical potential μ_s of condensate. E_k is the quasiparticle excitation energy, positive on both electron like ($\xi_k > 0$) and hole-like ($\xi_k < 0$) branches of excitation spectrum, and Δ is the BCS gap parameter. When the system is in thermal equilibrium, the distribution function f_k is simply the Fermi function f_k^0 and (1) may be expressed as the sum of

$$Q_n = \sum_{k\sigma} (U_k^0 - V_k^0)\, f_{k\sigma}, \tag{4}$$

and $$Q_s = \sum_{k\sigma} V_k^2. \tag{5}$$

Here U_k^0 and V_k^0 are the equilibrium values of coherence factors defined by (2). When normal and superfluid components differ from their equilibrium values, one may write for first order deviations (Pethick and Smith 1979)

$$\delta Q_{\text{tot}} = \delta Q_n + \delta Q_s, \tag{6}$$

where $$\delta Q_n = \sum_{k\sigma} q_k^0 \, \delta f_{k\sigma}, \tag{7}$$

and $$\delta Q_s = \sum_{k\sigma} (1 - 2 f_{k\sigma}^0) \, \delta V_k^2. \tag{8}$$

Here $\delta f_{k\sigma}$ being the first order deviation in f_k from its equilibrium value, which is caused either by direct injection of quasiparticles, quasiparticle diffusion from adjacent regions or by inelastic scattering processes, and q_k^0 is the equilibrium value of q_k, referred to as effective charge of quasiparticles,

$$\begin{aligned} q_k &= U_k^2 - V_k^2 = \xi_k/E_k = \pm (E_k^2 - \Delta^2)^{1/2}/E_k \\ &= \pm N_s^{-1}(E_k), \end{aligned} \tag{9}$$

where $N_s(E_k)$ is the normalised BCS density of states. It is worth noting that δQ_n is the deviation of Q_n from its global equilibrium value.

One can discuss many non-equilibrium phenomena in a physically appealing way by defining the net charge density of the quasiparticle system representing the deviation of Q_n from its local equilibrium value, also referred to as δQ_n^{le}, called the charge imbalance Q^*:

$$Q^* = \sum_{k,\sigma} q_k \{f_{k\sigma} - f_{k\sigma}^0(E_k)\}. \tag{10}$$

In a situation where μ_s and hence q_k has its equilibrium value, like in steady-state experiments, the change in f_k alone corresponds to change in Q_n. Hence for small variations from equilibrium one may linearize (10) and write

$$Q^* = \sum_{k\sigma} q_k \, \delta f_{k\sigma}. \tag{11}$$

However, in situations where μ_s and hence q_k also change, as in pulsed experiments, (10) takes the general form:

$$Q^* = \sum_{k,\sigma} \{q_k \, \delta f_{k\sigma} + \delta q_k f_{k\sigma}\}, \tag{12}$$

where δq_k is the first order deviations in the quasiparticle charge q_k from its equilibrium value. The main contribution of δq_k to Q^* is due to the shift in the chemical potential μ_s and hence response of superfluid part of density δQ_s to a chemical potential change, which in turn is related to the variations in the coherence factors V_k^2. Equations (2) and (3) represent the dependence of V_k on μ through ξ_k which includes the Fermi liquid effects. The change in coherence factors V_k^2 is given by:

$$\delta V_k^2 = (\Delta^2/2 E_k^3)\, \delta E_F, \tag{13}$$

where $\delta E_F = \delta \mu_s - \delta \epsilon_k$. In the absence of Fermi-liquid effects, (13) along with (8) leads to

$$\delta Q_s = X_s^0\, \delta \mu_s, \tag{14}$$

where
$$X_s^0 = \sum_{k\sigma} (1 - 2 f_{k\sigma}^0)\, (\Delta^2/2E_k^3) \simeq 2\, N(0)\, [1 - Z(T)]. \tag{15}$$

called the susceptibility of superfluid component in the absence of Fermi-liquid effects. The temperature-dependent function $Z(T)$ is discussed by Clarke *et al* (1979) and $N(0)$ is the usual density of states in normal metal at the Fermi energy. In thermal equilibrium, a change in μ_s alters the quasiparticle energy and hence the quasiparticle distribution, resulting in a change in quasiparticle charge by an amount:

$$\delta Q_n = X_n^0\, \delta \mu_s, \tag{16}$$

where the susceptibility of normal fluid in the absence of Fermi liquid effects is:

$$X_n^0 = \sum_{k\sigma} q_k \frac{\partial f_k^0}{\partial E_k} \frac{\partial E_k}{\partial \mu_s} = \sum_{k\sigma} - q^2 \frac{\partial f_k^0}{\partial E_k} = 2\, N(0)\, Z(T). \tag{17}$$

