Pramāṇa, Vol. 19, No. 2, August 1982, pp. 175–182. © Printed in India.

Dibaryons as six-quark states

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MS received 2 January 1982; revised 31 May 1982

Abstract. In this paper we consider the experimentally observed dibaryons as six-quark states. The mass spectrum of *S*-wave six-quark states is investigated in the recently developed variable pressure bag model. There is very good qualitative agreement between theory and experiment.

Keywords. Dibaryons; multiquark states; variable pressure bag model.

1. Introduction

Ever since the development of the quark model which describes ordinary mesons and baryons as quark-antiquark ($q\bar{q}$) and three-quark (q^3) composites, the question of multiquark states with configurations containing additional quarks and antiquarks has been considered as a possibility. But lack of experimental evidence discouraged early quark model enthusiasts from giving serious attention to this problem. However, recent experimental observations of "baryonium" and "dibaryon" states have resulted in a revival of interest in this sector of hadron sepctroscopy.

Growing experimental evidence is available for the existence of what are described as dibaryon states as enhancements in BB channels such as NN, $N\Delta$, $N\Lambda$, $\Delta\Delta$ and so on. The conventional explanation that these might be bound states of two baryons (de Swart *et al* 1971; Kamae and Fujita 1977) is difficult to understand in the quark picture incorporating colour. Jaffe's pioneering works (Jaffe 1977a, b, c) on multiquark states in bag model have led to the exiciting possibility of considering them as colourless six-quark (q^6) states. Following Jaffe (1977a, b, c), Mulders *et al* (1980) have worked out the spectrum of orbitally-excited q^6 dibaryons in the MIT bag model (de Grand *et al* 1975, Johnson and Thorn 1976) on the assumption that the quarks form clusters sitting at the ends of an elongated bag. The present work aims at understanding the experimentally observed dibaryon resonances as colour singlet six-quark states in the variable pressure bag model (Babu Joseph and Sreedharan Nair 1981) which proved to fare better than the traditional MIT model (de Grand *et al* 1975) in the low mass sector of hadron spectroscopy where the constituent quarks are light.

2. The variable pressure bag model

This is a variant of the fixed sphere MIT bag model (de Grand *et al* 1975) which is a highly successful model for hadron spectroscopy. It differs from the original MIT model in two respects:

(i) The bag pressure term B is not a universal constant as with the original model. It is dependent on the energy density of the bag. B has been shown to be given by

$$B = \frac{\rho}{3} \tag{1}$$

where ρ represents the contribution to the energy density of the bag from sources other than the volume tension.

(ii) The bag size does not vary from hadron to hadron as in the case of the MIT model.

The phenomenology of the variable pressure bag model is developed along the same lines as the MIT model wherein the mass of a baryon is derived from four different sources: the volume energy E_v, the quark rest and kinetic energy E_q, the colour magnetic interaction energy E_m and the zero-point energy E_0,

$$M = E_v + E_q + E_m + E_0. \tag{2}$$

The kinetic energy of the quark of mass m_i moving in a spherical cavity of radius R is given by

$$\omega\,(m_i\, R) = \left[\left(\frac{x}{R}\right)^2 + m_i^2\right]^{1/2}, \tag{3}$$

with $x = x(m_i R)$ being the smallest positive root of the transcendental equation

$$\frac{x}{\tan x} = 1 - m_i\, R - (x^2 + m_i^2\, R^2)^{1/2}. \tag{4}$$

Thus $$E_q = \sum_i N_i\, \omega\,(m_i\, R), \tag{5}$$

where N_i is the number of quarks and antiquarks of the ith flavour. The colour-magnetic interaction energy for the various quark pairs in a baryon is of the form

$$E_m = \frac{-\alpha_c}{R} \sum_a \sum_{i>j} (F^a\, \sigma)_i\, (F^a\, \sigma)_j\, M_{ij}, \tag{6}$$

$$M_{ij} = \frac{1}{12}\, \mu'\,(m_i\, R)\, \mu'\,(m_j\, R)\, I\,(m_i\, R, m_j\, R) \tag{7}$$

$$\mu'\,(m\, R) = \frac{4\omega R + 2mR - 3}{2\omega R\,(\omega R - 1) + mR}. \tag{8}$$

σ_i is the spin vector and α_c is the quark-quark colour coupling constant; $I(m_i R, m_j R)$ is a slowly varying function of $m_i R$ and $m_j R$. The zero-point energy

$$E_0 = -Z/R, \tag{9}$$

where Z is assigned the phenomenologically determined value of 1·84.

