Pramāṇa, Vol. 19, No. 4, October 1982, pp. 357-366. © Printed in India.

Violation of the second law of black hole physics by tachyons

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MS received 30 March 1982; revised 17 August 1982

Abstract. It is shown that the interaction of a class of positive energy tachyons decreases the area of the horizon of the *T-S*, $\partial = 3$ metric even in the case of a reversible transformation (M_{ir} = constant). This is a violation of the second law of black hole physics.

Keywords. Tachyon; black hole; event horizon.

1. Introduction

The behaviour of space-like or time-like geodesics in Schwarzschild, Reissner-Nordstrom, Kerr and Kerr-Newman space-times has been discussed by several authors and their investigations have been reviewed by Sharp (1976).

Christodoulou (1970) introduced the concept of irreducible mass for those black holes by direct examination of the test particle orbits and showed that irreducible mass is connected with reversible and irreversible transformations which are important for black hole physics. It is also relevant because there exists a one-to-one connection between the irreducible mass and the proper surface area of the horizon which as proved by Hawking (1971) never decreases for classical process. Bekenstein (1973) found a rather formal analogy between the horizon area and the black hole entropy but Hawking (1975) showed that this analogy hides a deep physical meaning concerning the quantum processes in black hole physics. Nevertheless it turns out that the concepts of area of the horizon, irreducible mass and the entropy are no doubt deeply connected for a black hole.

Tachyons, contrary to ordinary matter, behave anomalously near a black hole. Raychaudhuri (1974) and Dhurandhar-Narlikar 1976 have independently shown that infalling tachyons may incur repulsive effects even under normal situations in Schwarzschild or Kerr black holes. Dhurandhar and Narlikar (1978) further showed that the usual proof of the second law of black physics breaks down when the infalling matter is made of tachyons and in this situation the horizon area of a Kerr black hole even decreases classically violating the second law of black hole physics meant for ordinary particles in normal physical processes.

The odd-∂ Tomimatsu and Sato 1973 metrics of which the Kerr metric is the first member ($\partial = 1$) possess an event horizon like the Kerr but naked singularities, unlike the Kerr and irreducible mass concept like the Kerr, are applicable in this case (Calvani *et al* 1979). Calvani *et al* have shown that reversible transformations (irreducible mass constant) are not connected any more with isoareal transformations for the $\partial=3$ metric, although the odd ∂ metrics could well describe black holes surrounded by rings of matter replacing the singularities. They opined that variation of the horizon area in $\partial=3$, *T-S* metric for ordinary particle interaction is perhaps due to the presence of naked ring singularities and therefore it cannot be interpreted at this moment, as a violation or contradiction of the second law of black hole physics.

That tachyons behave differently has been shown by Dhurandhar and Narlikar (1978) while Calvani *et al* (1979) have shown that even material particles can behave like tachyons in regard to variation in the horizon area in odd ∂ T-S black holes. It is therefore interesting to investigate as to what happens when a T-S black hole interacts with a field of tachyons. In this paper we investigate this problem and show that under certain general restrictions on the parameters of the interacting tachyons and the black holes the area of the horizon may decrease classically without changing the irreducible mass of the black hole *i.e.* reversible transformation is not connected with the isoareal transformation even when the infalling particles are tachyons instead of material particles as shown by Calvani *et al* (1979).

In § 2 we briefly recall the T-S $\partial=3$ metrics and their relevant properties. In § 3 we develop necessary mathematical equations to tackle the problem of tachyon trajectories and establish a condition necessary for decreasing the horizon area. In the last section we show that reversible transformation is not connected with isoareal transformation. This result is derived by using a computer as the $\partial=3$ metric components are very complicated in nature.

