Pramāṇa, Vol. 19, No. 3, September 1982, pp. 249-254. © Printed in India.

The ($\alpha-d$) cluster model of ^{6}Li and muon capture

I AHMAD and S K SINGH
Department of Physics, Aligarh Muslim University, Aligarh 202 001, India

MS received 3 October 1981; revised 5 June 1982

Abstract. The ($\alpha - d$) cluster model with parameters determined from electron scattering and pion photoproduction processes is used to calculate the muon capture rate in ^{6}Li. The result is found to be better than the results calculated in other models and is in agreement with the experimental data.

Keywords. Muon capture; cluster model; form factors; impulse approximation.

1. Introduction

The process of muon capture in light nuceli has been widely studied by using either the impulse approximation (IA) or the elementary particle model approach (Primakoff 1975; Walecka 1975; Mukhopadhyay 1977). While both these models give similar results for the partial muon capture rate in ^{6}Li none of these can be said to be in good agreement with the experimental result (Hwang 1978; Cammarata and Donnelly 1976; Bergstrom *et al* 1975; Donnelly and Walecka 1973; Delorme 1970: Kim and Mintz 1970). When the question of the existence of second class currents seems to have been settled now (Wu 1977; Garvey 1977), the disagreement is in general attributed to the uncertainties in the knowledge of the nuclear wave function in ^{6}Li and other light nuclei (Primakoff 1977). In the case of ^{6}Li no attempts have been made to calculate the effect of meson exchange currents —an effect suspected to play an important role in weak and electromagnetic interactions of nuclei even at low energies and momentum transfers (Parthasarathy and Waghmare 1979; Ivanov and Trublik 1978, 1979, Guichon *et al* 1978, 1977, Porrmann 1981, Henley and Hwang 1980, Jaus and Woolcock 1981). In nuclei, the study of meson exchange currents is generally related with the description of basic nuclear potential which is used to calculate the nucleon wave functions. A better wave function for the nucleus may therefore simulate to some extent the effects of meson exchange currents. Many authors have recently explored various wave functions of ^{6}Li while studying the threshold pion photoproduction and have used these wave functions to analyse other weak and electromagnetic processes in this nucleus (Cammarata and Donnelly 1976; Bergstrom *et al* 1975, Cannata *et al* 1974; Koch and Donnelly 1974, 1973). These studies have shown that no wave function of ^{6}Li in the shell model using harmonic oscillator basis is capable of giving a consistent description of electron scattering, muon capture, pion photoproduction and radiative pion capture processes.*

*In an improved analysis by Mukhopadhyay (1977) it is shown that the experimental capture rate can be reproduced in shell model with harmonic oscillator basis with the oscillator parameter (b) in the range of $1{\cdot}33 < b < 1{\cdot}75$. The values of this parameter b from electron scattering and threshold pion photoproduction data are higher than 1·75 fm.

The $(\alpha-d)$ cluster model foi the ^{6}Li nucleus has attracted considerable attention during the last several years. A number of studies show that the model works quite well for the description of low lying states of this nucleus. In particular it is very successful in accounting for the various electric and magnetic from factors of ^{6}Li. It has also explained quite successfully the data on p-^{6}Li scattering from the Saclay group (Ahmad and Khan 1979; Bruge 1978, Aslanides *et al* 1975) especially at low momentum transfer range which concerns us here. This model has however not been used so far to analyse the present data on weak processes in ^{6}Li (Ahmad and Khan 1979; Noble 1974; Raphael 1973; Kudeyarov 1971; Neudatchin and Smirmov 1964). We have earlier shown that this model gives a better description of the threshold pion photoproduction process in ^{6}Li (Singh and Ahmad 1977). In this paper we calculate the partial muon capture rate in ^{6}Li using the $(\alpha - d)$ clustei model wave function of ^{6}Li with the parameters determined from the electron scattering experiments and show that our result for the capture ıate is better than those calculated in the various shell models and is in fair agreement with the experimental result.

