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The quadrupole plus pairing interaction model in large configuration space for W, Os and Pt nuclei

ASHOK KUMAR and M R GUNYE
Theoretical Reactor Physics Section, Bhabha Atomic Research Centre, Trombay, Bombay 400 085, India

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Abstract. The equilibrium deformations of tungsten, osmium and platinum nuclei are studied with the self-consistent quadrupole plus pairing interaction model by considering all the nucleons in nucleus explicitly. It is shown that similar results can be obtained by performing calculations with or without the assumption of an inert core. The only difference is in the strength of the quadrupole and pairing interactions to be employed in the respective calculations. The experimental static quadrupole moments and the B ($E2$) values are correctly reproduced by performing calculations with bare nucleon charge for all the nucleons.

Keywords. Equilibrium deformations; B ($E2$) values; tungsten; osmium; platinum; quadrupole plus pairing model.

1. Introduction

The equilibrium deformations of heavy nuclei in the mass region $150 \lesssim A \lesssim 190$ were studied for the first time by employing an axially symmetric Nilsson model without any residual interactions (Mottelson and Nilsson 1959) and also by incorporating residual pairing interactions (Bes and Szymanski 1961). The restriction of axially symmetric deformations was subsequently removed in later calculations (Das Gupta and Preston 1963; Gunye *et al* 1964). These generalized Nilsson model calculations led to the conclusion that all the rare-earth nuclei favour an axially symmetric prolate shape ($\gamma_0 = 0$) with the deformation magnitude $\beta_0 \simeq 0{\cdot}3$, whereas in the still heavier nuclei in the transition region ($A \sim 190$), the deformation is small ($\beta_0 \simeq 0{\cdot}15$) and could very well be axially asymmetric ($\gamma_0 \neq 0$). The equilibrium deformation of heavy nuclei in the mass region $150 \leqslant A \leqslant 200$ were also studied extensively (Kumar and Baranger 1968) with the self-consistent quadrupole plus pairing ($Q + P$) interaction model by considering only the valence nucleons outside an "inert core" of 40 protons ($N = 0$ to $N = 3$ major shells) and 70 neutrons ($N = 0$ to $N = 4$ major shells). The equilibrium deformation parameters β_0 and γ_0 obtained from the $Q + P$ model (Kumar and Baranger 1968) and the generalized Nilsson model (Das Gupta and Preston 1963; Gunye *et al* 1964) are found to be very similar over the entire mass region $150 \leqslant A \leqslant 200$. There is, however, a clear distinction between the calculations based on the Nilsson and the $Q + P$ models to obtain equilibrium deformations. In the Nilsson model, all the nucleons must be taken into account explicitly (Das Gupta and Preston 1963) while in $Q + P$ model,

one can assume an inert core and obtain correct equilibrium deformations by a proper choice of the strengths of the quadrupole and pairing interactions. The $Q + P$ model has been recently applied to investigate the properties of the high spin states of the nuclei in the rare-earth (Faessler *et al* 1974; Warke and Gunye 1975; Gunye and Warke 1979; Ashok Kumar and Gunye 1980a) and the transition (Gunye and Ashok Kumar 1980a, b; Ashok Kumar and Gunye 1980b) region. It is found from these detailed investigations that the assumption of an inert core necessitates the modification of the nucleon charges and excitation energies. The bare nucleon charges have to be replaced by the effective charges to simulate the effect of core-polarization in order to reproduce the experimental static quadrupole moments and the $B(E2)$ values. The observed energy spectra could be reproduced satisfactorily in all the nuclei only after renormalizing the calculated energy spectra by introducing a parameter to account for the moment of inertia of the core (Gunye and Warke 1979). It is thus necessary to investigate the effect of the neglected core on the equilibrium deformations, static quadrupole moments, $B(E2)$ values and the energy spectra in order to justify the renormalization prescription.

A brief report of our investigations regarding the equilibrium deformations of nuclei in the rare-earth and transition region was given recently (Ashok Kumar and Gunye 1981). In this paper, we report the results of our detailed calculations on equilibrium deformations, static quadrupole moments and $E2$ transition probabilities in the doubly even tungsten, osmium and platinum nuclei by employing $Q + P$ model in a large configuration space of first seven major shells for protons as well as for neutrons. The theoretical formulation is outlined in § 2. The results and discussion are presented in § 3 and the conclusions in § 4.

2. Theoretical formulation

The many-body Hamiltonian H for the nuclear system can be written as

$$H = \sum T_{\alpha\beta}\, a_\alpha^\dagger\, a_\beta + \tfrac{1}{4} \sum V_{\alpha\beta\gamma\delta}\, a_\alpha^\dagger\, a_\beta^\dagger\, a_\delta\, a_\gamma, \tag{1}$$

where T is the kinetic energy operator and $V_{\alpha\beta\gamma\delta}$ is the antisymmetrized matrix element of the nucleon-nucleon (NN) interaction. The spherical basis states are denoted by the Greek letters and the deformed states will be denoted by Latin letters. The subscript α in (1) denotes all the quantum numbers $(n_\alpha, l_\alpha, j_\alpha, m_\alpha)$ necessary for the specification of a single particle state $|\alpha\rangle$. The general Hartree-Fock-Bogoliubov (HFB) equations can be obtained by introducing an auxiliary Hamiltonian $H' = H - \lambda \hat{N}$ where $\hat{N}$ is the number operator and λ is the Lagrangian multiplier to conserve the number of nucleons in the system. The general HFB equations can be written as (Baranger 1961)

$$\sum (W_{\alpha\gamma}\, A_\gamma^i + \Delta_{\alpha\gamma}\, B_\gamma^i) = E_i\, A_\alpha^i,$$