The value of μ_s is determined by requiring that the total electronic charge density be the same in the presence of charge imbalance as in thermal equilibrium. Thus, the first order changes in V_k and $f_{k\sigma}$ from their equilibrium values $f_{k\sigma}$ and V_k^0 are constrained by:

$$\sum_{k,\sigma} \{(1 - 2 f_{k\sigma}^0)\, \delta V_k^2 + (1 - 2\, V_k^{02})\, \delta f_{k\sigma}\} = 0. \tag{18}$$

But the δV_k^2 is determined from the shift, δE_F, in the Fermi energy of the condensate in equation (13). The net charge density of the quasiparticle system Q^* is also proportional to this shift since it is equal and opposite to the change in condensate charge. Waldram (1975) has shown that this condition gives

$$Q^* = -\, 2\, N(0)\, \delta E_F. \tag{19}$$

Using (13) to (19), one obtains

$$Q^* = \delta\, Q_n/\lambda = \delta\, Q_n/[1 - Z(T)], \tag{20}$$

where λ is the dimensionless parameter, representing the reduced susceptibility of superfluid component, X_s^0/X_{tot}^0. In situations where change in q_k plays the essential role, the significance of (20) may be understood from the relation

$$\delta\, q_k = \delta\, \mu_s \frac{\partial}{\partial \mu_s}(\xi_k/E_k) = -\,\delta\, \mu_s \frac{\Delta^2}{E_k^3}. \tag{21}$$

Using (19) in the absence of the Fermi liquid effects along with (12) and (21), one obtains

$$Q^* = \sum_{k\sigma} q_k\, \delta\, f_{k\sigma}/\lambda. \tag{22}$$

Thus the response of superfluid in maintaining electroneutrality acts to enhance the change in Q^* due to change in distribution function alone. A similar argument for time variations leads to (Kadin *et al* 1980):

$$\dot{Q}^* = \sum_{k,\sigma} q_k \dot{f}_{k\sigma}/\lambda. \tag{23}$$

In the absence of spatial variations of the f_k, the normal component is changed only by collision processes and its equilibrium value is determined from:

$$\partial Q_n/\partial t = (\partial Q_n/\mathrm{d}t)_{\text{coll.}} \tag{24}$$

However, in the presence of spatial variations, Q_n can also change because of transport of normal charge from adjacent regions. Thus (24) should be generalised to

$$\partial\, Q_n/\partial t + \nabla \cdot J_n^Q = (\mathrm{d}Q_n/\mathrm{d}t)_{\text{coll.}} \tag{25}$$

Similarly the continuity equation for superfluid part has the form:

$$\partial Q_s/\partial t + \nabla . J_s^Q = (\mathrm{d}Q_s/\mathrm{d}t)_{\text{coll.}} = -\,(\mathrm{d}Q_n/\mathrm{d}t)_{\text{coll.}} \tag{26}$$

Here J_n^Q and J_s^Q are the currents associated with Q_n and Q_s, referred to as normal and superfluid charge currents respectively. These currents are related to the conventional currents J_n and J_s associated with normal and superfluid components (Pethick and Smith 1979) by:

$$J_n^Q = (1 - \lambda)\, J_n, \tag{27}$$

and

$$J_s^Q = J_s + \lambda\, J_n. \tag{28}$$

The above discussion has been for clean superconductors. In dirty superconductors (27) and (28) may still hold, with $\lambda \sim \Delta/k_B T$ near T_C, but in evaluating J_s one must calculate n_s and the coefficients λ from microscopic considerations.