2. Capture rate

The transition probability for muon capture is expressed as:

$$dw = 2\pi\,\delta\,(E_\nu + E_i - m_\mu - E_f)\,\frac{d\mathbf{p}_\nu}{(2\pi)^3}\,|\,m\,|^2, \tag{1}$$

where the matrix element m is given by (Commins 1973)

$$m = \frac{G\cos\theta}{\sqrt{2}}\int d\mathrm{x}\,\bar{\psi}_n(x)\,[f_1(q^2)\,\gamma^\mu + if_2(q^2)\,\sigma^{\mu\nu}\,q_\nu$$

$$+\,g_1(q^2)\,\gamma^\mu + g_3(q^2)\,\gamma^5\,q^\mu]\,J_\mu\,\psi_p(x)$$

where $$J_\mu = \bar{\psi}_\nu(x)\,\gamma_\mu\,(1+\gamma_5)\,\psi_\mu(x). \tag{2}$$

Using the standard methods for nonrelativistic reduction of the covariant single nucleon operator applicable to nuclei the matrix element m is derived to be (Commins 1973, Singh 1972, 1974, 1975).

$$m = \sum_i \int d\mathrm{x}\,\langle f\,|\,\chi_\nu\frac{(1-\sigma\cdot\hat{p}_\nu)}{2}\,\tau_i\,[G_V(q^2) + G_A(q^2)\,\sigma\cdot\sigma_i$$

$$+\,G_P(q^2)\,\sigma_i\cdot\hat{p}_\nu]\,\chi_\mu\,\exp\,(-i\,\mathbf{p}_\nu\cdot\mathbf{x}_i)\,\phi_\mu(\mathbf{x}_i)\,\delta\,(\mathbf{x}-\mathbf{x}_i)\,|\,i\rangle, \tag{3}$$

where $$G_V(q^2) = G\cos\theta\, f_1(q^2)\,(1+E_\nu/2M),$$

$$G_A(q^2) = G\cos\theta\,[-g_1(q^2) - [f_1(q^2)+f_2(q^2)]\,(E_\nu/2M)],$$

$$G_P(q^2) = G\cos\theta\left[-m_\mu\,g_3(q^2) + g_1(q^2) - [f_1(q^2)+f_2(q^2)]\,\frac{E_\nu}{2M}\right] \tag{4}$$

$g_p(q^2) = m_\mu\, g_3(q^2)$ and momentum transfer $\mathbf{q} = \mathbf{p}_\nu$ $|i\rangle$ and $|f\rangle$ are the initial and final nuclear wave functions described in ($\alpha - d$) cluster model through a function ξ_{JM} given by

$$\xi_{JM}(X_1, X_2, \ldots, X_6) = A\left[\sum \begin{pmatrix} j & l & J \\ m_j & m_l & m_J \end{pmatrix} \phi^0_{\alpha,0}(X_1, \ldots, X_4)\right.$$

$$\phi^{m_j}_j(X_5, X_6)\, \psi^{m_l}_l(X_1, \ldots, X_4; X_5, X_6), \tag{5}$$

where A stands for antisymmetrisation. $\phi^0_{\alpha,0}$ and $\phi^{m_j}_j$ are the wave functions of α and dinucleon cluster in the initial and final nuclei; $\psi^{m_l}_l$ is the wave function which describes the relative motion of the α and d cluster inside ^{6}Li.

Neglecting the effect of antisymmetrisation* and D state of the deuteron, in d cluster, the matrix element (3) is calculated to be

$$m = \int \chi^\dagger_\nu \left(\frac{1 - \sigma \cdot \hat{p}_\nu}{2}\right) (G_A(q^2) + G_P(q^2)\, \hat{p}_\nu)\, \chi_\mu\, \phi^0_0(\mathbf{x} + 2\mathbf{X}/3)$$

$$\cdot (\sigma(5) - \sigma(6))\, \phi^{m_j}_1(\mathbf{x} + 2\mathbf{X}/3)\, |\psi^0_0(X)|^2 \exp(-i\, \mathbf{P}_\nu \cdot \mathbf{x})$$