The mass of a baryon excluding the volume energy is written

$$A(R) = \sum_i N_i\, \omega(m_i R) - \frac{\alpha_c}{R} \sum_a \sum_{i>j} (F^a \sigma)_i (F^a \sigma)_j M_{ij} - Z/R. \tag{10}$$

To this the volume energy

$$E_v = \frac{4}{3}\pi R^3 B = \frac{1}{3} A(R) \tag{11}$$

is added to get the total mass in the form

$$M(R) = \frac{4}{3} A(R). \tag{12}$$

The model has proved to make better predictions of the static properties of hadrons than the original MIT model. It has been possible to reproduce the mass spectrum of the light hadrons, with an exact fit with the pion mass. Also, the magnetic moments of baryons have been computed and found to be in far closer agreement with measured values than the MIT model.

3. Six-quark states in the bag

In the bag model the exotic six-quark states are constructed by populating the cavity eigen modes with quarks just the same way as the familiar $q\bar{q}$ mesons and q^3 baryons are obtained. The allowed six-quark states are determined by Pauli's exclusion principle which requires that the total wavefunction must be antisymmetric. It is also required that the colour part of the wavefunction corresponds to vanishing net colour for the hadronic state.

We assume the quarks to be in $S_{\frac{1}{2}}$ states of the bag. Since up to 18-coloured quarks can be accommodated in these states, we can regard these states as belonging to an SU (18) representation. It is the direct product of SU $(3, F)$, SU $(2, J)$ and SU $(3, C)$:

$$\begin{aligned} \mathrm{SU}(18) &\supset \mathrm{SU}(3, F) \otimes \mathrm{SU}(2, J) \otimes \mathrm{SU}(3, C) \\ &\supset \mathrm{SU}(6, FJ) \otimes \mathrm{SU}(3, C) \end{aligned} \tag{13}$$

in which F, J and C stand for flavour, spin and colour respectively. It follows that the q^6 states belong to the irreducible representation [490] of SU(6, *FJ*). The decomposition of SU(6, *FJ*) into irreducible representations in flavour and spin is

$$[490] = [1, 0] \oplus [8, 1] \oplus [8, 2] \oplus [10, 1]$$
$$\oplus [10^*, 1] \oplus [27, 0] \oplus [27, 2] \oplus [10^*, 3]$$
$$\oplus [35, 1] \oplus [28, 0]. \tag{14}$$

Table 1 contains some of the allowed q^6 states and their quantum numbers. States are designated as $Q^6(Y, I)$. The quark content of a state is expressed by specifying the number of nonstrange quarks N_n and the number of strange quarks N_s in that state.

4. Masses of six-quark states

Since the mass of a hadron varies roughly in proportion to the number of constituent quarks, multi-quark hadrons are expected to have relatively higher masses than the conventional mesons and baryons. In § 2 we considered various phenomenological contributions to the mass of a hadron in the variable pressure bag model. Denoting the sum of the quark kinetic energies E_q and the zero point energy E_0 by M_0, we write for the mass of a multiquark hadron

$$M = \frac{4}{3}\left[M_0 + \frac{\alpha_c}{R} M_{\text{av}}\, \Delta\right], \tag{15}$$