2. The $\partial=3$, T-S metric

The general form of the stationary axisymmetric line element is given by

$$ds^2 = f^{-1}\left[e^{2\gamma}(d\rho^2 + dz^2) + \rho^2 d\phi^2\right] - f(dt - \omega d\phi)^2, \tag{1}$$

where f, ω, γ are functions of ρ and z only. In prolate spheroidal coordinates defined by

$$\rho = k(x^2-1)^{1/2}(1-y^2)^{1/2}, \quad z = kxy. \tag{2}$$

the metric coefficients for $\partial=3$ given by Tomimatsu and Sato (1973) are as follows:

$$f = \frac{A}{B} = \frac{u^2+v^2-m^2-n^2}{(u+m)^2+(v+n)^2}, \quad \omega = \frac{2\,mq(1-y^2)}{u^2+v^2-m^2-n^2}\,c, \tag{3}$$

where
$$\begin{aligned}
u &= pxa^3(x^2+3) - pq^2x(a-b)^3(x^2+3y^2),\\
v &= -qyb^3(y^2+3) - p^2qy(a-b)^3(y^2+3x^2),\\
m &= p^2a^3(3x^2+1) + q^2b^3(3y^2+1),\\
n &= 12\,pq\,ab\,xy(a-b),\\
a &= x^2-1,\\
b &= y^2-1,\\
p^2 + q^2 &= 1
\end{aligned} \tag{4}$$

$$c = F(8)\,[(3px+9) + a^{-1}(16\,px+24) + a^{-2}(16\,px+16)]$$
$$+ b\,F(7)\,[(4px+18) + a^{-1}(40\,px+64) + a^{-2}(48\,px+48)],$$
$$- b^2 F(6)\,[(2px+10) + a^{-1}(24\,px+40) + a^{-2}(32\,px+32)],$$
$$F(8) = 16\,(p^2a^5 - q^2)(p^2a^3 - q^2) - 15\,(p^2a^4 + q^2)^2,$$
$$F(7) = -\,5\,(p^2a^4+q^2)(p^2a^3-q^2) + 6\,(p^2a^5-q^2)(p^2a^2+q^2),$$
$$F(6) = -\,8\,(p^2a^3-q^2)^3 + 9\,(p^2a^4+q^2)(p^2a^2+q^2).$$

In writing these we have used the notations of Yamazaki (1976) for computational facilities as and when necessary. However before using the Yamazaki notations, expressions for A, B and C have been compared with the T-S and are found correct. ∂ is the distinguishing parameter of the family and is a positive integer: in this case it always equals 3. The dimensionless parameter q is defined as $q = a/M$, where a is the specific angular momentum of the source and M the mass. A, B and C are polynomials in x and y respectively of degree $2\partial^2$, $2\partial^2$, $2\partial^2 - 1$. For $q<1$, we have considered in this paper the spacetime is described by the following main properties.

(i) The surfaces $|x| = 1$ are event horizons for the $\partial = 3$ when $a < M$ (Calvani *et al* 1979 and Tomimatsu and Sato 1973).
(ii) The surface $A = 0$ are ergosurfaces and their number is 6 for $\partial = 3$.
(iii) There are 3 ring singularities for $\partial = 3$ metric in the equatorial plane where $B = 0$; also $A = C = 0$ on them.
(iv) There are ring singularities outside the surfaces $|x| = 1$, but $\partial = 1$ solution (Kerr) has no such singularity outside.

As regards the property (iii) Calvani (1980) showed graphically that for $q>1$, $\partial=3$, there are six ring singularities outside the equatorial plane ($y=0$) and three on the equatorial plane. The case $q<1$ could not be analytically studied but computer calculations for different q values showed no ring singularity outside the equatorial plane. The infalling tachyons out of the way of the equatorial plane would not incur anomalous effect due to ring singularities. In our calculations we have investigated the trajectories both in the equatorial as well as in the y=constant directions. Absence of singularities outside the equatorial plane simplifies the interpretation of trajectories even in the equatorial plane for we may consider the result of equatorial trajectories to hold good just a little distance apart from it since the singularity thickness is zero as shown by computer calculations in the vicinity of $y=0$ plane (say $y=0{\cdot}1$).

