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Two-body effects on baryon magnetic moments and radiative decays of mesons

R C VERMA
Department of Physics, Kumaun University, Nainital 263 002, India

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Abstract. We demonstrate that the two-body interaction effects involving spectator quark can account for the discrepancy between theory and experiment for the baryon magnetic moments and meson radiative decays.

Keywords. Magnetic moments; radiative decays; quark model; baryons; mesons.

1. Introduction

The discrepancy between theoretical and experimental results on the magnetic properties of hadrons has motivated a great deal of work so as to bring them closer. In the case of $VP\gamma$ decays, various approaches namely, the symmetry-breaking effects (Boal *et al* 1976; Edwards and Kamal 1976; Verma 1980*a*), addition of unitary singlet piece to electromagnetic current (Bohm and Teese 1972; Bajaj and Khanna 1977), extended VMD hypothesis (O'Donnell 1977, 1981) and the quark model approaches (Ono 1975; Isugr 1976), etc have been considered. But none of these attempts has been completely successful in explaining $V \rightarrow P\gamma$ decay rates. The main source of plague has been the anomalous large ratio $\Gamma(\omega \rightarrow \pi\gamma)/\Gamma(\rho \rightarrow \pi\gamma)$ and a small $K^{*-} \rightarrow K^-\gamma$ decay width (Gobbi *et al* 1974, 1976). Recent measurements (Berg *et al* 1980) on $\Gamma(\rho \rightarrow \pi\gamma)$ yielding almost twice the previous value has greatly reduced the difficulties of its understanding. The first measurements of $\Gamma(K^{*-} \rightarrow K^-\gamma)$ (Berg *et al* 1980) and η' decay width (Binnie *et al* 1979, Abrams *et al* 1979) have provided other checks on various models.

In the case of baryon magnetic moments, all possible experimental values are now available. Theoretically, the prediction of $\mu(p)/\mu(n)$ ratio was the cornerstone of the simple SU(6) quark model. With quark mass breaking (de Rujula *et al* 1975, Lipkin 1978), the quark model predicted $\mu(\Lambda) = -0{\cdot}61$ which was confirmed by experiment (Schachinger *et al* 1978). But the same quark mass parameters led to a value for $\mu(\Xi^\circ)$ four standard deviations from experiment (Bunce *et al* 1979). Later the low observed value for $\mu(\Sigma^+)$ moment (Settles 1979) demonstrated that the magnetic moments cannot be fitted within the quark model (Franklin 1979, 1980). Even the current quark (Bucella *et al* 1978) and the bag model (De Grand *et al* 1975) suffer from similar violations. The bag model with variable pressure (Joseph and Nair 1981) gives a closer agreement with experiment, though the Σ^+ and Ξ° moment remain larger than observed values. The new measurement of $\mu(\Xi^\circ)$ and $\mu(\Sigma^+)$, therefore, implies that the simple picture of baryon structure and magnetic interaction is incomplete. Configuration mixing (Isgur and Karl 1980) worsens the situation as long as u/d quark moment ratio is proportional to charge ratio of (-2) (Geffen and Wilson 1981). The effects of gluon exchange on baryon wave function seem to improve the situation (Verma and Khanna 1981).

Thus the present situation of magnetic interaction, specially for the hadrons containing s quark, demands some kind of SU(6) breaking interaction in the wave functions and /or the Hamiltonian. Ideas about the nature of SU(6) breaking forces inside the hadron have long been suggested (De Rujula *et al* 1975). The hyperfine interaction arising because of the gluon exchange nicely explains the ordinary mass spectrum.

In this paper, we study the problem of magnetic moments and the $VP\gamma$ transitions using the quark model. In addition to the main contribution to magnetic interaction arising from single quark, we include nonspectator type two-body interaction Hamiltonian, which may appear as a result of gluon exchange phenomena. We see that these two-body interaction effects can account for the discrepancy between theory and experiment.