3. Dynamics of charge imbalance

When the frequency (ω) and wave-vector (κ) dependent electromagnetic perturbation, described by vector potential $A(\omega, \kappa)$ and a scalar potential ϕ (ω, κ) is applied to a superconductor, it causes the quasiparticle distribution function to deviate from its equilibrium value. For a weak applied perturbation and for first order changes, this deviation may be written as:

$$f_k \rightarrow f_k + \delta f_k(\omega, \kappa). \tag{29}$$

We take the deviation δf_k to be odd with respect to inversion through the local Fermi surface, so as to excite the charge imbalance mode. A distribution of this type implies that the gap parameter Δ has its equilibrium value and only deviations in the phase of order parameter is excited. For first order deviations, we define

$$\varphi \rightarrow \varphi + \delta\varphi(\omega, \kappa), \tag{30}$$

where $\delta\varphi$ (ω, κ) is the deviation in phase of order parameter associated with current and charge density of perturbed state of the superconductor.

In a steady-state situation, the relaxation time for this mode, very near T_C, is shown to be (Pethick and Smith 1979):

$$\tau_{Q^*} = \frac{4k_B T}{\pi\Delta}\tau_{\text{in}}(0), \tag{31}$$

and is called the charge imbalance relaxation time. Here $\tau_{\text{in}}^{-1}(0)$ is the relaxation rate due to inelastic scattering processes of normal state quasiparticle with $\xi=0$ at T_C. When the frequency of applied perturbation is less than the rate of inelastic scattering processes ($\omega \ll \tau_{\text{in}}^{-1}(0)$), the charge imbalance relaxation becomes frequency-dependent and is discussed in §4. However, when $\omega \gg \tau_{\text{in}}^{-1}(0)$ there is a negligible conversion of quasiparticle charge into charge associated with pairs. The charge densities on the other hand execute local oscillations. The total density oscillates slightly since variations in it are suppressed by Coloumb interactions. To get further insight of the density fluctuations we return to (12). For slow-time variations the quasiparticle distribution function may be written as (Pethick and Smith 1979)

$$f_k = f_k^{\text{le}} + q_k(\partial f_k^0/\partial E_k)\,\delta\mu_s, \tag{32}$$

where f_k^{le} is the local equilibrium distribution equal to the Fermi function $f_k^0(E_k)$ evaluated for local equilibrium value of quasiparticle energy. Using (32) and (12), one finds:

$$\begin{aligned} Q^* &= \delta Q_n - (\partial Q_n/\partial\mu_s)\,\delta\mu_s, \\ &= \delta Q_n - (X_n^0/X_s^0)\delta\mu_s, \end{aligned} \tag{33}$$

which on using definition of λ along with $\delta Q_s = \delta Q_{tot.} - \delta Q_n$ becomes

$$Q^* = (1/\lambda)(\delta Q_n - \delta Q_n^{eq.}) \tag{34}$$

where

$$\delta Q_n^{eq.} = (X_n^0/X_s^0)\, \delta Q_{tot.}$$

is the value of δQ_n for equilibrium in presence of density fluctuations. For $T \sim T_C$, $X_n/X_s^0 = (1-\lambda)/\lambda$ is very large as the susceptibility of superfluid is small. The positive δQ_n is accompanied by equal and opposite δQ_s so that $\delta Q_{tot.}$ vanishes. The change in δQ_s leads to large change in μ_s which in turn shifts the local value of δQ_n. This counterflow idea implies that the superfluid moves in a weak manner without dissipation. The resulting space charge induces a counterflow of the normal part, acting as the dissipating agent. In a normal state deviations of the total charge density vanishes for frequencies below the plasma frequency ω_p by the requirement $\nabla . J_n = 0$. However, in superconducting state, $\delta Q_{tot.}$ is of the order of $\omega/\omega_p^2\tau$. This is because $\nabla . J_n$ cannot be zero in this case as we have (Entin-Wholman and Orbach 1978)

$$\nabla . J_n + \nabla . J_s = 0 \tag{35}$$

where τ is the life-time due to scattering by non-magnetic impurities (also called the momentum relaxation time). In the presence of applied perturbation, where there is direct coupling between superfluid motion and applied perturbation, one may write supercurrent as*:

$$J_s(\omega, \kappa) = \frac{n_s}{em}\left[\nabla \phi(\omega, \kappa) + \frac{2\, e\, A(\omega, \kappa)}{c}\right]. \tag{36}$$

The supercurrent is driven by the effective chemical potential $\mu_s = e\phi + \delta\mu_s$. Using the Josephson relation $\mathrm{d}/\mathrm{d}t\, \varphi = 2\mu_s$, along with (30) and (36), we get:

$$\frac{\mathrm{d}}{\mathrm{d}t} J_s(\omega, \kappa) = \frac{n_s}{em}\,[eE + \nabla\delta\mu_s] \tag{37}$$