Area of the event horizon ($x=1$) turns out to be

$$\mathrm{A} = 16\pi/\partial\,(L^2/(M^4 - L^2))^{\frac{\partial-1}{2}}\,M_{\mathrm{ir}}^2, \tag{5}$$

where $$M_{\mathrm{ir}}^2 = \tfrac{1}{2}\,[M^2 + (M^4 - L^2)^{\frac{1}{2}}]. \tag{6}$$

L is the total angular momentum and M, the mass of the black hole. M_{ir} is known as irreducible mass introduced by Christodoulou (1970) and equation (5) shows that for the Kerr black hole we cannot change the area of the event horizon keeping irreducible mass constant *i.e.* by reversible transformation. Dhurandhar and Narlikar (1978) showed that the area of the horizon decreases for infall of tachyons by changing the irreducible mass. In our investigation, however, for T-S, $\partial=3$ black hole we maintained the constancy of irreducible mass throughout and arrived at the same result as that of Dhurandhar and Narlikar.

3. Necessary mathematical expressions

3.1 *Mass formula*

The method of direct examination of test particle orbits was used by Christodoulou to introduce the concept of irreducible mass and reversible transformation for a black hole. Calvani *et al* have shown that the same mass formula holds for all $\partial>1$ members of the T-S family solutions.

$$M^2 = M_{\text{ir}}^2 + L/(4\, M_{\text{ir}}^2). \tag{7}$$

M_{ir} has the same meaning for any ∂ *i.e.* the irreducible contribution to mass that is left when the rotational energy is extracted by reversible transformation.

Any interacting incoming particle will change the mass and angular momentum of the black hole when absorbed. If one wants to keep $M_{\text{ir}}=$ constant, certain relations between the energy (which adds to M) and angular momentum (adds to L) of the infalling particle are to be satisfied. This is obtained by varying eq. (7)

$$dL/dM = 4\, c_1\, M/L, \tag{8}$$

where $c_1 = M_{\text{ir}}^2$; dL and dM are the change in angular momentum and the mass of the black hole concerned.

3.2 *Decrease of horizon's area*

Equation (5) gives the area of the horizon in terms of the parameters M, L and M_{ir}. For reversible transformation M_{ir} is kept constant and the variation in the area of the horizon is estimated by differentiating equation (5). However for $\partial=1$, T-S metric, the area of the horizon can never be changed without altering the irreducible mass. Dhurandhar and Narlikar (1978) therefore had no other alternative than to change it.

$$dA = 16\pi/\partial\, M_{\text{ir}}^2\, \frac{\partial-1}{2}\cdot\left(\frac{L^2}{M^4-L^2}\right)^{\frac{\partial-3}{2}} \cdot \frac{1}{(M^4-L^2)^2}\,[2M^4\, L\, dL - 4M^3\, L^2\, dM]. \tag{9}$$

Decrease in area needs

$$M\, dL < 2L\, dM, \tag{10}$$

i.e.

$$dL/dM = 2L/(M\, c_2), \tag{11}$$

where c_2 is greater than 1.

3.3 *Restrictions*

To achieve the desired result (8) and (11) must be simultaneously satisfied. This gives

$$L = (2\, c_1\, c_2)^{\frac{1}{2}}\, M, \tag{12}$$

i.e.

$$dL = (2\, c_1\, c_2)^{\frac{1}{2}}\, dM. \tag{13}$$

We put $q\,(= L/M^2) = (2c_1\, c_2)^{1/2}$ without loss of generality: then dL and dM are connected by the relation.

$$dL = q \, . \, dM \qquad \text{or} \quad l = q\, \epsilon, \tag{14}$$

where dL $(=l)$, the change in L, and dM $(=\epsilon)$ the change in and M are nothing but the contribution of the incoming particles to the corresponding parameters of the black hole. Equation (14) is very important because it simplifies the geodesic equation of the incoming particles to great extent. As we are dealing with $q<1$ it puts a certain restriction on the irreducible mass of the black hole:

$$0 < M_{\text{ir}}^2 < 1/(2c_2). \tag{15}$$

Restrictions (10), (13) and (15) have not been given by earlier workers although these are essential.