2. Electromagnetic Hamiltonian

In this section, we construct a phenomenological electromagnetic Hamiltonian, modified in the presence of two-quark interaction. The present understanding of the hadron mass spectrum (strong interaction) requires a significant term arising due to single gluon exchange between two quarks (De Rujula *et al* 1975). This Fermi-Breit type interaction, for the case of baryon and mesons ($l=0$) can be reduced to

$$H^{ij}_{\text{strong}} = \alpha_s(\lambda_i \cdot \lambda_j)\left[\frac{1}{|\mathbf{r}|} - \frac{1}{2m_i m_j}\left(\frac{\mathbf{p}_i\cdot\mathbf{p}_j}{|\mathbf{r}|} + \frac{\mathbf{r}\cdot(\mathbf{r}\cdot\mathbf{p}_i)\mathbf{p}_j}{|\mathbf{r}|^3}\right) - \frac{\pi}{2}\,\delta^3(\mathbf{r})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16\mathbf{S}_i\cdot\mathbf{S}_j}{3m_i m_j}\right)\right], \tag{1}$$

where $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$ and $\mathbf{S}_i$, m_i and $\mathbf{p}_i$ represent spin, mass and momentum of ith quark respectively, α_s is the strong coupling constant and the factor $(\lambda_i\cdot\lambda_j)$ appears as the residue of the non-abelian nature of gauge coupling differentiating between $q\bar{q}$ and qq pair interaction.

In the presence of such strong Hamiltonian the electromagnetic Hamiltonian would get modified to:

$$H^{\text{modified}}_{\text{em}} = \sum_i H^i_{\text{em}} + \sum_{i>j} H^{ij}\,(\text{em} \times \text{strong}), \tag{2}$$

where the conventional matrix elements are obtained from

$$H^i_{\text{em}} = \frac{e_i}{2m_i} q_i^\dagger \boldsymbol{\sigma}\, q_i\cdot\mathbf{B} \tag{3}$$

and the matrix elements of H_{ij} are obtained from (Okubo 1963):

$$\langle f | T(H^{i}_{\mathrm{em}}, H^{ij}_{\mathrm{st}}) | i \rangle. \tag{4}$$

Unfortunately these matrix elements cannot be evaluated as the strong interaction dynamics is not exactly known for the low energy sector. We therefore reduce the two-body Hamiltonian H_{ij}, keeping terms upto first order of perturbation, to the following:

$$H^{ij} = A_j \frac{e_i}{2m_i} (\lambda_i \cdot \lambda_j)(q_i^{+} \sigma q_i)(q_j^{+} q_j) \cdot \mathbf{B}, \tag{5}$$

where the parameter A_j includes all the effects through (1) arising due to the presence of nonspectator jth quark. We have kept $(\lambda_i \cdot \lambda_j)$ term separately, which takes care of colour interactions between qq and $q\bar{q}$. This is needed in order to relate the baryon magnetic moments with meson decays.

3. Calculation of magnetic moments and transition rates

In general, for the uncharmed sector, the interaction (5) will introduce three parameters for $j = u, d$, and s quarks respectively. In the limit of isospin invariance the contribution involving u and d quarks can be absorbed in conventional moments. The effective piece arising as a result of SU(3) breaking would be

$$A_s \frac{e_i}{2m_i} (q_i^{+} \sigma q_i)(s^{+} s). \tag{6}$$

Taking the quark mass ratio (Lipkin 1978) $m_u/m_d = 1$ and $m_u/m_s = 0{\cdot}63$ and fixing A_s, from $\mu(\Xi^0)$, we obtain various magnetic moments as displayed in column 2 of table 1. The calculated value $\mu(\Sigma^+) = 2{\cdot}43$ is consistent with the experimental value of

Table 1. Magnetic moments of baryons (n.m)