where $$M_0 = N_n\, \omega\, (m_n R) + N_s\, \omega\, (m_s R) - Z/R, \tag{16}$$

Table 1. Some of the allowed six-quark states

SU(3, *F*) multiplet	J^P	State $Q^6(Y, I)$	Quark content (N_n, N_s)
1	0^+	$Q^6(0, 0)$	(4, 2)
8	1^+	$Q^6(1, \frac{1}{2})$	(5, 1)
	2^+	$Q^6(1, \frac{1}{2})$	(5, 1)
10	1^+	$Q^6(1, 3/2)$	(5, 1)
10*	1^+	$Q^6(2, 0)$	(6, 0)
	1^+	$Q^6(1, \frac{1}{2})$	(5, 1)
27	0^+	$Q^6(2, 1)$	(6, 0)
	0^+	$Q^6(1, \frac{1}{2})$	(5, 1)
28	0^+	$Q^6(2, 3)$	(6, 0)
	0^+	$Q^6(0, 2)$	(4, 2)
	0^+	$Q^6(-1, 3/2)$	(3, 3)
	0^+	$Q^6(-4, 0)$	(0, 6)
35	1^+	$Q^5(1, 5/2)$	(5, 1)
	1^+	$Q^6(-2, 1)$	(2, 4)
	1^+	$Q^6(-3, \frac{1}{2})$	(1, 5)

and where $-\sum_a \sum_{i>j} (F^a \sigma)_i (F^a \sigma)_j M_{ij}$ has been replaced by $M_{\text{av}} \Delta$. M_{av} results from an approximation to the mass dependence of $M(m_i R, m_j R)$ introduced to render the calculations simpler (Mulders *et al* 1979):

$$M_{\text{av}} = [\tfrac{1}{2} N_n (N_n - 1) M_{nn} + \tfrac{1}{2} N_s (N_s - 1) M_{ss} + N_n N_s M_{ns}] [\tfrac{1}{2} N (N - 1)]^{-1}, \tag{17}$$

in which $N = N_n + N_s$. By the averaging procedure the flavour-dependent factor M_{ij} which determines the strength of the colour magnetic interaction has been factored out from E_m. The remaining spin- and colour-dependent group theoretic factor Δ is given by

$$\Delta = -\sum_a \sum_{i>j} (F^a \sigma)_i (F^a \sigma)_j. \tag{18}$$

Introducing a new group called colour-spin SU(6) for the description of multiquark systems, Jaffe (1977 a, b) has evaluated the expression on the rhs of (18) in terms of the Casimir invariants of spin SU(2), colour SU(3) and colourspin SU(6), for an N quark/antiquark system

$$\Delta = 8N + \tfrac{1}{2} C_6(\text{tot}) - \frac{4}{3} J_{\text{tot}} (J_{\text{tot}} + 1) + C_3(q) + \frac{8}{3} J_q (J_q + 1) - C_6(q) + C_3(\bar{q}) + \frac{8}{3} J_{\bar{q}} (J_{\bar{q}} + 1) - C_6(\bar{q}) \tag{19}$$

The lables q, $\bar{q}$ and tot refer to representations of quarks, antiquarks and the entire system respectively. Knowing the SU(3) ⊗ SU(2) content of a given representation the Casimir invariants of SU(3) and SU(6) can be evaluated. Eight λ matrices and three σ matrices generate SU(6). For states which do not contain antiquarks, the operator Δ gets simplified: it reduces to the form

$$\Delta = -\tfrac{1}{4} N(N - 10) + \tfrac{1}{3} J^2 + F_f^2 + \tfrac{1}{2} F_c^2, \tag{20}$$

in which F_c^2 is the Casimir operator for the colour SU(3) irreducible representation, F_f^2 that of flavour SU(3) and J the total spin of the multiplet. The eigen values of F_f^2 and F_c^2 are evaluated using group theoretic techniques. For colour singlet states $F_c^2 = 0$. Hence for the six-quark states the factor Δ becomes

$$\Delta = -6 + \tfrac{1}{3} J (J + 1) + F_f^2. \tag{21}$$

Values of Δ, along with allowed flavour (F) and spin (J) combinations are presented in table 2.

