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Forward hadron-hadron scattering at high energy

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Abstract. The high energy elastic scattering of pp, $p\bar{p}$, $p\pi^+$, $p\pi^-$, pk^+ and pk^- processes is studied at forward directions. The expressions for total scattering cross-sections and the ratios of real-to-imaginary parts of the forward amplitude are derived in $P+f$ model. For the Regge part of the scattering amplitude, the standard form is taken. For the pomeron part, the Harari-Freund conjecture is assumed. The background is assumed to get dominant contributions from the multiparticle exchanges in that channel. These contributions are obtained by parametrising the branch cuts by conformal mapping and polynomial expansion methods. The agreement with the experiment is good. The fits suggest strong exchange degeneracy for pp and kp which in turn is consistent with the Harari-Freund duality.

Keywords. Cross-sections; Regge poles; pomerons; duality; hadron-hadron scattering; elastic scattering.

1. Introduction

In our earlier paper (Badatya and Patnaik 1980) a model was proposed for high energy proton proton forward scattering in $P+f$ frame work. For the Regge pole part, a very standard form (Collins *et al* 1974) with one dominant pair of exchange degenerate Regge trajectory was taken. To choose the pomeron part of the amplitude the Harari-Freund conjecture (Harari 1968, Freund 1968) that the Pomeron was built out of s-channel background, was postulated. A further assumption was made that the s-channel branch cuts resulting out of multiparticle exchanges in that channel dominated this background. As a variance of the standard dispersion-relation technique, the conformal mapping method (Erdelyi 1953) was used to parametrise the branch cut contributions. With the help of a conformal mapping the whole of anlyticity plane was transformed into a strip along real axis with branch cuts forming the boundary. Then the pomeron amplitude was expressed as a series of Hermite polynomials. By imposing Froissart bound (Froissart 1961 on this the series was truncated and the amplitude was obtained.

In this paper we repeat the procedure for pp, $p\bar{p}$, $p\pi^+$, $p\pi^-$, pk^+ and pk^- processes. We determine the forward amplitudes for these processes and then obtain expressions for total cross-sections and the ratios of real to imaginary part. In § 2 we briefly explain the model and the notations. In § 3 this is compared with the experiment.

2. The model

As in the previous paper (Badatya and Patnaik 1980) the spin effects are ignored. The amplitude is defined by,

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi} |f(s, t)|^2, \tag{1}$$

$$\sigma_T = \operatorname{Im} f(s, o), \tag{2}$$

and
$$\rho = \operatorname{Re} f(s, o) / \operatorname{Im} f(s, o) \tag{3}$$

The amplitude $f(s, t)$ can be written as

$$f(s, t) = f_P(s, t) + f_R(s, t), \tag{4}$$

where (Collins *et al* 1974)

$$f_R(s, t) = -G_R [s \exp(-i\pi/2)]^{\alpha_R(t)}$$
$$\exp(a_R t) [1 - i\beta \exp(a_3 t)(1 + t/t_0)]. \tag{5}$$

We are interested in the forward direction only. Let the corresponding amplitudes be named as $f(s)$, $f_P(s)$ and $f_R(s)$. As described in § 1 $f_P(s)$ is obtained by parametrising s-channel branch cuts. For pp the branch cuts run from $s=4m^2$ to $S=\infty$ and from $s=4m^2 - 4\mu^2$ to $-\infty$. The conformal mappings

$$x = \frac{s - 4m^2 + 2\mu^2}{2\mu^2}, \tag{6}$$

and
$$z = -i \sin^{-1} x, \tag{7}$$

transform the whole of anlyticity plane into a strip along real axis, with the cuts forming the boundary at $\pm i\pi/2$. Thus, having a strip along the real axis as the domain of anlyticity (in z variable) of the amplitude $f(s)$, and an anticipated unbounded growth rate at infinity (most probably logarithmic) we expand $f(s)$ in a series of Hermite polynomials in z. (Erdelyi 1953; Badatya and Patnaik 1980)

$$f(s) = \sum_{n=0}^{\infty} a_n H_n(z), \tag{8}$$

For
$$s \geqslant 4m^2,$$

we have
$$z = y - i\pi/2, \tag{9}$$

where
$$y = \ln(x + \sqrt{x^2 - 1}). \tag{10}$$

For large s

$$z = \ln \frac{s}{\mu^2} - i\frac{\pi}{2} = \ln\left(\frac{S \exp(-i\,\pi/2)}{\mu^2}\right). \tag{11}$$

The Froissart bound requires that the series in (8) must be truncated after $n=2$. We rewrite the expression for $f(s)$ as

$$f(s) = i\,(b_0 + b_1\, z + b_2\, z^2). \tag{12}$$

Keeping high energy behaviour in mind the constants b_i's are chosen to be real.