3.4. *Geodesic motion:*

The Hamilton-Jacobi equation for geodesic motion reads:

$$\frac{\dot{x}^2}{x^2-1} + \frac{\dot{y}^2}{1-y^2} + p^{2(\partial-1)} \frac{\partial^2\, \mu^2\, (x^2-y^2)^{\partial^2-1}}{B}$$

$$= \frac{\partial^2\, (x^2-y^2)^{\partial^2-1}}{p^{2(2-\partial)}\, (x^2-1)\, B^2}\, [D\epsilon^2 - 4\, \partial\, qcl\epsilon - \partial^2\, Al^2/(1-y^2)], \tag{16}$$

Where the constants of motion, μ, ϵ, $p_\phi = lM$ denote respectively the rest mass of the particle, its energy and the z-components of its angular momentum: the dot denotes differentiation with respect to an affine parameter.

$$DA = p^2\, (x^2-1)\, B^2 - 4\, \partial^2\, q^2\, (1-y^2)\, c^2. \tag{17}$$

As yet no detailed study of geodesic motions for $\partial=3$, T-S metric has been published, perhaps due to its very complicated and lengthy nature of the metric coefficients. However, useful remarks can be made qualitatively from (16) even for tachyon trajectories. Equation (16) is for the general case irrespective of the nature of the incoming particles and for tachyons we replace μ^2 by $-\,M_0^2$, where M_0 is the metamass of tachyon (Dhurandhar and Narlikar, 1978) and normalise M by putting it equal to unity. In general the energy of the infalling tachyon, in terms of its location (x, y), constants μ, l and momenta $p^x = \dot{x}$, $p^y = \dot{y}$ is given by (Calvani *et al* 1979):

$$\epsilon_{\pm} = \frac{-2\partial^2 qcl \pm \Delta^{1/2}}{D}, \tag{18}$$

where
$$\Delta = \frac{\partial^2 p^2 (x^2-1) B^2 l^2}{1-y^2} + D\Big[- M_0^2 p^2 B (x^2-1)$$
$$+ \frac{p^{2(2-\partial)} B^2 (x^2-1)}{\partial^2 (x^2-y^2)^{\partial^2-1}} \left(\frac{\dot{x}^2}{x^2-1} + \frac{\dot{y}^2}{1-y^2} \right)\Big]. \tag{19}$$

To produce a reversible transformation, zero separation between positive and negative root states is required; for equation (19) one can easily see that such a requirement can always be achieved on the $|x| = 1$ surface for any value of ∂.

Moreover by rearranging (19) we obtain for $\dot{x} = 0$, $\dot{y} = 0$ and $y = 0$,

$$l = \frac{-4\partial^2 q c \epsilon \pm \Delta^{1/2}}{2\partial^2 A}, \tag{20}$$

where
$$\Delta = 4\partial^2 p^2 (x^2-1) B^2 \left(\epsilon^2 + \frac{A M_0^2}{B} \right). \tag{21}$$

Regarding A and B it is expected that these two terms can never be negative for well-behaved solutions as that of the Kerr. But we know that the nature of A may behave anomalously (negative, zero or infinity) near or within the singularities so far as the singularities are concerned. With this limitation in mind it is natural to conclude from (21) that the motion is allowed for all values of x (Calvani and Catenacci 1976). In this paper, however, we have not considered qualitative properties at all, and the above is therefore of academic interest only and not in any way connected with our final conclusion.

For $\dot{y}=0$, the governing geodesic equation in our case turns out to be:

$$\frac{p^{2(2-\partial)} B^2}{\partial^2 (x^2-y^2)^{\partial^2-1}} \dot{x}^2 = \epsilon^2 D - 4\partial^2 qcl\epsilon - \frac{\partial^2 A l^2}{1-y^2} + M_0^2 p^2 (x^2-1) B. \tag{22}$$