$$\phi_\mu(\mathbf{x})\, d\mathbf{X}\, d\mathbf{x}, \tag{6}$$

where $\mathbf{X} = (1/4) \sum_{i=1}^{4} \mathbf{x}_i - (1/2) \sum_{i=5,6} \mathbf{x}_i$ is the relative coordinate between α and dinucleon clusters. Taking the muon wave function to be

$$\phi_\mu(\mathbf{x}) = \frac{Z^{3/2}}{(\pi a_0)^{1/2}} \exp(-Zr/a_0)\, U_\mu, \tag{7}$$

the capture rate is calculated to be

$$W = (G^2 \cos^2\theta/\pi^2)\, R\, (m^2_\mu\, E^2_\nu/1 + E_\nu/3)\, a^3 Z^3 F^2_d(p_\nu)\, F^2_{\alpha-d}(2p_\nu/3)$$

$$\{G^2_A(p^2_\nu) + 1/3\, [G^2_P(p^2_\nu) - 2G_A(p^2_\nu)\, G_P(p^2_\nu)]\}, \tag{8}$$

where m is the reduced mass of the muon and R is the reduction factor needed to properly take into account the effect of the charge distribution of ^{6}Li nucleus and is taken to be 0·95 following Walecka (1975, 1976). This corresponds to an effective charge $Z_{\text{eff}} = 2{\cdot}94$ extrapolated by Eckhouse *et al* (1963) following Ford and Willis (1962).

*It has been shown by Kudeyarov *et al* (1971) in a similar transition that because of small overlap of the two clusters the intercluster nucleon exchange effects and antisymmetrisation effects are not significant at low momentum transfers ($q = 1{\cdot}3$ fm^{-1}) if we take $\psi^0_0(\mathbf{R})$ to have no modes.

The form factors $F_d(q)$ and $F_{\alpha-d}(q)$ in (8) are given by

$$F_d(q) = \int \phi_{nn}^{*}(\mathbf{x}) \exp(-i\,\mathbf{q}\cdot\mathbf{x})\, \phi_d(\mathbf{x})\, d\mathbf{x}, \tag{9}$$

$$F_{\alpha-d}(q) = \int |\psi_0^0(\mathbf{x})|^2 \exp(i\,\mathbf{q}\cdot\mathbf{X})\, d\mathbf{X}, \tag{10}$$

$\phi_{nn}(\mathbf{x})$ and $\phi_d(\mathbf{x})$ being the radial part of $\phi_0^0(\mathbf{x})$ and $\phi_j^{m_j}(\mathbf{x})$ respectively.

Evaluation of $F_d(q)$ requires a knowledge of the wave function ϕ_d for the *n-p* cluster within ^{6}Li as well as for the wave function ϕ_{nn} of the *nn* cluster within ^{6}He. The wave function ϕ_d, although it describes a system with the same quantum number as the deuterons, need not necessarily be taken as the free deuteron wave function. As a matter of fact, the commonly used gaussian wave function for the deuteron within the nucleus ^{6}Li departs greatly from the realistic free deuteron wave function. This is understandable because the free deuteron being a loosely bound system is more amenable to polarization effects than a tightly bound cluster like the α-particle. This is the main reason why in almost all the studies of ^{6}Li nucleus in the α-*d* cluster model, the deuteron wave function parameter is treated as adjustable. In what follows, we also work in the same spirit and take $\phi_d(\alpha)$ as suggested from the electron scattering experiments.

As regards the *nn* cluster wave function ϕ_{nn}, unfortunately, we do not have sufficient information as yet. Therefore in the absence of anything to the contrary it does not appear to be a bad approximation to assume the bound *nn* cluster wave function in ^{6}He to be the same as the bound *np* cluster wave-function in ^{6}Li. Under this approximation which sounds reasonable for the present study, the form factor $F_d(q)$ is found to be

$$F_d(q) = \exp(-b_d^2\, q^2/8)\cdot F_p(q), \tag{11}$$

where $F_p(q)$ is the proton form factor (Raphael 1973). In order to calculate the relative form factor $F_{\alpha-d}(q)$ the $L = 0$ intercluster wave function $\psi_0^0(R)$ is taken to be