Definining the superfluid response factor $\gamma = n_s/n\tau$, which relates the superfluid density with total density through momentum relaxation time τ, and using (19), (37) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t} J_s(\omega, \kappa) = \gamma\, J_n(\omega, \kappa) - D\gamma\, Q^*(\omega, \kappa) \tag{38}$$

*This equation for supercurrent can also be obtained from equation (15) of Entin-Wholman and Orbach (1978), who relates the change in phase of order parameter by $\delta\varphi = \mathrm{i}\,\delta\Delta_2/\Delta$, $\delta\Delta_2$ being imaginary.

where we have used $\sigma = ne^2\tau/m$ as the normal state electron conductivity and $D = 1/3V_F^2\tau$ is the normal state diffusion constant. Combining (35), (38) and (27) we get:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \gamma\right)\nabla \cdot J_n^Q(\omega, \kappa) = (1-\lambda)\, D\gamma\nabla^2 Q^*(\omega, \kappa) \tag{39}$$

The quasiparticle charge current, J_n^Q, is obtained by writing the Boltzmann equation for excitation, which for a clean superconductor is discussed by Betbeder-Matibet and Nozieres (1969); when linearized this equation has the form:

$$\frac{\partial}{\partial t} f_k + v_k \cdot \nabla \delta f_k - \frac{\partial f_k}{\partial E_k} v_k \cdot \nabla \delta E_k = \left(\frac{\mathrm{d}f_k}{\mathrm{d}t}\right)_{\text{coll.}} \tag{40}$$

where v_k is the group velocity of an excitation and δE_k is the variation of quasiparticle energy in the absence of Fermi liquid effects. Defining $\delta f_k^{\text{le}} = \delta f_k - (\partial f_k/\partial E_k)\, \delta E_k$, as the deviation from local equilibrium, one can write (40) in a compact form as:

$$\frac{\partial t}{\partial f_k} + v_k \cdot \nabla \delta f_k^{\text{le}} = \left(\frac{\mathrm{d}f_k}{\mathrm{d}t}\right)_{\text{coll.}} \tag{41}$$

On multiplying both sides of this by q_k and summing over all possible states, we get:

$$\sum_{k,\sigma} q_k \frac{\partial f_k}{\partial t} + \nabla \cdot J_n^Q = \left(\frac{\mathrm{d}Q_n}{\mathrm{d}t}\right)_{\text{coll.}} \tag{42}$$

The relaxation of the quasiparticle charge is described phenomenologically as

$$(\mathrm{d}Q_n/\mathrm{d}t)_{\text{coll.}} = -\, Q^*/\tau_{Q^*}. \tag{43}$$

Using (34), we have:

$$(\mathrm{d} \cdot Q_n/\mathrm{d}t)_{\text{coll.}} = -\frac{1}{\lambda\tau_{Q^*}}(\delta Q_n - \delta Q_n^{\text{eq.}}) \tag{44}$$

Equations (43) and (44) predict that Q_n will relax to local equilibrium value at the rate $1/\tau_{Q^*}$, but to global equilibrium value at the rate $1/\lambda\tau_{Q^*} \sim 1/\tau_{\text{in}}(0)$ which remains finite in the limit $T \to T_C$. Using (43) and (23) in (42), one obtains:

$$\lambda\,(\mathrm{d}/\mathrm{d}t)\, Q^* + \nabla \cdot J_n^Q = -\, Q^*/\tau_{Q^*}. \tag{45}$$

On combining this with (39), we get:

$$\left[\mathrm{d}^2/\mathrm{d}t^2 + \left\{\gamma + \tau_{\text{in}}^{-1}(0)\right\}\mathrm{d}/\mathrm{d}t + \frac{1-\lambda}{\lambda} D\,\gamma\,\nabla^2 + \gamma\,\tau_{\text{in}}^{-1}(0)\right] \times Q^*(\omega, \kappa) = 0 \tag{46}$$

This is the desired general differential equation in space and time for charge imbalance. If the charge imbalance is a wave-propagation of the form exp $[i(k \cdot \gamma - \omega t)]$ then (46) represents a dispersive wave whose dispersion relation for large ω is:

$$\omega^2 - \frac{(1-\lambda)}{\lambda} D \gamma \kappa^2 + i \omega \gamma = 0. \tag{47}$$