Equations (14) and (22) are of fundamental importance in our study of area decrease. The former simplifies calculations for obtaining the values of $\dot{x}^2$ at different x values and the latter decides whether the tachyons will interact permanently with the black holes entailing decrease of area of the horizon. In the Kerr case, Dhurandhar and Narlikar (1978) considered, from their previous studies (Narlikar and Dhurandhar (1976)) that the incoming tachyons may bounce at a certain value of x where the right side of their simple equation (equivalent to our (22)) is zero or negative. They, therefore, drew curves ϵ versus R (their radial coordinate) making the right side of their equation zero and argued that as points outside the curves drawn exist for positive values of ϵ, the area of the Kerr horizon decreases undoubtedly. In our calculation, however, we have determined the values of the right side of (22) in conjunction with (14) for different values of x ranging from -40 to -1 and 40 to 1 giving preassigned values of q and y. In every step a particular realistic value of ϵ^2 gives the right side of equation (22) positive. Hence it is concluded that the area of the horizon decreases even in classical reversible transformation when tachyons fall into the black hole. Ranges of x chosen are sufficient to support the above remark because prolate spheroidal coordinates x and y are connected with the cylindrical coordinates r, θ, as follows (Glass 1973)

$$x = \partial/M_p\,(r - M),\; y = \cos\,\theta,$$

$$\text{for } r \to 0,\; x \to -\,\partial/M_p \text{ and at } r \to \infty,\quad x \to \infty. \tag{23}$$

In our calculations it is interesting to note that to make $\dot{x}^2$ in (22) positive the square of the energy of tachyons becomes smaller and smaller as the distance from the $x=1$ surface decreases and on $x=1$ surface, all values of ϵ^2 are permissible.

4. Results

The coefficient of $\dot{x}^2$ on the left side of (22) is always positive for all values of x and y except at the positions where B vanishes *i.e.* at the singularities on $y=0$. With equation (14), (22) simplifies to

$$\frac{p^{-2}\,B^2}{9\,(x^2 - y^2)^6}\,\dot{x}^2 = \epsilon^2\left[D - \frac{9\,q^2\,A}{1 - y^2} - 36\,q^2\,c\right] + p^2\,M_0^2\,(x^2 - 1)\,B. \tag{24}$$

Here we have put $\partial = 3$.

As $|x|>1$ the second term $p^2\,M_0^2\,(x^2-1)\,B$ in (24) is always positive except at the points where B is zero. Now the coefficient of ϵ^2 on the right side of (24) may take positive, negative or zero value; no remarks therefore, can be made about its nature in general. If it is positive then whatever may be the value of ϵ^2, the right side of (24) is positive and hence the area of the horizon decreases in reversible transformation. If it is negative, the value of ϵ^2 can suitably be adjusted so that the right side of (24) becomes positive, and this can be done easily. Thus even qualitatively we can conclude that $\dot{x}^2$ can be made positive for a particular value of ϵ^2 and hence the area of the horizon decreases even when the irreducible mass remains invariant. But the question arises whether that value of ϵ^2 is physically realistic. To ensure this, the coefficient values of ϵ^2 *i.e.* the sum of all the terms in the third bracket, have been calculated for different x values keeping q and y constant and the highest limit of ϵ^2 has been determined for each case to maintain the right side of (24) positive. Calculation shows that for $q=0{\cdot}8$ and $y=0$, the highest limit of ϵ^2 at very near to the surface $x=1$ is of the order of $10^{-10}\,M_0^2$ and it attains almost constant value of the order of $10^{-1}\,M_0^2$ as x increases. Calculations for $\Delta x = 0{\cdot}1$ has been made starting from 1 in the positive side and -1 at the negative side. This is continued up to 40 and -40 respectively. Only a synopsis of the result is given in tables 1 to 4 with reference to the special features regarding the nature of A, B, C and ϵ^2. From $x = 4{\cdot}6$ to $x = 40$, A, B and C remain positive throughout. ϵ^2 is of the order of 10^{-1}