$$\psi_0^0(R) = NX^2 \exp(-2X^2/3b^2). \tag{12}$$

This form of $\psi_0^0(R)$ is specially suitable at low q^2 as the antisymmetrisation effects with this wave-function are shown to be small (Raphael 1973; Kudeyarev *et al* 1971). With this wave function $F_{\alpha-d}$ is calculated to be

$$F_{\alpha-d}(2\,p_\nu/3) = (1/15)\, e^{-y^2}(4y^4 - 20\,y^2 + 15);\; y^2 = (1/12)\, p_\nu^2\, b^2. \tag{13}$$

3. Results and discussion

Results for the partial muon capture rate is calculated from (8), (11) and (13) for $E_\nu = 100{\cdot}7$ MeV and $q^2 = 1{\cdot}072 \times 10^{-2}$ GeV^2 applicable to the muon capture in ^{6}Li. The following values have been used for the various form factor used in (4)

$$g_{1,p}(q^2) = \frac{g_{1,p}(0)}{(1 + q^2/M_A^2)^2} \qquad \text{and} \qquad f_{1,2}(q^2) = \frac{f_{1,2}(0)}{(1 + q^2/M_v^2)^2}$$

with $g_1(0) = 1{\cdot}23$; $g_p(0) = 7g_1(0)$; $f_1(0)+f_2(0) = 4{\cdot}706$, $M_A = 0{\cdot}95$ GeV and $M_v = 0{\cdot}92$ GeV. The results for the capture rate are shown in figure 1 for various values of b_d^2 and b^2 suggested by electron scattering and pion photoproduction experiments (Singh and Ahmad 1977). In order to compare our results with other calculations in literature we have also shown in this figure the muon capture rates calculated by various authors. It is clear that the results obtained in this model are better than the results in all other models when compared with the experimental value (Deutsch *et al* 1968). For fixed b^2, a lower value of b_d^2 (*i.e* $b_d^2/b_0^2 < 1$) as suggested by inelastic electron scattering experiments on ^{6}Li gives even better results. If we compare these calculations with our earliei calculation (Singh and Ahmad 1977) of threshold pion photoproduction which also measures the matrix elements of the isovector axial current between the same nuclear states but in a different kinematic region, then we find that in muon capture the matrix element is higher than the corresponding results in the shell model while in the threshold pion photoproduction, the matrix element in the cluster model is smaller than the corresponding results in the shell model. The difference is not much (at least for $b_d^2/b^2 \approx 1$) but it is in the right direction of better agreement with experimental results. This is because of the different dependence of the matrix element in these models. The comparison with the experimental value suggests that the ($\alpha - d$) cluster model wave function provides a better description of the ^{6}Li nucleus than the shell model with the harmonic oscillator basis. An analysis of various weak interaction processes in ^{6}Li using improved cluster model wave functions including the effects of antisymmetrisation and internucleon cluster exchanges, etc is presently under consideration.

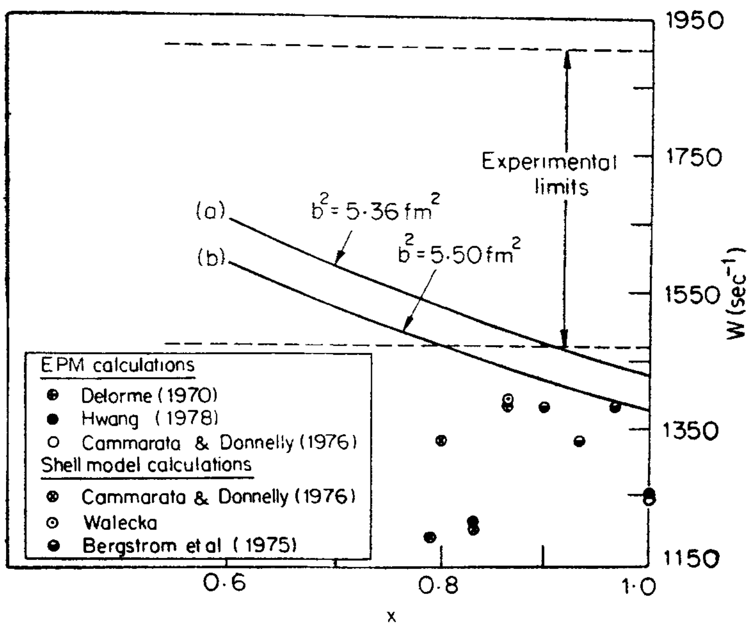

Figure 1. Muon capture rates calculated in various models.