This describes a propagating mode, with mode velocity

$$V^2_{\text{mod}} = \frac{(1-\lambda)}{\lambda} D \gamma. \tag{48}$$

It is worth noting that (46) is a general one in the sense that it accounts for many results obtained by using other techniques. For example, at $T \to T_C$, (47) and (48) reduce to the results obtained by Entin-Wholman and Orbach (1978), in the gap regime. Using the dirty and clean limit values of superfluid response factor:

$$\gamma = \begin{cases} \pi \Delta^2 / 2\, k_B T & \text{for dirty limit} \\ 2\left(1 - \dfrac{T}{T_C}\right) \Big/ \tau & \text{for clean limit.} \end{cases} \tag{49}$$

In (48) we get the results obtained by Schmid and Schon (1975b) and Artemenkov and Volkov (1976) for propagating modes in dirty and clean superconductors, respectively. Further application of (46) is discussed in the next section.

4. Frequency Dependent of charge imbalance relaxation rate

In a steady-state experiment in which Q^* is uniformly distributed over the volume Ω, one can define relaxation rate $\tau^{-1}_{Q^*} = \dot{Q}^*/Q^*_i$. Here $\dot{Q}^*_i$ is the rate of injection of quasiparticle charge. Since in actual experiments one measures Q^*_i in terms of injection current I_i ($Q^*_i = I_i/e\Omega$) rather than Q^*_i it is convenient to present the measurements in the form (Pethick and Smith 1979)

$$I/F^* \tau_{Q^*} = I_i/e\Omega\; Q^*, \tag{50}$$

where F^* is the dimensionless calculable function, and I_i is the current through injection junction. The injection of quasiparticles create a charge density which is carried inside the superconductor by the superfluid and normal component current, *i.e.*;

$$I_i/e\Omega = -(\nabla \cdot J_n + \nabla \cdot J_s). \tag{51}$$

In the presence of applied perturbation, the superfluid component gets directly coupled, satisfying the acceleration equation (39). Assuming the applied perturbation to be spatially uniform, (39) simplifies to:

$$\frac{\mathrm{d}}{\mathrm{d}t} J_s(\omega) = \gamma J_n. \tag{52}$$

Using continuity equation $\partial Q_n/\partial t = -(\nabla \cdot J_n + \nabla \cdot J_s)$ inside the superconductor and combining (51) and (52) one obtains:

$$I_i/e\Omega = (1 + 1/\gamma\, \mathrm{d}/\mathrm{d}t)\left\{(\mathrm{d}/\mathrm{d}t)\, Q_n(\omega) + \nabla \cdot J_n(\omega)\right\}. \tag{53}$$

As mentioned in the previous section, the quasiparticle current in a case of clean superconductor is obtained from the Boltzmann equation (40). Since we are dealing with injection of quasiparticles in thin films, the spatial variations of quasiparticle energy δE_k is negligible. Thus one can write (40) in simplified form as:

$$\frac{\partial f_k}{\partial t} + v_k \cdot \nabla\, \delta f_k = \left(\frac{\mathrm{d}f_k}{\mathrm{d}t}\right)_{\text{coll.}} \tag{54}$$

Multiplying both sides by q_k^0 and summing over all possible states, we have:

$$\frac{\partial Q_n}{\partial t} + \nabla \cdot J_n = \sum_{k\sigma} q_k^0 \left(\frac{\mathrm{d}f_k}{\mathrm{d}t}\right)_{\text{coll.}} \tag{55}$$

[Strictly, a current differing slightly from the normal current $J_n = \sum_{k\sigma} \frac{\hbar\kappa}{m}\, \delta f_k$ appears in this equation. However, close to T_C this difference can be neglected. If this difference is taken into account, one gets second order correction terms (Mattoo and Singh 1982)]. Combining (29), (53) and (55), one obtains

$$I_i/e\Omega = (1 + 1/\gamma\, \mathrm{d}/\mathrm{d}t)\left[\sum_{k\sigma} q_k^0 (-1/\tau_{\text{in}} + \mathrm{d}/\mathrm{d}t)\, \delta f_k(\omega)\right], \tag{56}$$

where the relaxation of quasiparticle distribution function due to inelastic scattering processes has been written as

$$(\mathrm{d}f/\mathrm{d}t)_{\text{coll.}} = -\,\delta f_k(\omega)/\tau_{\text{in}}. \tag{57}$$