Then we have

$$\sigma_T = \frac{G_R\,(1-\beta)}{\sqrt{2s}} + b_0 + b_1\, y + b_2\,(y^2 - \pi^2/4), \tag{13}$$

and

$$\rho = \left(-\frac{G_R\,(1+\beta)}{\sqrt{2\,s}} + \frac{\pi}{2}\,b_1 + \pi\, b_2\, y\right)\Big/\,\sigma_T \tag{14}$$

For $p\pi$ the branch cuts run from $s=(m+\mu)^2$ to ∞ and from $s=(m-\mu)^2$ to $-\infty$ and

$$x = \frac{s-(m^2+\mu^2)}{2m\,\mu}. \tag{15}$$

The same is also true for pk^+ process except that the pion mass μ is replaced by kaon mass M. Except for these differences the expressions for the σ_T and ρ remain same for all the processes. However the constants are different which are to be obtained from fits to experiment.

3. Comparison with experiment

The σ_T fits are shown in figure 1. The values of the parameters and the χ^2 are given in table 1. The agreement with experiment is quite good. For pp and kp β comes out to be 1 signifying strong exchange degeneracy. For $p\pi$ it is different from 1. Figure 2 shows σ_T for pp at cosmic ray energies. The agreement with our model is significant. This is a prediction of the model. There are some disagreement with antiparticle crosssections at lower energies. This can be cured with inclusion of additional trajectories, particularly baryonium trajectories (Roy and Gavai 1978). In figure 3 to 8 we plot the predicted ρ values along with data for all processes. In view of the minimal number of Regge trajectories used, the predictions are extremely good. The experimental data for σ_T are from Denisov *et al* (1971, 1973), Carrol *et al* (1974, 1976, 1979) Ayer's *et al* (1977), Foley *et al* (1967), Galbraith *et al* (1965), Amaldi *et al* (1973) and Amendolia *et al* (1973). The experimental data for ρ are taken from Fajardo *et al* (1981) and other sources cited in this reference.

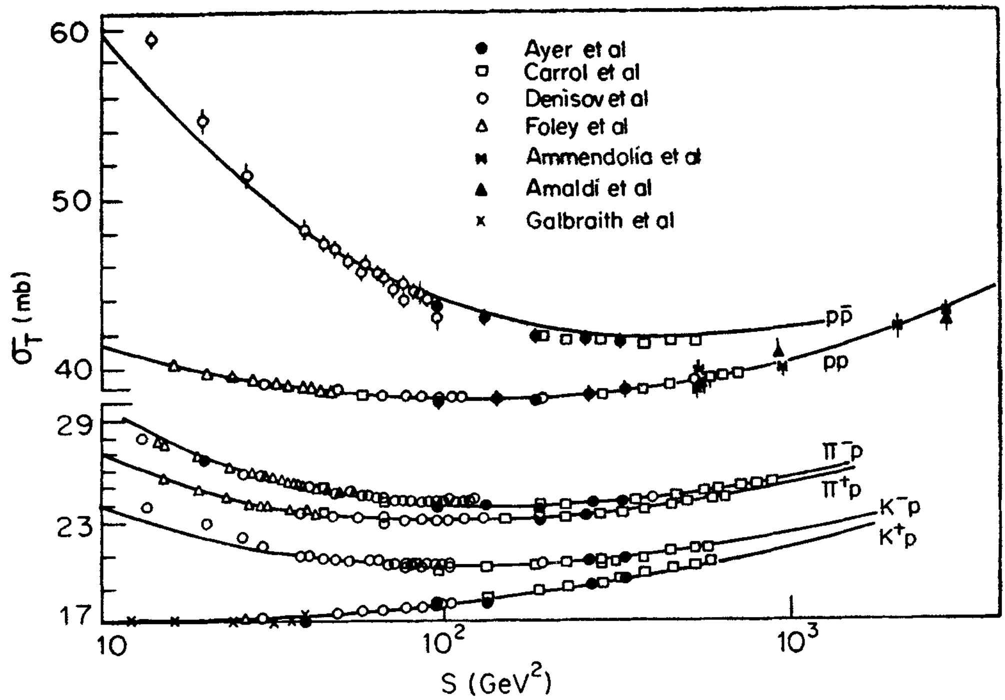

Figure 1. Total cross-sections for pp, pp^-, $p\pi^+$, $p\pi^-$, pk^+ and pk^-.