Moment	Theory		Experiment (n.m.) (Rosner 1980)
	$m_u/m_d = 1$, $m_u/m_s = 0{\cdot}63$	$m_u/m_d = 0{\cdot}85$ $m_u/m_s = 0{\cdot}63$	
p	2·79†	2·79†	2·793
n	−1·86	−1·91†	−1·913
Λ	−0·61	−0·61	$-0{\cdot}6138 \pm 0{\cdot}0047$
Σ^+	2·43	2·26	$2{\cdot}33 \pm 0{\cdot}13$
Σ^0	0·76	1·14	—
Σ^-	−0·91	−0·92	$-0{\cdot}89 \pm 0{\cdot}14$*
Ξ^0	−1·23†	−1·23†	$-1{\cdot}236 \pm 0{\cdot}014$
Ξ^-	−0·50	−0·65	$-0{\cdot}75 \pm 0{\cdot}07$
$\Lambda\Sigma^0$	1·45	1·61	$1{\cdot}82 \begin{cases} +0{\cdot}25 \\ -0{\cdot}18 \end{cases}$

†input *Theberge and Thomas (1982).

2.33 ± 0.13 nm. The $\mu(\Sigma^-)$ is not disturbed much from its symmetric value of -0.93nm and agrees well with latest value -0.89 ± 0.14 nm (mentioned by Theberge and Thomas (1982)). The $\mu(\Xi^-)$ and Σ-Λ transition moments are lowered like any other model incorporating SU(3) breaking only (Franklin 1979; Verma 1980b). Next we calculate the mesonic decay widths using the formula:

$$\Gamma(V \to P\gamma) = (4\alpha/3)k^3 \, |A_{VP\gamma}|^2 \, \Omega^2, \tag{7}$$

where Ω is the overlap integral and k is the photon energy. With the ideal $\omega\phi$ mixing and the physical $\eta - \eta'$ mixing, the quark picture gives $\langle \pi\gamma \,|\, \omega \rangle = \mu(p)$ yielding, if Ω is taken to be unity, $\Gamma(\omega \to \pi\gamma) = 1.18$ MeV which is larger than the experimental value. Therefore we take $\Gamma(\omega \to \pi\gamma)$ as input to fix the overall scale and use $\Gamma(\omega \to \pi\gamma) = 789 \pm 92$ keV, a value claimed in a recent analysis (Oshima 1980). The calculated decay widths for other modes are given in table 2. The $\phi \to \pi\gamma$ decay widths vanish as a result of ideal $\omega\phi$ mixing and a nonvanishing value can be obtained by slight variation from ideal mixing without producing significant change in other numbers. All the decay widths are in good agreement with experiment except that $\rho \to \pi\gamma$ decay width calculation gives 82 ± 8 keV, slightly larger than the experimental value. In fact, in the present model $\Gamma(\omega \to \pi\gamma)/\Gamma(\rho \to \pi\gamma)$ ratio has remained undisturbed. Experimentally, the new ratio

$$\frac{\Gamma(\omega \to \pi\gamma)}{\Gamma(\rho \to \pi\gamma)} = 11.7 \pm 2.5 \tag{8}$$

has come close to VMD or quark model prediction of 9·5. Geffen and Wilson (1980) have remarked that by allowing quark moments to be arbitrary, a better agreement can be obtained. So far, we have avoided such effects mainly to

Table 2. Radiative widths of mesons (ideal $\omega\phi$ mixing and physical $\eta - \eta'$ mixing)

Decay	Theory (keV)	Experiment (keV)	
$\rho \to \pi\gamma$	82	67 ± 7 (Berg *et al* 1980)	
$\rho \to \eta\gamma$	49	52.5 ± 13.7	(Oshima 1980)
$\omega \to \pi\gamma$	789†	789 ± 92	(Oshima 1980)
$\omega \to \eta\gamma$	6·3	$3.2 \begin{cases} +2.6 \\ -1.9 \end{cases}$	(Oshima 1980)
$\phi \to \pi\gamma$*	0	5.7 ± 2.1	(Oshima 1980)
$\phi \to \eta\gamma$	42	65 ± 15	(Oshima 1980)
$\phi \to \eta'\gamma$	0·19	—	
$K^{*-} \to K^-\gamma$	41	40 ± 15 (Berg *et al* 1980)	
$K^{*0} \to K^0\gamma$	97	75 ± 35 (Gobbi *et al* 1974)	
$\eta' \to \rho\gamma$	105	94.1 ± 25.1	(Binnie *et al* 1979; Abrams *et al* 1979)
$\eta' \to \omega\gamma$	9·6	8.4 ± 2.7	(Binnie *et al* 1979; Abrams *et al* 1979)
$\eta' \to \rho\gamma/\eta' \to \omega\gamma$	11	14.0 ± 3.4	(Binnie *et al* 1979; Abrams *et al* 1979)
$\pi^0 \to \gamma\gamma$	7·35 (eV)	7.8 ± 0.9 (eV)	(Kelly *et al* 1980)
$\eta \to \gamma\gamma$	0·51	0.323 ± 0.046	(Kelly *et al* 1980)
$\eta' \to \gamma\gamma$	4·65	5.4 ± 2.1 (Binnie *et al* 1979)	