$$\sum (\Delta_{\alpha\gamma}\, A_\gamma^i + W_{\alpha\gamma}\, B_\gamma^i) = -\, E_i\, B_\alpha^i, \tag{2}$$

where, A^i_γ and B^i_γ are the transformation coefficients of deformed states in terms of the spherical basis states, E_i is the quasi particle energy and

$$W_{\alpha\gamma} = T_{\alpha\gamma} + \Gamma_{\alpha\gamma} - \lambda\delta_{\alpha\gamma}, \tag{3a}$$

$$\Gamma_{\alpha\gamma} = \sum V_{\alpha\beta\gamma\delta}\, \rho_{\delta\beta}, \tag{3b}$$

$$\Delta_{\alpha\gamma} = \tfrac{1}{2} \sum V_{\alpha\gamma\beta\delta}\, \sigma_{\delta\beta}. \tag{3c}$$

The density matrices ρ and σ are given by

$$\rho_{\alpha\gamma} = \sum B^i_\alpha\, B^i_\gamma, \tag{4a}$$

$$\sigma_{\alpha\gamma} = \sum A^i_\alpha\, B^i_\gamma. \tag{4b}$$

The expectation value E^{HFB} of the Hamiltonian in (1) with respect to the variational HFB state is then given by

$$E^{\text{HFB}} = \Sigma \left\{ (T_{\alpha\beta} + \tfrac{1}{2}\, \Gamma_{\alpha\beta})\, \rho_{\beta\alpha} + \tfrac{1}{2}\, \Delta_{\alpha\beta}\, \sigma_{\beta\alpha} \right\} \tag{5}$$

The general HFB equations in large configuration space with realistic NN interactions are very involved. They can be simplified by taking recourse to simple NN interactions. We take the quadrupole (V^Q) and pairing (V^P) interactions in the form

$$V^Q(1, 2) = -\sum_{\tau\tau'\mu} \sqrt{\chi_\tau\, \chi_{\tau'}}\, q^\tau_\mu(1)\, (-)^\mu\, q^{\tau'}_{-\mu}(2) \tag{6}$$

where τ (τ') stands for neutron or proton, χ_τ ($\chi_{\tau'}$) is the strength constant and q_μ is the quadrupole operator given by

$$q_\mu = r^2\, \gamma^2_\mu. \tag{7}$$

The matrix elements of the quadrupole and pairing interactions can now be written as

$$V^Q_{\alpha\beta\gamma\delta} = -\sum_{\tau\tau'\mu} \sqrt{\chi_\tau\, \chi^{\tau'}} \left\{ \left(q^\tau_\mu\right)_{\alpha\gamma} \left(q^{\tau'}_\mu\right)_{\delta\beta} - \left(q^\tau_\mu\right)_{\alpha\delta} \left(q^{\tau'}_\mu\right)_{\gamma\beta} \right\}, \tag{8}$$

and

$$V^P_{\alpha\beta\gamma\delta} = -\sum_\tau G_\tau\, \delta_{\alpha\bar{\beta}}\, \delta_{\gamma\bar{\delta}}, \tag{9}$$

where G_τ is the strength of the pairing interaction and the state $\bar{\beta}$ is obtained from the state β by time reversal operator. To simplify HFB equations still further, Baranger and Kumar (1965, 1968) have made the following approximations of

(i) neglecting the exchange term in calculating $V^Q_{\alpha\beta\gamma\delta}$,

(ii) neglecting the contribution of $V^{Q}_{\alpha\beta\gamma\delta}$ to pairing potential $\Delta_{\alpha\beta}$ and

(iii) neglecting the contribution of $V^{P}_{\alpha\beta\gamma\delta}$ to Hartree-Fock potential $\Gamma_{\alpha\gamma}$.

The validity of these three approximations has been justified (Gunye and Das Gupta 1966) by performing the exact HFB calculations and compairing their results with those of the approximate HFB calculations. With these simplifying approximations, the HF potential $\Gamma^{\tau}_{\alpha\gamma}$ in (3) reduces to

$$\Gamma^{\tau}_{\alpha\gamma} = -(\chi_\tau)^{1/2} \sum_{\mu} (q^{\tau}_{\mu})_{\alpha\gamma} \sum_{\tau'} (\chi_{\tau'})^{1/2} Q^{\tau'}_{\mu}, \tag{10}$$

where
$$Q^{\tau}_{\mu} = \sum_{\beta\delta} (q^{\tau}_{\mu})_{\beta\delta} \rho^{\tau}_{\delta\beta}. \tag{11}$$

The relationship between χ_p and χ_n can be established by comparing the deformation potential $\Gamma_{\alpha\gamma}$ with the corresponding Nilsson model potential in the axially symmetric ($\mu = 0$) case.

$$\Gamma^{p}_{\alpha\gamma} = -(\chi_p Q^p + (\chi_p \chi_n)^{1/2} Q^n)(q^p_0)_{\alpha\gamma} = -\tfrac{1}{3} m \delta_p \omega_p^2 (q^p_0)_{\alpha\gamma},$$

and
$$\Gamma^{n}_{\alpha\gamma} = -(\chi_n \chi_p{}^{1/2} Q^p + \chi_n Q^n)(q^n_0)_{\alpha\gamma} = -\tfrac{1}{3} m \delta_n \omega_n^2 (q^n_0)_{\alpha\gamma}.$$