Assuming τ_{in} to be energy-independent, and using (7), (56) takes the form;

$$I_i/e\,\Omega = (1 + 1/\gamma\, \mathrm{d}/\mathrm{d}t)\,(-1/\tau_{\text{in}}^{(0)} + \mathrm{d}/\mathrm{d}t)\, \delta Q_n(\omega). \tag{58}$$

Dividing both sides by Q^* and using (20), we have

$$1/F^* \tau_{Q^*} = [1 - Z(T)]\left\{1 - \frac{i\omega}{\gamma}\right\}\left\{\frac{1}{\tau_{\text{in}}(0)} - i\omega\right\} \tag{59}$$

where $F^* = F^* (\Delta/k_B T, eV_t/k_B T)$
is a known function of temperature and applied voltage and has limiting form:

$$F^* \simeq \left(1 - \frac{\pi\Delta}{|eV_t|}\right) \text{with } (|eV_t| \gg k_B T).$$

At temperatures close to T_C, $F^* \sim 1$ and remains close to unity for arbitrary temperatures provided the injection voltage is much larger than the energy gap. Thus (59) is identical with the result obtained by Entin-Wholman and Orbach (1980). In the absence of applied perturbation, (59) reduces to (31). The comparison of the two equations clearly demonstrate the decrease in the charge imbalance relaxation rate. To calculate this decrease in charge imbalance relaxation rate we write (59) for low frequencies as:

$$|\tau_{Q^*}^{-1}(\omega)| = |\tau_{Q^*}^{-1}(0)| \{1 + \omega^2 (\tau_{\text{in}}^2 + 1/\gamma^2)\}. \tag{60}$$

Under the stationary but spatially inhomogeneous condition the normal charge may diffuse over a characteristic length, which from (46) is seen to be:

$$\lambda_{Q^*}^2 = \frac{(1-\lambda)}{\lambda} D \tau_{\text{in}}^{-1}. \tag{61}$$

At temperature close to T_C, (61) and (31) leads to

$$\lambda_{Q^*}^2 = D \tau_{Q^*}, \tag{62}$$

in agreement with the result that quasiparticle diffusion length involves the charge imbalance relaxation time τ_{Q^*} (Waldram 1975; HSiang and Clarke 1980) instead of τ_{in} as earlier predicted by Pippard *et al* (1971). In the presence of applied perturbation, the superfluid currents get accelerated, resulting in increase of δQ_s and hence change in μ_s. This in turn shifts the local equilibrium value of δQ_n. Thus in the presence of applied perturbation the quasiparticle charge penetration is morei n comparison to that in steady state condition. To a first approximation, this change in quasiparticle diffusion length may be obtained by using (59) in (62), *i.e.*

$$\lambda_{Q^*}^2(\omega) = \lambda_{Q^*}^2(0) \{(1 - i\,\omega/\gamma)(1 - i\,\omega\,\tau_{\text{in}}(0))\}^{-1}, \tag{63}$$

where $\lambda_{Q^*}(0)$ represents zero frequency value given by (62). For low frequencies (63) may be written as

$$|\lambda_{Q^*}(\omega)| = \lambda_{Q^*}^2(0) \{1 - 1/4\, \omega^2 (\tau_{\text{in}}^2 + 1/\gamma^2)\}. \tag{64}$$

At temperature close to T_C, the excess boundary resistance at normal-superconductor interface due to diffusion of quasiparticle charge is given by

$$R = \lambda_{Q^*} R_n. \tag{65}$$

Here R_n is the normal state resistivity of superconducting metal. This result is a consequence of the exponential decay of the quasiparticle charge over a region inside the superconductor where the normal current is converted into supercurrent. The result (65) was derived microscopically by Artemonkov *et al* (1978) and also discussed by Waldram (1975). Recently HSiang and Clarke (1980) have reported a simple derivation of (65). Since in the presence of applied perturbation the effective length over which the quasiparticle charge diffuse increases, the boundary resistance should also change accordingly. Using (63) and (65) we get

$$Z(\omega) = Z(0)\,\{(1 - i\omega/\gamma)\,(1 - i\omega\tau_{\text{in}})\}^{1/2}, \tag{66}$$