Table 1. $q = 0{\cdot}8$, $y = 0$ (equatorial trajectories) $x \geqslant 1$

x	A	B	C	Highest limit of $M_0^{-2}\,\epsilon^2 <$
1	$-0{\cdot}262144$	$1{\cdot}04858$	$-0{\cdot}65536$	All values
1·0001	$-0{\cdot}26187835$	$0{\cdot}10492841 \times 10$	$-0{\cdot}6879761 \times 10$	do
1·1001	$0{\cdot}38504495$	$0{\cdot}226469 \times 10$	$-0{\cdot}7844221 \times 10^4$	$0{\cdot}465019 \times 10^{-10}$
1·2001	$0{\cdot}26971301 \times 10$	$0{\cdot}47812587 \times 10$	$-0{\cdot}20273894 \times 10^4$	$0{\cdot}216101 \times 10^{-7}$
1·4001	$0{\cdot}1564133 \times 10^2$	$0{\cdot}72661594 \times 10$	$-0{\cdot}11096298 \times 10^3$	$0{\cdot}160307 \times 10^{-3}$
1·5001	$0{\cdot}74616144 \times 10$	$0{\cdot}5198811$	$0{\cdot}16345224 \times 10^2$	$0{\cdot}188027 \times 10^{-3}$
1·6001	$-0{\cdot}88640966 \times 10^2$	$0{\cdot}26083355 \times 10^2$	$-0{\cdot}3249001 \times 10^3$	All values
1·8001	$-0{\cdot}18282831 \times 10^4$	$0{\cdot}17928659 \times 10^4$	$0{\cdot}72411359 \times 10^4$	do
4·5001	$-0{\cdot}4336156 \times 10^9$	$0{\cdot}99769874 \times 10^{11}$	$0{\cdot}28095320 \times 10^{13}$	do
4·6001	$0{\cdot}78298188 \times 10^9$	$0{\cdot}14630936 \times 10^{12}$	$0{\cdot}42462466 \times 10^{13}$	$0{\cdot}200182 \times 10^{-5}$
10·9001	$0{\cdot}17533105 \times 10^{18}$	$0{\cdot}46108518 \times 10^{18}$	$0{\cdot}40277433 \times 10^{20}$	$0{\cdot}105822 \times 10^{-1}$
...	...	...	...	Of same nature.

Table 2. $q = 0{\cdot}8$, $y = 0$. $x \leqslant -1$

x	A	B	C	Highest limit of $M_0^{-2}\,\epsilon^2 <$
-1	$-0{\cdot}26214406$	$0{\cdot}065536$	$-0{\cdot}16384$	All values
$-1{\cdot}0001$	$-0{\cdot}26187835$	$0{\cdot}65359002$	$-0{\cdot}17199402 \times 10^{10}$	All values
$-1{\cdot}1001$	$0{\cdot}38504495$	$0{\cdot}065465742$	$-0{\cdot}19620771 \times 10^4$	$0{\cdot}2151159 \times 10^{-10}$
$-1{\cdot}6001$	$-0{\cdot}88640966 \times 10^2$	$0{\cdot}30103505 \times 10^3$	$-0{\cdot}12563526 \times 10^3$	All values
$-1{\cdot}8001$	$-0{\cdot}18282831 \times 10^4$	$0{\cdot}18644 \times 10^4$	$0{\cdot}15597292 \times 10^4$	All values
$-4{\cdot}6001$	$0{\cdot}78298201 \times 10^9$	$0{\cdot}41001684 \times 10^7$	$0{\cdot}62412717 \times 10^{12}$	$0{\cdot}2649859 \times 10^{-8}$
$-7{\cdot}3001$	$0{\cdot}91322641 \times 10^{14}$	$0{\cdot}19045107 \times 10^{14}$	$-0{\cdot}12377345 \times 10^4$	$0{\cdot}1751250 \times 10^{-1}$
$-11{\cdot}5001$	$0{\cdot}47156121 \times 10^{18}$	$0{\cdot}18959974 \times 10^{18}$	$-0{\cdot}20430844 \times 10^{20}$	All values
$-40{\cdot}0001$	Positive	Positive	Negative	All values