In order that the deformations δ_p for proton system and δ_n for neutron system be the same, one must have

$$(\chi_p/\chi_n)^{1/2} = (\omega_p/\omega_n)^2. \tag{12}$$

The oscillator frequencies ω_p and ω_n can be expressed in terms of the corresponding oscillator size parameters b_p and b_n by the relation

$$\omega_p / \omega_n = b_n^2 / b_p^2 \tag{13}$$

Moreover, it is convenient to introduce a τ-independent size parameter b as

$$b_p^2 = b^2/\theta_p \text{ and } b_n^2 = b^2/\theta_n, \tag{14}$$

where
$$b^2 = \hbar/m\omega = \tfrac{4}{5}(2/3)^{1/3} r_0^2 A^{1/3}, \tag{15}$$

$$\theta_p = (2Z/A)^{1/3};\ \theta_n = (2N/A)^{1/3}. \tag{16}$$

Hence one obtains the relations

$$(\chi_p/\chi_n)^{1/2} = (\omega_p/\omega_n)^2 = (b_n/b_p)^4 = (\theta_p/\theta_n)^2. \tag{17}$$

Introducing new variables

$$\chi = b^4 \chi_\tau / \theta_\tau^4, \tag{18}$$

and
$$\tilde{q}^{\tau} = \theta_\tau^2 q^\tau / b^2, \tag{19}$$

where χ has the dimensions of energy and $\tilde{q}$ is dimensionless operator, one can express VQ (1, 2) in (6) as

$$VQ(1,2) = -\chi \sum_{\tau\tau'\mu} \tilde{q}^{\tau}_{\mu} (-)^{\mu} \tilde{q}^{\tau'}_{-\mu}. \tag{20}$$

The HF potential $\Gamma^{\tau}_{\alpha\gamma}$ and the pairing potential $\Delta^{\tau}_{\alpha\gamma}$ in (3), after making the three approximations (Baranger and Kumar 1965) of the $Q+P$ model can now be expressed as

$$\Gamma^{\tau}_{\alpha\gamma} = -\chi \sum_{\mu} Q_{\mu} (\tilde{q}^{\tau}_{\mu})_{\alpha\gamma}, \tag{21}$$

where $$Q_{\mu} = \sum_{\tau\beta\delta} (\tilde{q}^{\tau}_{\mu})_{\delta\beta} \rho^{\tau}_{\beta\delta} \tag{22}$$

and $$\Delta^{\tau}_{\alpha\gamma} = \delta_{\gamma\bar{\alpha}} \Delta_{\tau} \tag{23}$$

where $$\Delta_{\tau} = G_{\tau} \sum_{\beta} \sigma^{\tau}_{\beta\bar{\beta}} \tag{24}$$

The HFB eqs in (2) for the $Q+P$ model can be simplified (Baranger 1961) by substituting

$$A^{i}_{\alpha} = u_i \; C^{i}_{\alpha} \text{ and } B^{i}_{\alpha} = v_i \; C^{i}_{\alpha}. \tag{25}$$

The orthonormality condition of the transformation coefficients A and B yields

$$u_i^2 + v_i^2 = 1 \text{ and } \Sigma (C^{i}_{\alpha})^2 = 1. \tag{26}$$

The HFB eqs in (2) can now be cast in the form

$$\sum_{\gamma} W^{\tau}_{\alpha\gamma} \; C^{i\tau}_{\gamma} = \eta^{\tau}_{i} \; C^{i\tau}_{\alpha}, \tag{27}$$

$$(1 - 2v^2_{i\tau}) \; \eta^{\tau}_{i} + 2u_{i\tau} \; v_{i\tau} \; \Delta_{\tau} = E^{\tau}_{i}. \tag{28}$$

The occupation probability $v^2_{i\tau}$ can be easily obtained from (28) and is given by

$$v^2_{i\tau} = \frac{1}{2}\left(1 - \frac{\eta^{\tau}_{i}}{E^{\tau}_{i}}\right),$$

$$E^{\tau}_{i} = [(\eta^{\tau}_{i})^2 + \Delta^2_{\tau}]^{1/2}. \tag{29}$$

The chemical potentials λ_τ and the pairing gaps Δ_τ are determined from the number-conservation condition $N_\tau = \sum_i v_{i\tau}^2$ and (24). Thus the equations

$$N_\tau = \tfrac{1}{2} \sum_i [1 - (\eta_i^\tau / E_i^\tau)], \tag{30}$$

and

$$G_\tau^{-1} = \tfrac{1}{4} \sum_i (E_i^\tau)^{-1} \tag{31}$$

determine λ_τ and Δ_τ necessary to compute the occupation probability in (29). The HFB energy in (5) can now be expressed as

$$E^{\text{HFB}} = \sum_{i\tau} T_{ii}^\tau v_{i\tau}^2 - \tfrac{1}{2} \chi \sum_\mu Q_\mu^2 - \sum_\tau \Delta_\tau^2 / G_\tau, \tag{32}$$

where the intrinsic quadrupole moment Q_μ obtained from (22) and (25) is given by

$$Q_\mu = \sum_{i\tau} v_{i\tau}^2 (\tilde{q}_\mu^\tau)_{ii}. \tag{33}$$

The HFB calculations in $Q + P$ model are to be carried out self-consistently to determine the chemical potentials λ_τ, pairing gaps Δ_τ and intrinsic quadrupole moment Q_μ at the minimum of energy E^{HFB}. It can be easily seen that the equilibrium deformation parameters β_0 and γ_0 of the generalized Nilsson model (Das Gupta and Preston 1963) are related to the intrinsic quadrupole moment Q_μ by the relations

$$\begin{aligned} \chi Q_0 &= \hbar \omega \beta_0 \cos \gamma_0 \\ \chi Q_2 &= \hbar \omega \beta_0 \sin \gamma_0 \end{aligned} \tag{34}$$

where $\hbar \omega$ is given by (15).