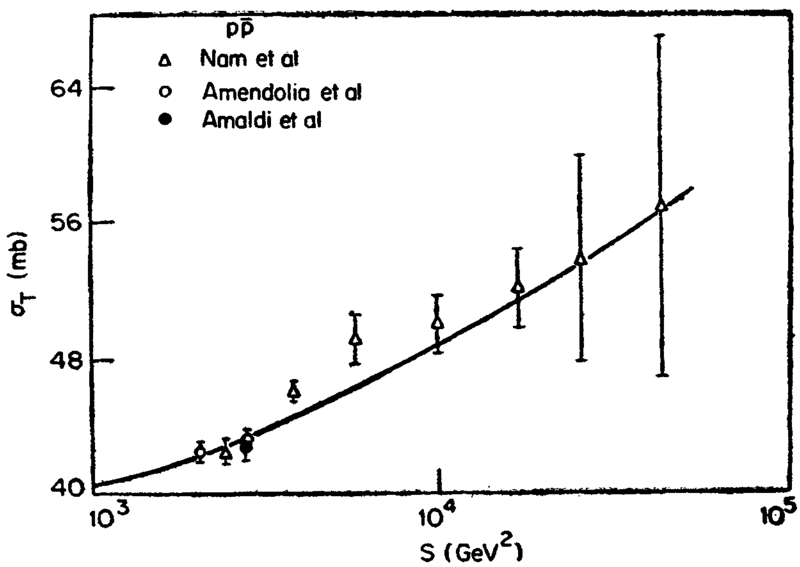

Figure 2. Total cross-section for pp at cosmic ray energies.

Table 1. Parameters for hadron-hadron scattering

Processes	G_R (in mb)		α (0)	b_0 (in mb)	b_1 (in mb)	b_2 (in mb)	$\frac{\chi^2}{\text{NDF}}$
pp	29·16	1	0·5	80·00	−9·255	0·529	33/40
p	36·25	0·13	0·5	17·022	−0·556	0.1584	42/35
kp	10·80	1	0·5	20·244	−1·1513	0·2328	45/28

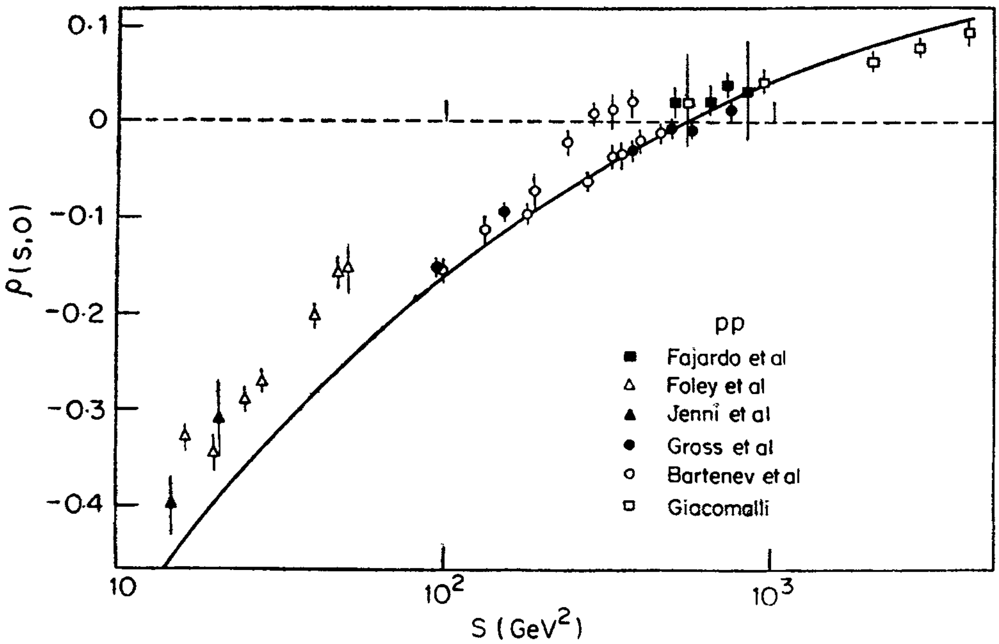

Figure 3. Ratio of real to imaginary parts of scattering amplitude for *pp*.