Table 3. $q = 0{\cdot}6$, $y = 0{\cdot}1$ $x > 1$ and $x < -1$

1·0001	$0{\cdot}42483177 \times 10^{-1}$	$0{\cdot}42365687$	$-0{\cdot}1989467 \times 10^{10}$	All values
1·2001	$0{\cdot}11675213 \times 10$	$0{\cdot}1972284 \times 10$	$-0{\cdot}44529398 \times 10^3$	$0{\cdot}2556848 \times 10^{-6}$
1·4001	$-0{\cdot}11439269 \times 10^2$	$0{\cdot}15269755 \times 10^2$	$-0{\cdot}77313808 \times 10^3$	All values
3·2001	$0{\cdot}20873557 \times 10^8$	$0{\cdot}10667613 \times 10^{10}$	$0{\cdot}26937723 \times 10^{11}$	$0{\cdot}141435 \times 10^{-4}$
20·8001	$0{\cdot}13475762 \times 10^{24}$	$0{\cdot}19356696 \times 10^{24}$	$0{\cdot}47334351 \times 10^{26}$	$0{\cdot}8711408 \times 10^{-1}$
$-1{\cdot}0001$	$-0{\cdot}42483177 \times 10^{-1}$	$0{\cdot}91231883 \times 10^{-2}$	$-0{\cdot}22105190 \times 10^9$	All values
$-1{\cdot}2001$	$0{\cdot}11675113 \times 10$	$0{\cdot}11246550 \times 10$	$-0{\cdot}49579085 \times 10^2$	$0{\cdot}1201465 \times 10^{-4}$
$-1{\cdot}6001$	$-0{\cdot}36885754 \times 10^3$	$0{\cdot}18365232 \times 10^3$	$0{\cdot}8670852 \times 10^2$	All values
$-3{\cdot}0001$	$-0{\cdot}12281972 \times 10^7$	$0{\cdot}40712066 \times 10^6$	$0{\cdot}62994392 \times 10^9$	All values
$-3{\cdot}2001$	$0{\cdot}20873657 \times 10^8$	$0{\cdot}15403159 \times 10^7$	$0{\cdot}19960639 \times 10^{10}$	$0{\cdot}368064 \times 10^{-5}$
$-10{\cdot}4001$	$0{\cdot}4648394 \times 10^8$	$0{\cdot}22290787 \times 10^{18}$	$-0{\cdot}31099883 \times 10^{20}$	All values
$-20{\cdot}8001$	$0{\cdot}134762 \times 10^{24}$	$0{\cdot}33807491 \times 10^{23}$	$-0{\cdot}25762055 \times 10^{26}$	All values

Table 4. $q = 0{\cdot}6$, $y = 0{\cdot}89$. $x > 1$ and $x < -1$

1·0001	$-0{\cdot}14801105 \times 10^{-7}$	$0{\cdot}25822691 \times 10^{-3}$	$-0{\cdot}12018424 \times 10^{-9}$	All values
1·6001	$-0{\cdot}14879426$	$0{\cdot}16829040 \times 10^{4}$	$0{\cdot}36321769 \times 10^{4}$	All values
1·8001	$0{\cdot}14693621 \times 10^{3}$	$0{\cdot}22326235 \times 10^{5}$	$0{\cdot}74890203 \times 10^{4}$	All values
10·8001	$0{\cdot}96146648 \times 10^{18}$	$0{\cdot}19134702 \times 10^{19}$	$0{\cdot}77670728 \times 10^{19}$	$0{\cdot}14970084 \times 10^{-1}$
20·8001	$0{\cdot}13605107 \times 10^{24}$	$0{\cdot}19488126 \times 10^{24}$	$0{\cdot}17151182 \times 10^{25}$	$0{\cdot}21331103 \times 10$
$-1{\cdot}0001$	$-0{\cdot}14801105 \times 10^{-7}$	$0{\cdot}28678370 \times 10^{-4}$	$-0{\cdot}13353804 \times 10^{8}$	All values
$-1{\cdot}8001$	$0{\cdot}14693621 \times 10^{3}$	$0{\cdot}21476362 \times 10^{4}$	$0{\cdot}67912103 \times 10^{3}$	All values
$-2{\cdot}2001$	$0{\cdot}33062987 \times 10^{5}$	$0{\cdot}10501296 \times 10^{6}$	$0{\cdot}20138985 \times 10^{5}$	$0{\cdot}732762 \times 10$
$-8{\cdot}0001$	$0{\cdot}40359018 \times 10^{16}$	$0{\cdot}17045406 \times 10^{16}$	$-0{\cdot}43321793 \times 10^{16}$	All values
$-20{\cdot}8001$	$0{\cdot}13605108 \times 10^{24}$	$0{\cdot}97100098 \times 10^{23}$	$-0{\cdot}92016455 \times 10^{24}$	All values