It is also necessary to investigate whether the experimental data on static quadrupole moments and $E2$ transition probabilities can be reproduced by ascribing bare nucleon charges in the present calculations performed without the assumption of an inert core. Hence the $Q(J)$ and $B(E2; J_i \to J_f)$ values are computed employing the number-conserved projected wave function Ψ^J for which the energy $\langle \Psi^J | H | \Psi^J \rangle$ is minimum (Gunye and Warke 1979).

3. Results and Discussion

The equilibrium deformations from the $Q + P$ model (Kumar and Baranger 1968) and the generalized Nilsson model (Das Gupta and Preston 1963; Gunye *et al* 1964) are found to be very similar. The former calculations in $Q + P$ model differ markedly from the latter calculations in the Nilsson model as regards the role played by the completely filled shells. Kumar and Baranger (1968) assume an inert core of

three major proton shells and four major neutron shells and obtain the equilibrium deformations by suitably fixing the strengths of the quadrupole and pairing interactions. However, the effect of the neglected core has to be properly incorporated in order to explain the experimental energy spectra and the $E2$ transition probabilities in the rare-earth and transition nuclei (Gunye and Warke 1979; Gunye and Ashok Kumar 1980 a, b; Ashok Kumar and Gunye 1980a, b). It is found that the observed static quadrupole moments $Q(J)$ and the $B(E2, J_i \to J_f)$ values can be reproduced after replacing the bare nucleon charges by the effective charges. These effective charges can be understood (Gunye and Das Gupta 1966) in terms of the renormalization of the strength of the quadrupole force so as to obtain the same equilibrium deformations with or without the assumption of an inert core. The renormalized strength χ' necessary in the equilibrium deformation calculations performed by assuming an inert core is expected to be larger than the strength χ necessary in the corresponding calculations performed without an assumption of an inert core and the effective charge is related to the ratio χ'/χ (Gunye and Das Gupta 1966). This was verified by doing calculations in a single major shell with one kind of nucleons. It is worthwhile to investigate whether the renormalization prescription is valid in a realistic case where both kinds of nucleons are present in a large configuration space of many major shells. The motivation of this paper is to show that it is indeed possible, in $Q + P$ model, to reproduce the equilibrium deformations, static quadrupole moments and $B(E2)$ values in tungsten, osmium and platinum nuclei by performing calculations in large configuration space without assuming an inert core.

The HFB calculations with $Q + P$ model reported here are carried out in a configuration space of first seven ($N = 0$ to $N = 6$) major shells for both kinds of nucleons to obtain the deformation parameters for nuclei in the transition region. The minimum of energy E^{HFB} and the corresponding deformation parameters β_0 and γ_0 are obtained by solving (27)–(34) self-consistently by an iterative procedure. The iterations are triggered by ascribing the trial values to the variational coefficients $C_\alpha^{i\tau}$ and the occupation probabilities $v_{i\tau}^2$. The single particle energies corresponding to the spherical basis states employed in the present calculations are the same as those employed by Das Gupta and Preston (1963) and by Gunye *et al* (1964). The present realistic calculations in large configuration space show that the strength of the quadrupole interaction required to obtain appropriate equilibrium deformations in the nuclei under consideration depends sensitively on the mixing caused by the quadrupole operator across the major shells. It is necessary to include the mixing across the major shells in order to investigate the renormalization effects caused by the assumption of an inert core. The HFB calculations are, however, substantially involved if one includes the mixing across the major shells since the dimensions of the matrices W in (27) increase. For the configuration space of seven major shells employed in the present calculations, the dimensions of the matrix W for both protons and neutrons are 50×50 for positive parity states and 34×34 for negative parity states.

Following Kumar and Baranger (1968) we assume the same A-dependence of the strengths of the quadrupole and pairing interactions as

$$\chi = \chi_0 A^{-1.4},$$

$$G_p = G_{po} A^{-1},$$

$$G_n = G_{no} A^{-1}.$$

By assuming an inert core of 40 protons and 70 neutrons and the configuration space of $N = 4$ and 5 shells for protons and $N = 5$ and 6 shells for neutrons Kumar and Baranger (1968) obtain reasonable values for the deformation parameters and pairing gaps Δ_p and Δ_n by employing $\chi_0 = 70$ MeV, $G_{po} = 27$ MeV and $G_{no} = 22$ MeV. The values of these three constants χ_0, G_{po} and G_{no} have to be determined in order to get similar results from our variational calculations in the large configuration space. We have performed a series of self-consistent calculations to obtain the equilibrium deformation parameters and pairing gaps Δ_p and Δ_n by varying the strengths χ_o, G_{po} and G_{no}. We find that the equilibrium deformation β_0 and both the pairing gaps Δ_p and Δ_n are very sensitively dependent on the strength χ_0 of the quadrupole interaction. The variation of β_0, Δ_p and Δ_n with χ_0 for some typical nuclei ^{176}W, ^{184}Os and ^{192}Pt is shown in figure 1. The results displayed in figure 1 are obtained by varying χ_0 and keeping G_{po} and G_{no} constant. For the same nuclei, the variation of β_0 and Δ_p with G_{po} by keeping χ_0 and G_{no} fixed is shown in figure 2. A similar variation of β_0 and Δ_n with G_{no} by keeping χ_0 and G_{po} fixed is plotted in figure 3. It should be mentioned here that the pairing gap Δ_n (Δ_p) remains almost unaltered by the variation of G_{po} (G_{no}) for a fixed value of χ_0 and hence we have not plotted these variations in figures 2 and 3. By scrutinizing the variations shown in figures 1, 2 and 3, we find that one has to employ $\chi_0 = 37{\cdot}5$ MeV, $G_{po} = 18{\cdot}5$ MeV and $G_{no} = 17{\cdot}0$ MeV to obtain results similar to those of Kumar and Baranger (1968) for the nuclei under investigation.