where $Z(\omega)$ and $Z(0)$ are the frequency dependent and zero frequency characteristic impedance of the *N-S* junction respectively. For low frequencies (66) can be written as:

$$R(\omega) = R(0)\left\{1 + \frac{i\,\omega\,L}{R\,(0)}\right\}. \tag{67}$$

Thus in the presence of applied perturbation the N-S junction acts like a series junction of the normal resistance R and an effective inductance L due to the usual kinetic inductance of supercurrent. The ratio

$$L/R(0) = 1/2\,\{\tau_{\text{in}}\,(0) + 1/\gamma\}, \tag{68}$$

is the characteristic response time for the supercurrent.

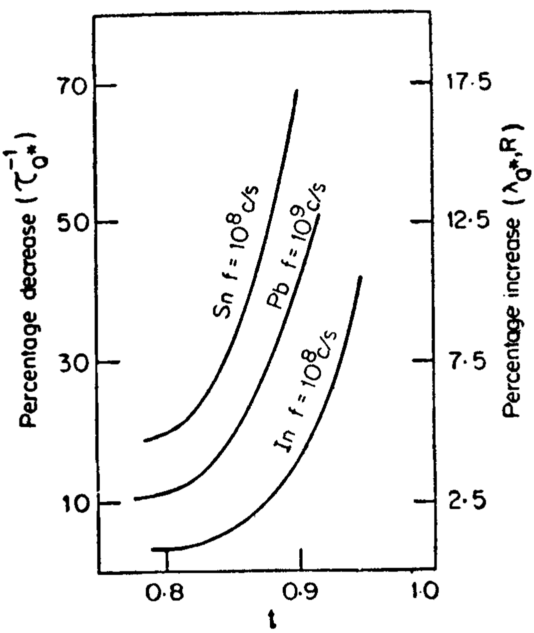

Figure 1. Detectable percentage effects *versus* reduced temperature for clean limit.

Table 1. Detectable percentage effects for dirty limit.

	Frequency in Cycles/sec.	Temperature $t = T/T_C$	$\omega^2 (\tau_{in}^2 + 1/\gamma^2)$	Percentage decrease in τ_{Q*}^{-1}	Percentage increase in λ_{Q*} and R
Sn	10^8	0·9 – 0·8	0·02634	2·63	0·65
Pb	10^9	0·9 – 0·8	0·079	7·9	1·97
In	10^9	0·9 – 0·8	0·00478	47·8	11·95

Using equation (49) for γ and average value of $\tau_{in}(0)$ from H Siang and Clarke (1980), we calculate the percentage effects from (60), (64) and (67) for Sn, Pb, and In. These are shown in figure 1 for clean limit along with their temperature dependence. The estimate indicates that these effects can be detected within a narrow range of temperature between 0·9 to 0·8 T_C, in accordance with expectations. Very close to T_C the fluctuations in the gap parameter dominates the scattering processes and far from T_C the approximations made in the text are not valid. Further, it is worth noting that in calculating percentage effects for clean limit it is assumed that the two relaxation times τ_{in}. (0) and τ (introduced in superfluid response factor) are not different. This also is valid in the neighbourhood of T_C. The percentage effects for dirty limit are listed in table 1. Since the impurity scattering processes are temperature-independent and $\tau_{in} \ll 1/\gamma$ (for dirty limit) so the percentage effects will be almost temperature-independent. We expect in the near future that these effects will be seen experimentally as in both cases the detectable effects lie in microwave region.

5. Conclusion

The approach outlined in this paper is one that combines the simplicity of two-fluid from work with a microscopic foundation. We have derived an expression for space and time variation of charge imbalance which is a general one in the sense that it accounts for many results obtained previously by using other techniques. Kadin *et al* (1980) exploited the electric transmission line anology to (46) to study charge imbalance wave and showed that transmission lines on either side of the case acts like a shunted impedance in the high frequency limit. We also studied the low frequency response of charge imbalance and showed an increase in the quasiparticle diffusion length. This in turn decreases the charge imbalance relaxation rate and increase in *N-S* interface boundary resistance. The *N-S* interface is shown to act like a series junction of normal resistance and an effective impedance of superfluid component due to the usual kinetic impedance of supercurrent. The predicted frequencies for detectable changes lie in the microwave region and thus can be verified experimentally.

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