To estimate M_{av} and hence the colour magnetic interaction energy we need the bag size and other parameters of the model. The quark masses m_n (nonstrange), m_s

(strange) and the quark-gluon coupling constant α_c and the constant Z are the parameters of the theory. For these we choose from the phenomenological values obtained by de Grand *et al* (1975)

$$m_n = 0, m_s = 290 \text{ MeV}, \alpha_c = 0{\cdot}55, Z = 1{\cdot}84. \tag{22}$$

To fix the bag size the traditional approach is to minimise the hadron mass with respect to bag radius in the zero-quark mass limit. However, we follow here the alternative approach of adopting a parametrization based on the assumption that the bag volume is proportional to the number of constituent quarks

$$R = R_0 N^{1/3}, \tag{23}$$

with $R_0 = 3{\cdot}467 \text{ GeV}^{-1}$ chosen to reproduce the nucleon bag size in the MIT model. For the six-quark states this yields $R = 6{\cdot}3 \text{ GeV}^{-1}$. The above parameter values are used to estimate values of M_{av} for various quark configurations. These are then used to predict the masses of a number of six-quark states. The results are presented in table 3.

Table 2. Values of Δ

F	1	8	8	10	10*	27	27	28	35
J	0	1	2	1	1	0	2	0	1
F_f^2	0	3	3	6	6	8	8	18	12
Δ	−6	−7/3	−1	2/3	2/3	2	4	12	20/3

Table 3. Masses of six-quark states

State $Q^6(Y, I, J^P)$	Flavour multiplet	Mass in GeV	
		Present result	Aerts *et al*
$Q^6(2, 3, 0^+)$	28	2·45	2·79
$Q^6(2, 1, 0^+)$	27	2·25	2·24
$Q^6(2, 1, 2^+)$	27	2·28	2·36
$Q^6(2, 0, 1^+)$	10*	2·217	2·16
$Q^6(2, 0, 3^+)$	10*	2·28	2·34
$Q^6(1, 5/2, 1^+)$	35	2·566	2·69
$Q^6(1, 3/2, 1^+)$	10	2·45	2·38
$Q^6(1, 1/2, 0^+)$	27	2·48	2·38
$Q^6(1, 1/2, 1^+)$	8	2·40	2·21
$Q^6(1, 1/2, 2^+)$	8	2·39	2·29
$Q^6(0, 2, 0^+)$	27	2·705	2·62
$Q^6(0, 0, 0^+)$	1	2·558	2·20
$Q^6(0, 0, 2^+)$	27	2·656	2·43
$Q^6(-1, 3/2, 0^+)$	28	3·098	3·16
$Q^6(-2, 1, 1^+)$	35	3·238	3·06

5. Discussion

On the basis of the variable pressure bag model, we have presented an evaluation of the mass spectrum of several q^6 states which are interpreted as dibaryons. Some of these resonances have already been experimentally observed. The predicted masses of the dibaryons, regarded as q^6 states, are found to be in good agreement with the observed masses of the corresponding resonances (table 4). These identifications refer to extensive experimental study carried out in the past relating to the deuteron B^2 (1·875; $Y = 2, I = 0, J^P = 1^+$) and other resonances such as B^2 (2·17; $Y = 2, I = 1, J^P = 2^+$); B^2 (2·4; $Y = 1, J^P = 0^+$); B^2 (2·495; $Y = 1, I = 3/2$); B^2 (2·32; $Y = 1$) and B^2 (2·37; $Y = 0, I = 0$). The deuteron B^2 (1·875) is a bound state in the $3S_1 + 3D_1$ NN wave is the best known dibaryon state. The state B^2 (2·17) was first observed as an enhancement at the $N\Delta$ threshold in the cross-section of the photo disintegration of the deuteron (Keck and Tollestrup 1956). The lowest of the experimentally observed resonances (Hoshizaki 1978; Grein and Kroll 1978) in the N-N scattering is a good candidate state, B^2 (2·4). Quite recently, Shabazian *et al* (1981) have reported an enhancement in the $\Lambda P\pi$ invariant mass spectrum at a mass $2{\cdot}495 \pm 8{\cdot}7$ GeV. Existence of a dibaryon resonance in the $\Sigma^- p$ system at mass 2·32 GeV in the reaction $K^- d \to \Sigma^- \pi^\circ p$ has been reported by Misra (1979). The above states B^2 (2·495) and B^2 (2·32) with $Y = 1$ occur as resonances in the nonstrange-strange baryon scattering. The enhancement in the $\Lambda\Lambda$ invariant mass spectrum at 2·37 GeV is interpreted as a possible dibaryon resonance produced in $\Xi \cdot N$ interaction (Beilliere *et al* 1972).