It is thus possible, within the framework of the $Q + P$ model, to arrive at similar results on equilibrium deformations of tungsten, osmium and platinum nuclei by performing calculations with or without the assumption of an inert core. The only difference is in the strengths of the quadrupole and pairing interactions to be employed

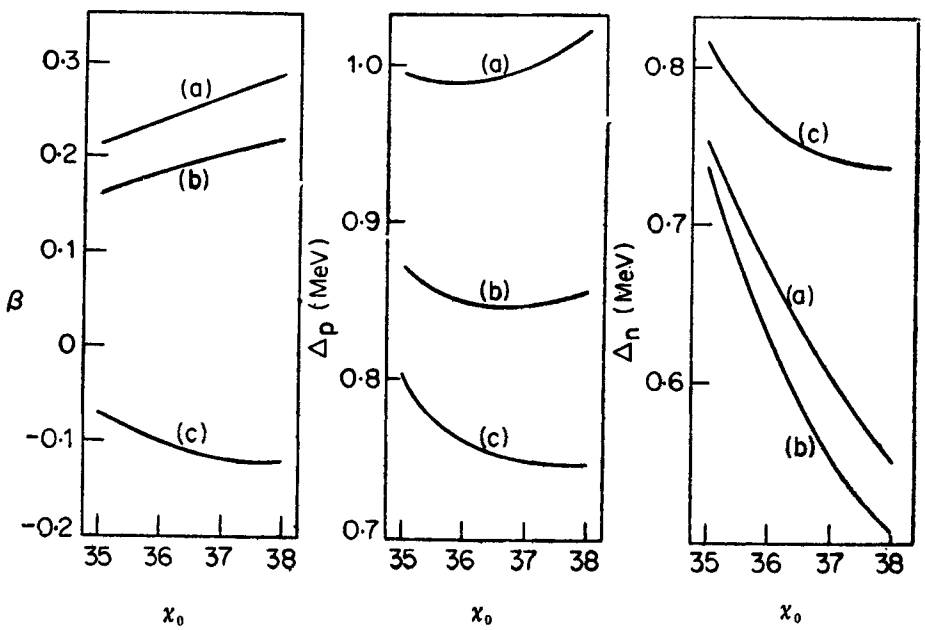

Figure 1. The variations of the equilibrium deformation β_0, the pairing gap Δ_p for protons and the pairing gap Δ_n for neutrons are plotted as a function of the strength of quadrupole force χ_o keeping G_{po} and G_{no} fixed. The curves a, b and c correspond to ^{176}W, ^{184}Os and ^{192}pt nuclei respectively.

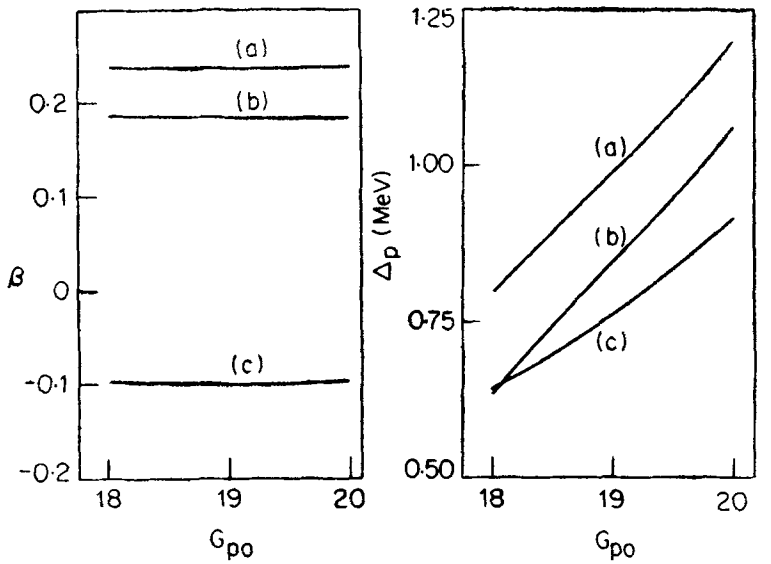

Figure 2. The variations of the equilibrium deformation β_0 and the pairing gap Δ_p for protons are plotted as a function of the strength of pairing force G_{p0} for proton keeping χ_0 and G_{n0} fixed. The curves a, b and c correspond to ^{176}W, ^{184}Os and ^{192}Pt nuclei respectively.

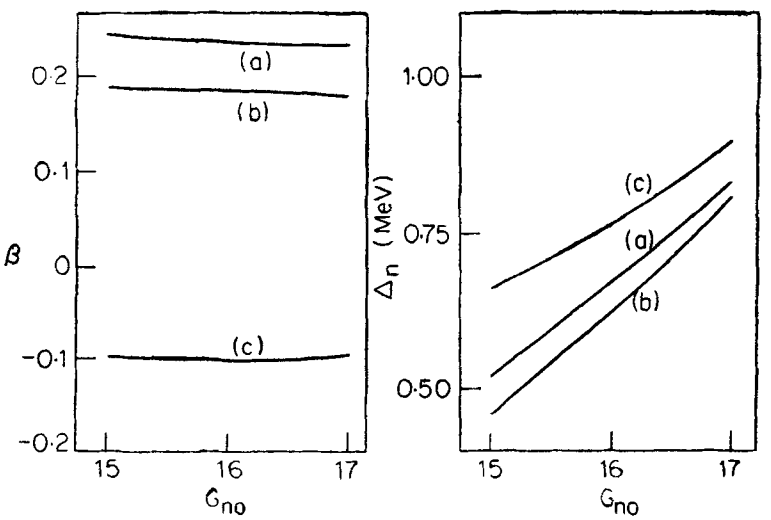

Figure 3. The variations of the equilibrium deformation β_0 and the pairing gap Δ_n for neutrons are plotted as a function of the strength of pairing force G_{n0} for neutrons keeping χ_0 and G_{p0} fixed. The curves a, b, and c correspond to ^{176}W, ^{184}Os and ^{192}Pt nuclei respectively.