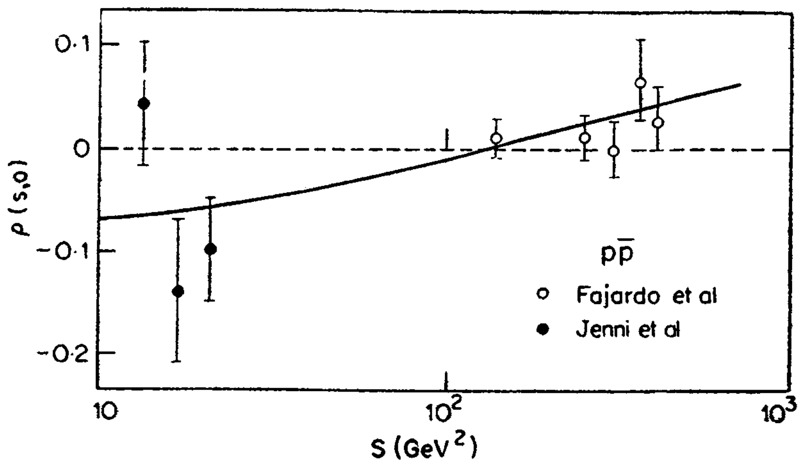

Figure 4. Ratio at real to imaginary parts of the scattering amplitude for $p\bar{p}$.

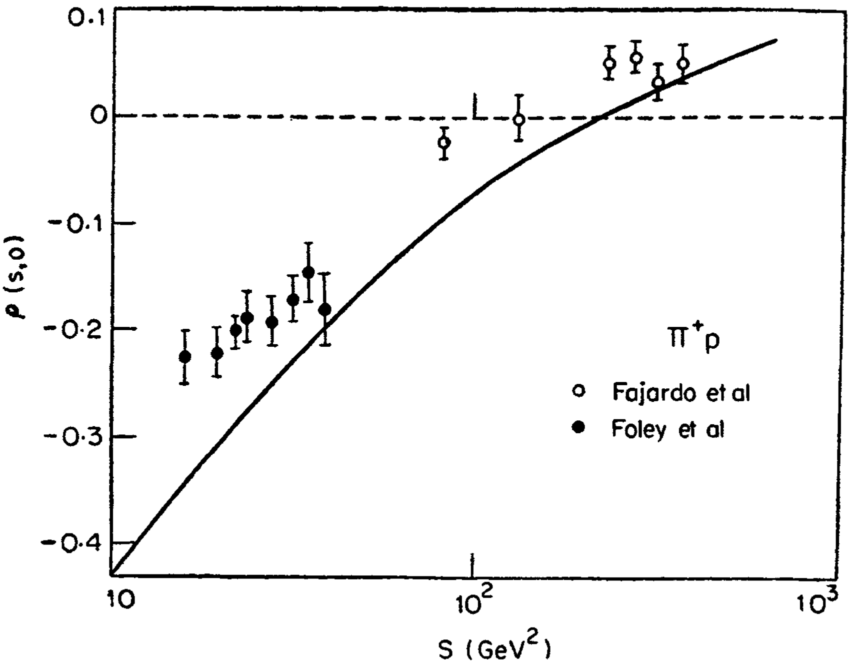

Figure 5. Ratio of real to imaginary parts of the scattering amplitude for $p\pi^+$.

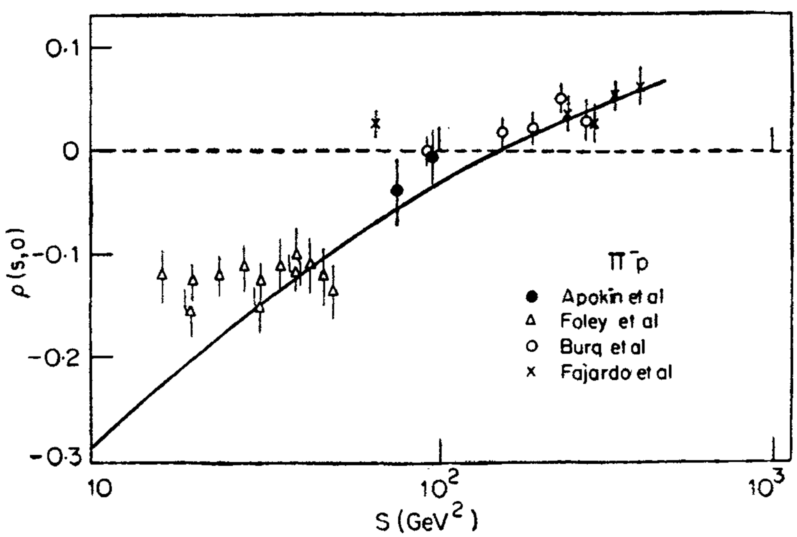

Figure 6. Ratio of real to imaginary parts of the scattering amplitude for $p\pi^-$.