M_0^2 from $x = 10{\cdot}9001$ and onward as shown in table 1. It is noteworthy that even for negative values of A, ϵ^2 can take any positive value to make $\dot{x}^2$ in (22) positive. All these fulfil the sufficient condition of decrement of the horizon's area when tachyons are considered as the incoming particles.

It is known that $y=0$ and $y=1$ are two special directions in which several new properties of the metric of T-S, $\delta = 3$, like the Weyl metric are observed. There is no ring singularity outside $y=0$ and $x=1$. Calculations, therefore, for different y values other than $y=0{\cdot}1$ have exposed the result sought for more clearly. The highest limit of ϵ^2 to make x^2 positive for $y=0{\cdot}1$, 0·89, 0·5 and $q=0{\cdot}6$, 0·4, 0·2 has been calculated and listed in the tables. Elaborate exposition requires too much of space and only the special features are therefore given in the table.

A comparison of tables 1 and 3 shows that the sign change of A for $x>1$ is more frequent in table 1 than table 3 as the former contains the result of equatorial region where the metric possesses ring singularities. Cases of $y=0{\cdot}5$, $q=0{\cdot}6$, 0·4, 0·2 do not show any deviation from the findings of the cases for $q=0{\cdot}6$ and $y=0{\cdot}89$. To save space, therefore, these are excluded. In table 4 we give the results for $q=0{\cdot}6$ and $y=0{\cdot}89$ only.

Tables 1 to 4 contain values of x, A, B, C and the highest possible values of ϵ^2 to make $\dot{x}^2$ in (22) positive. These are arranged such that one can know at what values of x the nature of A, C and the highest values of ϵ^2 change. Missing data for corresponding x values are considered insignificant since the nature of the said parameters remain the same with a change in values only.

5. Conclusions

In the course of our analysis through computer data we find frequent changes of the sign of A and C and variations in the highest positive values of ϵ^2. No general rule could be established here regarding their inter-relations, even then it is quite evident that the tachyons with suitable energy limit can reach the centre of the black hole and stop there entailing a decrease in the area of the horizon. This phenomenon seems to violate classically the second law of black hole physics. Dhurandhar and Narlikar (1978) in the Kerr case, remarked that the second law of black hole physics is no longer valid when infalling particles are tachyons. In this paper, we observe that in the T-S, $\delta=3$ space-times tachyons with sufficiently low energy can penetrate the black hole without rebounce and decrease the area of the horozon. It is not quite clear at this moment whether this apparent violation is due essentially to the presence of naked singularities or due to the breakdown of the proof of the second law of black hole physics in the presence of tachyons shown by Dhurandhar and Narlikar. Further analysis with different metrics may decide the fact in future and one may also calculate the entropy of the surrounding space-time to show whether the second law is preserved in general.

In the above investigation, with $M_{ir}=$constant we observed that tachyons with particular positive energy limit are suitable candidates for the decrease of the area of the horizon of a black hole whereas highly energetic tachyons may show points of inversion outside, inside or on the $x=1$ surfaces. Naturally the question arises what happens when M_{ir} is not considered constant. Under this condition, Dhurandhar and Narlikar studied the Kerr case only.

In conclusion it can only be said that further investigations could throw some light on the following topics: (a) role and properties of naked singularities in the new light of the laws of black hole physics. (b) Validity of the second law of black hole physics and the necessity of new laws for tachyon-interactions. (c) Hidden properties of the complicated metrics as regards to black hole energetics. (d) Future device for tachyon detection (Dhurandhar and Narlikar 1978).

Acknowledgements

Thanks are due to Prof S. Banerji for his constant encouragement and to Dr Pradip Kumar Datta for his indispensable help in computational work. Thanks also due to the referees for their valuable comments and the UGC for financial support.

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