in the two calculations. The results obtained by assuming an inert core can be reproduced without the assumption of an inert core after renormalizing suitably the strengths of the interactions. The assumption of an inert core necessitates larger strengths for the $Q + P$ interactions. The results on the equilibrium deformation parameters β_0 and γ_0 and the pairing gaps Δ_p and Δ_n in a few typical nuclei computed in large configuration space without assuming an inert core are given in table 1. In table 1, we have also tabulated the energy difference E_s by which the nuclei prefer

Table 1. The equilibrium deformation parameters β_0 and γ_0 (in degrees), the pairing gaps Δ_p and Δ_n, the intrinsic quadrupole moment Q and the deformation energy E_s for some typical W, Os and Pt nuclei are tabulated. These results are obtained with the values $\chi_o = 37{\cdot}5$ MeV, $G_{po} = 18{\cdot}5$ MeV and $G_{no} = 17{\cdot}0$ MeV in configuration space of $N = 0$ to 6 major shells for both protons and neutrons.

Nucleus	β_o	γ_o	Δ_p (MeV)	Δ_n (MeV)	Q (eb)	E_s (MeV)
^{172}W	0·28	0·0	0·99	0·87	6·41	8·81
^{176}W	0·27	0·0	0·91	0·75	6·16	8·15
^{180}W	0·24	0·0	0·83	0·66	5·68	6·56
^{184}W	0·19	0·0	0·81	0·79	4·72	3·98
^{182}Os	0·21	0·0	0·77	0·73	4·86	4·88
^{186}Os	0·18	0·0	0·72	0·81	4·17	2·84
^{188}Os	0·16	10·0	0·74	0·81	4·56	1·93
^{190}Os	−0·12	0·0	0·85	0·88	−2·84	1·02
^{192}Os	−0·11	0·0	0·83	0·78	−2·78	0·66
^{184}Pt	0·19	0·0	0·81	0·79	4·21	2·97
^{186}Pt	0·17	0·0	0·76	0·82	3·84	2·25
^{188}Pt	0·15	10·0	0·70	0·86	4·15	1·55
^{190}Pt	−0·12	0·0	0·70	0·93	−2·76	0·96
^{192}Pt	−0·11	0·0	0·69	0·88	−2·56	0·61

deformed shape to the spherical shape and the intrinsic quadrupole moment Q obtained from (22) by summing only over proton states. It can be seen that there is an overall agreement between our results obtained with the strengths $\chi_0 = 37{\cdot}5$ MeV, $G_{po} = 18{\cdot}5$ MeV and $G_{no} = 17{\cdot}0$ MeV and those obtained by Kumar and Baranger (1968) with the strengths $\chi_0 = 70$ MeV, $G_{po} = 27$ MeV and $G_{no} = 22$ MeV. The variation of the energy $E_s(\beta)$ with deformation β is found to be quite smooth and the typical curves for the nuclei ^{176}W and ^{192}Pt are shown in figures 4 and 5 respectively. It is seen from figure 4 that the nucleus ^{176}W prefers an axially symmetric prolate shape to the oblate shape by about 3·3 MeV. In contrast, the nucleus ^{192}Pt prefers the oblate shape to the prolate shape, though the energy difference between the two minima is quite small ($\sim 0{\cdot}3$ MeV) as seen from figure 5. The present results obtained without the assumption of an inert core are thus similar to those obtained (Kumar and Baranger 1968) with the assumption of an inert core. They are also similar in another vital aspect regarding the characteristic change from the axial to non-axial equilibrium shape for nuclei in the mass region $186 \lesssim A \lesssim 190$. This can be clearly seen from our results shown in table 1. The nuclei ^{186}Os and ^{186}Pt favour an axial prolate shape, ^{190}Os and ^{190}Pt favour an axial oblate shape whereas the ni between isotopes ^{188}Os and ^{188}Pt prefer a non-axial shape ($\gamma_0 \simeq 10°$). It should, however, be pointed out that the nuclei in this mass region $186 \lesssim A \lesssim 192$ have a characteristic γ-soft energy variation in contrast to the preceding nuclei with $A \lesssim 180$. We have illustrated this characteristic behaviour in figure 6 where the variations of energy $E_s(\beta_0, \gamma)$ with the shape parameter γ are displayed for the two representative nuclei ^{176}W and ^{188}Os for comparison. The energy difference between the $\gamma = 0$ (prolate) and $\gamma = 60°$ (oblate) shapes is quite large ($\sim 3{\cdot}5$ MeV) in ^{176}W whereas it is quite small ($\sim 0{\cdot}4$ MeV) in the case of ^{188}Os. It may also be mentioned here that, though ^{188}Os prefers a non-axial shape ($\gamma_0 = 10°$), the energy corresponding to the non-axial shape is only very slightly (~ 50 keV) lower than that corresponding