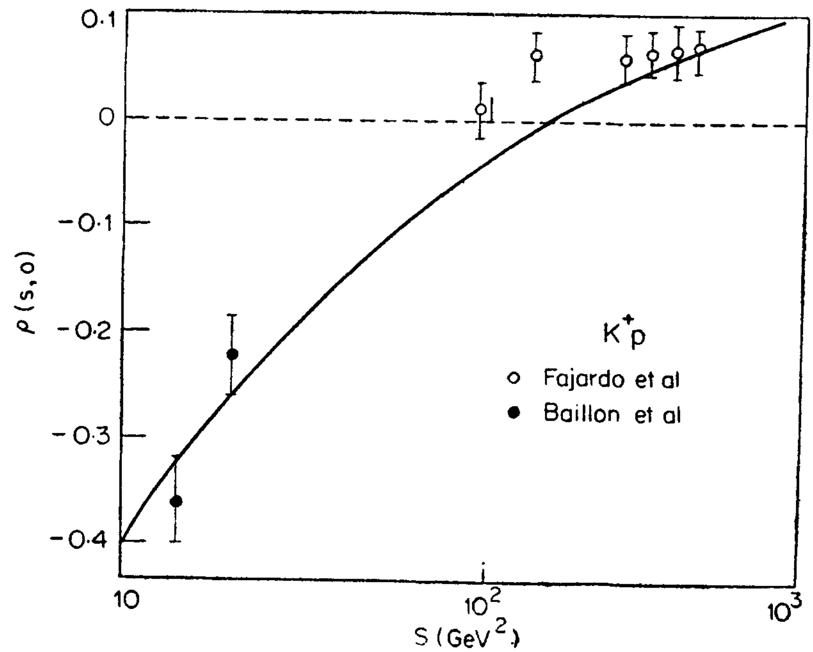

Figure 7. Ratio of real to imaginary parts of the scattering amplitude for k^+p.

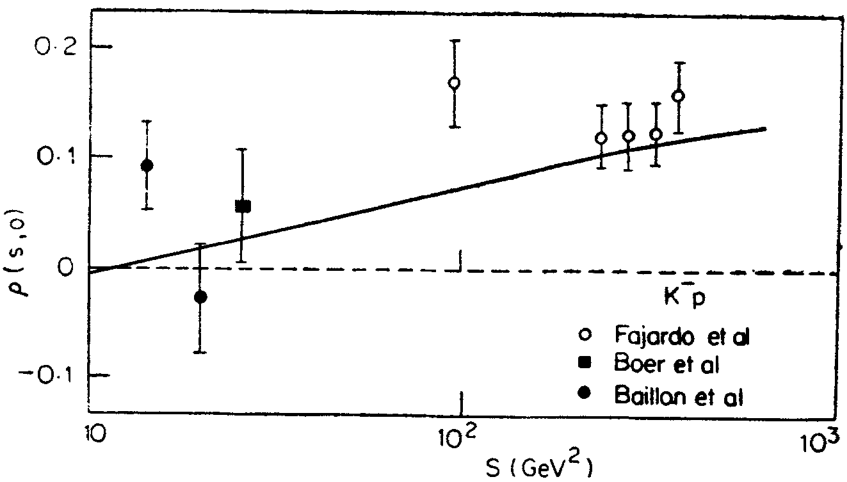

Figure 8. Ratio of real to imaginary parts of the scattering amplitude for k^-p.

4. Conclusion

A forward amplitude consisting of a Regge part and a pomeron part constructed using the Freund-Harari conjecture and the conformal mapping methods seems to describe the two-body hadronic scattering rather well. The best fits support strong exchange degeneracy and hence the two component duality. A similar result was earlier obtained for *kp* by Roberts *et al* (1978) in a different approach. However the pomeron so obtained stands for the totality of vacuum exchange phenomena, rather than a single pole in a complex *j* plane. This should be contrasted with the work of Dash *et al* (1980) who, taking a supercritical pomeron ($\alpha p(o) > 1$) come to the conclusion that the two-component duality is not correct in its present form.

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