References

Ahmad I and Khan Z A 1979 *Phys. Scr.* **20** 26
Aslanides E *et al* 1975 *Progress Report of the nuclear Physics department*, CEN Saclay, CEA-N 1861, 198
Bergstrom J C *et al* 1975 *Nucl. Phys.* **A251** 401
Bruge G 1978 Private Communication
Cammarata J B and Donnelly T W 1976 *Nucl. Phys.* **A267** 365
Cannata F *et al* 1974 *Phys. Rev. Lett.* **33** 1316
Commins E W 1973 *Weak interaction* (New York: McGraw Hill)
Delorme J 1970 *Nucl. Phys.* **B19** 573
Deutsch J P *et al* 1968 *Phys. Lett.* **B26** 315
Donnelly T W and Walecka J D 1973 *Phys. Lett.* **B44** 330
Eckhause M, Fillippan T A, Sutton R B and Welsh 1963 *Phys. Rev.* **132** 422
Ford K W and Wills J G 1962 *Nucl. Phys.* **35** 295
Garvey G T 1977 Presented at the Conference on 'Present status of weak interaction physics' in honour of the 65th birthday of E J Konopinski, Indiana University
Guichon P A M *et al* 1977 *Phys. Lett.* **B74** 15
Guichon P A M *et al* 1978 *Z. Phys.* **A285** 183
Henley E M and Hwang W P 1980 *Ann. Phys.* **129** 47
Hwang W P 1978 *Phys. Rev.* **C17** 1779
Ivanov E and Trublik 1979 *Nucl. Phys.* **A316** 451
Jaus W and Woolcock W S 1981 *Nucl. Phys.* **A365** 447
Kim C W and Mintz S L 1979 *Phys. Lett.* **318** 503
Koch J *H* and Donnelly T W 1973 *Nucl. Phys.* **B64**, 478
Koch J *H* and Donnelly T W 1974 *Phys. Rev.* **C10** 2618
Kudeyarev A *et al* 1971 *Nucl. Phys.* **A103** 316
Mukhopadhyay N C 1977 *Phys. Rep.* **C30** 1
Mukhopadhyay N C 1978 Private Communication
Neudatchin V G and Smirnov Yu. F 1964 *Prog. Nucl. Phys.* **10** 275
Noble J V 1974 *Phys. Rev.* **C9** 1209
Parthasarathy R and Waghmare Y R 1979 *Pramana* **13** 457
Porrmann M 1981 *Nucl. Phys.* **A360** 251
Primakoff H 1975 in *Muon Physics* (eds) V W Hughes and C S Wu (New York: Academic Press) Vol. 2
Primakoff H 1977 Presented at Conference on the 'Present status of weak interaction physics in honour of the 65th birthday of E J Konopinski, Indiana University
Raphael R B 1973 *Nucl. Phys.* **A201** 62
Singh S K 1972 *Nucl. Phys.* **B36** 419
Singh S K 1974 *Phys. Rev.* **D10** 833
Singh S K 1975 *Phys. Rev.* **D11** 2702
Singh S K and Ahmad I 1977 *Phys. Lett.* **B69** 422
Walecka J D 1975 in *Muon physics* (eds.) V W Hughes and C S Wu (: Academic Press) Vol. 2, p. 113
Walecka J D 1976 *Nucl. Phys.* **A258** 397
Wu C S 1977 Presented at Ben Lee Memorial International Conference on 'Parity nonconservation, weak neutral currents and gauge theories' FNAL, Batavia, Illinois