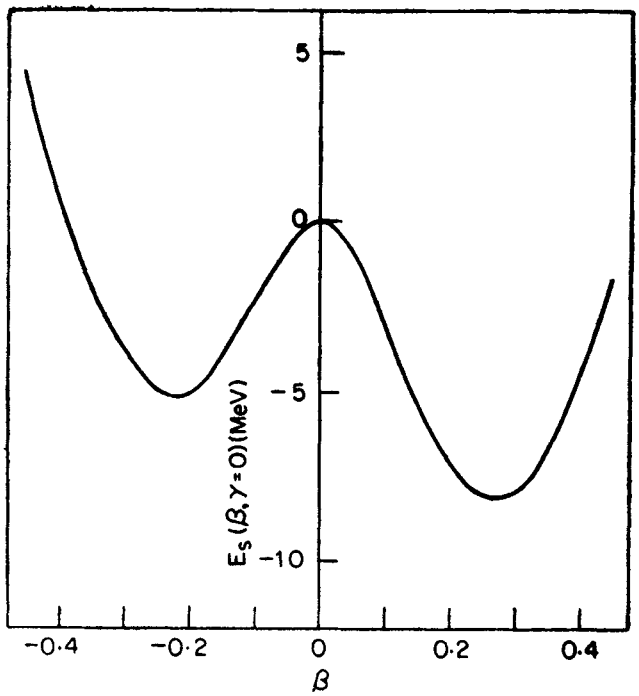

Figure 4. The deformation energy E_s is plotted against the deformation parameter β for ^{176}W nucleus.

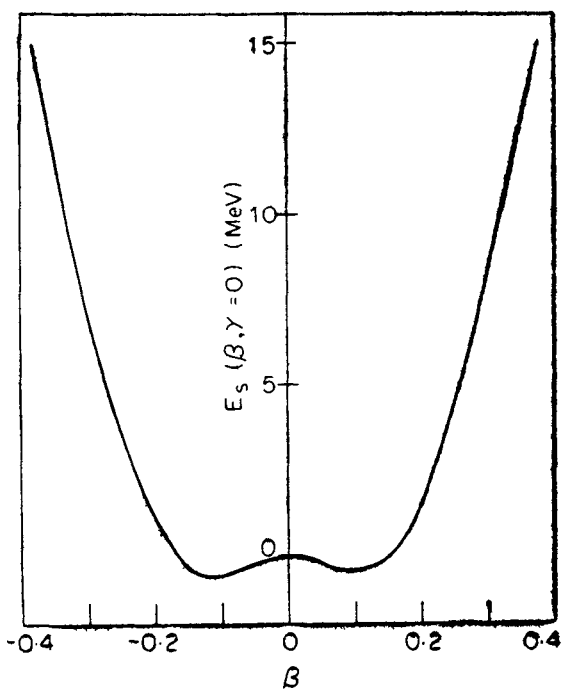

Figure 5. The deformation energy E_s is plotted against the deformation parameter β for ^{192}Pt nucleus.

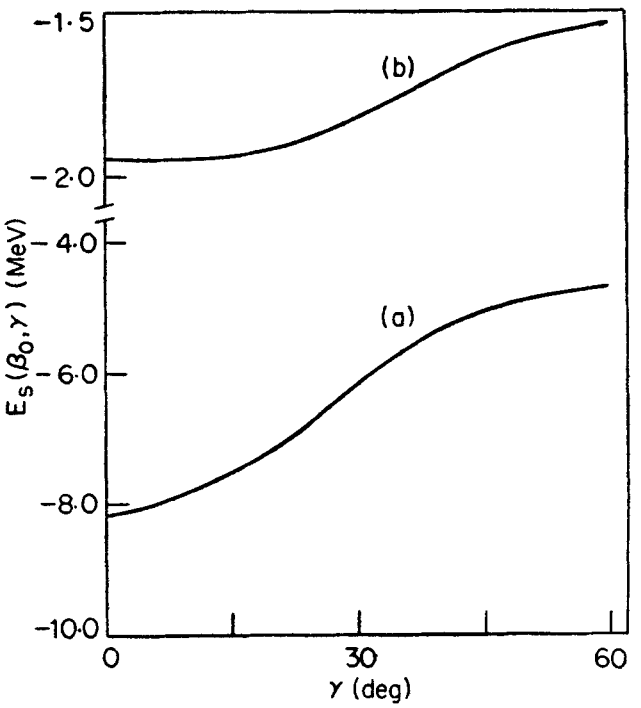

Figure 6. The deformation energy E_s at equilibrium deformation β_o is plotted against the shape parameter γ. The curves a and b correspond to ^{176}W and ^{188}Os nuclei respectively.

to the prolate shape (curve *b* in figure 6). All the nuclei in the mass region $186 \lesssim A \lesssim 192$ are thus characterized by such γ-soft potential energy surfaces.

It should be pointed out here that the earlier results (Kumar and Baranger 1968) with the assumption of an inert core are obtained by introducing an arbitrary reduction factor to decrease the influence of the uppermost shell ($N = 5$ for protons and $N = 6$ for neutrons). It may be worthwhile to examine the effect of such a reduction factor on the results of our calculations by introducing the same reduction factor $\tau = (2N + 1)/(2N + 3)$ for the uppermost shell for protons and neutrons. We find that the introduction of this reduction factor results in the increase of the value of χ_0 from 37·5 MeV to 40 MeV so as to obtain the same equilibrium deformations over the whole range of nuclei. The results obtained by employing the values (in MeV) $\chi_0 = 40$, $G_{po} = 18{\cdot}5$ and $G_{no} = 17{\cdot}0$ in the configuration space of $N = 0$ to 5 shells for protons and $N = 0$ to 6 shells for neutrons with reduction factor τ are displayed in table 2 for some typical nuclei.