†input *A nonzero $\phi \to \pi\gamma$ can be fixed from non-ideal $\omega\phi$ mixing.

emphasize that in magnetic interactions the quarks other than those emitting the photon, may not behave like spectator quarks and a simple two-quark interaction can account for a major discrepancy between theory and experiment. But the piece (6) is unable to explain the large observed $\mu(\Xi^-)$. In fact all the models involving SU(3) breaking only predict $\mu(\Xi^-)$ to be smaller than, $\mu(\Lambda)$ (Rosner 1980). It has been shown (Verma 1981) that a large $\mu(\Xi^-)$ can be obtained in the presence of isospin breaking which may also be held responsible for discrepancy between $\mu(n)/\mu(p)$ ratio. In the present picture, SU(2) breaking can be introduced by varying m_u/m_d from unity and by differentiating between $j=u$ and d contributions in (5). Taking $m_u/m_d=0{\cdot}85$ and fixing A_d from the $\mu(n)/\mu(p)$, we obtain the magnetic moments as shown in column 30f table 1. With this, the agreement for all the numbers is improved.

We also calculate the two-photon decay widths of pseudoscalar mesons, as the two processes can be related through VMD mechanism (Etim and Greco 1977). The calculated decay widths (table 2) agree with experiment.

The present model can also be extended to the charm sector. The OZI-violating decays like $\psi\to\pi/\eta/\eta'+\gamma$ and $\eta_c\to\rho/\omega/\phi+\gamma$ remain forbidden as a result of the ideal mixing. Here also the small observed value for ψ decays can be obtained by slight variation of standard $\eta-\eta'-\eta_c$ mixing (Fritsch and Jackson 1977). We find that the introduction of two-quark contributions as given in (5), tends to lower the $\Gamma(D^{*0}\to D^0\gamma)$ decay width to ~ 9 keV and suppress $\Gamma(D^{*+}\to D^+\gamma)/\Gamma(D^{*0}\to D^0\gamma)$ ratio by a factor of 10 or more. Though there is no experimental data available for charm meson radiative decay widths, the effects are in qualitative agreement with the measured branching fractions of $B(D^{*0}\to D^0\gamma)=45\pm 15\%$ and $B(D^{*+}\to D^+\gamma)=2\pm 1\%$ (Applequist *et al* 1978). Moreover, in case of charm particles the use of the same parameter, i.e. $A_c=A_s$ is not justified, as now the particles contain a heavy mass quark. Taking $B(D^{*0}\to D^0\gamma)/B(D^{*+}\to D^+\gamma)\sim 25$, we obtain $\Gamma(F^{*+}\to F^+\gamma)/\Gamma(D^{*+}\to D^+\gamma)\sim 1/10$ and $\Gamma(\psi\to\eta_c\gamma)$ decay width is reduced to 85 eV which is well below the experimental upper limit of 3·5 keV (Wiik and Wolf 1977). $\Gamma(\psi\to\eta_c\gamma)=85$ eV, in turn gives $\Gamma(\eta_c\to\gamma\gamma)=0{\cdot}6$ keV.

In summary, we have shown that the two-quark interaction effects seem to be responsible for the major discrepancy between theory and experiment for magnetic properties, specially, of low lying hadrons. Though a more general and complete treatment of the two-body interaction is desired, we find that the simple prescription considered here nicely explains most of the data on meson radiative decays and baryon magnetic moments.

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