The assumption of an inert core necessitates the introduction of effective nucleon charges to simulate the effect of core polarization. Our calculations are performed in large configuration space without introducing an inert core. It is thus worthwhile to investigate whether the experimental data on static quadrupole moments and electromagnetic $E2$ transitions can be reproduced by ascribing only bare nucleon charges. We have, therefore, calculated the electric quadrupole moments $Q(J)$ of the 2^+ state as well as the $B(E2; J_i \to J_f)$ values for the $E2$ transitions $2^+ \to 0^+$ and $4^+ \to 2^+$ in a number of nuclei of interest. These $Q(J)$ and $B(E2)$ values are calculated from the projected wave functions ψ^J for which the energy $\langle \psi^J | H | \psi^J \rangle$ is minimum. These

Table 2. Same as in Table 1. These results are obtained with the values $\chi_o = 40$ MeV, $G_{po} = 18{\cdot}5$ MeV and $G_{no} = 17{\cdot}0$ MeV in the configuration space of $N = 0$ to 5 major shells for protons and $N = 0$ to 6 major shells for neutrons with the reduction factor (see text) for the uppermost shell for protons and neutrons.

Nucleus	β_o	γ_o	Δ_p (MeV)	Δ_n (MeV)	Q (eb)	Es (MeV)
^{172}W	0·27	0·0	0·84	0·92	5·43	11·76
^{180}W	0·24	0·0	0·77	0·71	5·01	9·26
^{186}Os	0·18	0·0	0·70	0·83	3·83	4·46
^{188}Os	0·16	10·0	0·72	0·81	4·45	3·96
^{192}Os	−0·13	0·0	0·81	0·76	−2·97	1.52
^{184}Pt	0·19	0·0	0·70	0·84	3·66	4·66
^{188}Pt	0·16	10·0	0·68	0·84	4·20	3·10
^{192}Pt	−0·13	0·0	0·65	0·85	−2·86	1·49

Table 3. The $B(E2)$ values (in $e^2\,b^2$) for the γ-transitions $2^+\rightarrow 0^+$ and $4^+\rightarrow 2^+$ states and the static quadrupole moment $Q(J)$ (in eb) of the 2^+ state obtained by employing bare nucleon charges in a configuration space of seven major shells are displayed for a number of W, Os and Pt nuclei.

Nucleus	$B(E2;\ 2^+\rightarrow 0^+)$		$B(E2;\ 4^+\rightarrow 2^+)$		$Q(2^+)$	
	Calc.	Expt.[a]	Calc.	Expt.	Calc.	Expt.
^{182}W	0·58	0·84 ± 0·02	0·83	1·15 ± 0·08	1·55	
^{184}W	0·61	0·73 ± 0·03	0·87		1·58	
^{186}W	0·59	0·71 ± 0·04	0·85		1·56	
^{184}Os	0·50	0·64 ± 0·12	0·71		1·43	
^{188}Os	0·47	0·58 ± 0·08	0·66		1·38	
^{190}Os	0·43	0·50 ± 0·04	0·62		−1·34	
^{192}Os	0·40	0·42 ± 0·04	0·58		−1·29	
^{186}Pt	0·34	0·59 ± 0·03	0·49		1·19	
^{190}Pt	0·37	0·49 ± 0·14	0·52		−1·23	
^{192}Pt	0·34	0·34 ± 0·02	0·48	0·48 ± 0·03	−1·18	
^{194}Pt	0·34	0·33 ± 0·03	0·48	0·47 ± 0·02	−1·18	
^{196}Pt	0·28	0·30 ± 0·04	0·41		−1·09	

(a) Stelson and Grodzins (1965)

variational projection calculations (Gunye and Warke 1979) in the present case are very laborious in view of the fact that all the nucleons are taken into account explicitly in a large configuration space of seven major shells. The computed $Q(J)$ and $B(E2)$ values are listed in table 3. These values computed with bare nucleon charges are in good agreement with the experimental data wherever available. It is thus gratifying to note that in the framework of $Q + P$ model it is possible to perform calculations in large configuration space without assuming an inert core and to predict correctly the deformation parameters, static moments and $B(E2)$ values for the W, Os and Pt nuclei without resorting to any renormalization prescription for nucleon charges.

4. Conclusions

The variational HFB calculations with quadrupole plus pairing interactions are performed in a configuration space of first seven major shells for both kinds of nucleons to obtain the deformation parameters β_0 and γ_0 for the tungsten, osmium and platinum nuclei. We find that the mixing across the major shells must be taken into account in order to investigate the renormalization effects caused by the assumption of an inert core. It is seen that there is an overall agreement between the results obtained with the strengths $\chi_0 = 37{\cdot}5$ MeV, $G_{p0} = 18{\cdot}5$ MeV and $G_{n0} = 17{\cdot}0$ MeV for the interactions in the large configuration space without the assumption of an inert core and those obtained with the strengths $\chi_0 = 70$ MeV, $G_{p0} = 27$ MeV and $G_{n0} = 22$ MeV for the interactions in a truncated configuration space with the assumption of an inert core. It is thus possible, within the framework of $Q + P$ model, to arrive at similar results on equilibrium deformations of nuclei by performing calculations with and without the assumption of an inert core. The only difference is in the strengths of the quadrupole and pairing interactions to be employed in the two calculations. The static quadrupole moments and the $B(E2)$ values obtained in the present calculations in large configuration space with bare nucleon charges are in good agreement with the corresponding experimental data in the whole range of nuclei under investigation.

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