The identifications we have made differ from those of Aerts *et al* (1978) who, using the conventional MIT bag model, interpreted some of the resonances in NN, ΛN and ΣN scattering as six quark states. An experimental resonance structure found around 2·38 GeV in deuteron photo disintegration has been identified by them as D (2·36; $Y = 2, I = 0, J^P = 3^+$). The state D (2·36; $Y = 2, I = 1, J^P = 2^+$) is said to correspond to the resonance with $(I, J^P) = (1, 2^+)$ in the D_2 NN wave and in $NN\pi$ at $N\Delta$ threshold of mass 2·17 GeV. Two other assignments proposed by these workers are the following: (1) D (2·24; $Y = 1, I = \frac{1}{2}, J^P = 2^+$) $-$ B^2 (2·25, $Y = 1, I = \frac{1}{2}, J^P = 2^+$) and (2) D (2·430; $Y = 0, I = 0, J^P = 2^+$) $-$ B^2 (2·36; $Y = 0, I = 0, J^P = 2^+$). However, the values obtained by them for the states herein identified (table 4) do not agree well with the experimental values in most of the cases. Our

Table 4. Dibaryons and six-quark states compared

Experimentally observed dibaryons	Predicted six-quark states $Q^6(Y, I, J^P; M$ (GeV))
B^2 (1·875)	Q^6 (2, 0, 1^+, 2·217)
B^2 (2·4)	Q^6 (2, 1, 0^+, 2·25)
B^2 (2·17)	Q^6 (2, 1, 2^+, 2·28)
B^2 (2·495)	Q^6 (1, 3/2, 1^+, 2·45)
B^2 (2·32)	Q^6 (1, 1/2, 2^+, 2·39)
B^2 (2·37)	Q^6 (0, 0, 0^+, 2·558)

values for the resonances they have identified as dibaryon states are in qualitative agreement with the data. The divergence between the two sets of predictions stems partly from the fact that these are based on different bag models. Since the variable pressure bag model has been found to be superior to the standard model in the low mass regime, it is hoped that the present predictions pertaining to dibaryon spectra will be closer to the truth. Another source of error is the various uncertainties present in the experimental data which have not yet been subjected to an unambiguous analysis. Further experimentation is called for throwing more light on this exciting area of hadron spectroscopy.

Acknowledgement

One of the authors (VGS) is thankful to CSIR, New Delhi, for the award of a fellowship.

References

Aerts A T, Mulders P J G and de Swart J J 1978 *Phys. Rev.* **D17** 260
Babu Joseph K and Sreedharan Nair M N 1981 *Pramana* **16** 49
Beilliere P *et al* 1972 *Phys. Lett.* **B39** 5, 671
de Grand T, Jaffe R L, Johnson K and Kiskis J 1975 *Phys. Rev.* **D12** 2060
de Swart J J *et al* 1971 *Springer Tracts in Modern Physics* (Springer Verlag) **60** 138
Grein W and Kroll P 1978 *Nucl. Phys.* **B137** 173
Hoshizaki N 1978 *Prog. Theor. Phys.* **60** 1796
Jaffe R L 1977a *Phys. Rev.* **D15** 267
Jaffe R L 1977b *Phys. Rev.* **D15** 281
Jaffe R L 1977c *Phys. Rev. Lett.* **38** 195
Johnson K and Thorn C B 1976 *Phys. Rev.* **D13** 1934
Kamae T and Fujita T 1977 *Phys. Rev. Lett.* **38** 468
Keck J C and Tollestrup A V 1956 *Phys. Rev.* **101** 360
Misra J N 1979 *Prog. Theor. Phys.* **62** 1788
Mulders P J, Aerts A T and de Swart J J 1979 *Phys. Rev.* **D19** 2635
Mulders P J, Aerts A T and de Swart J J 1980 *Phys. Rev.* **D21** 2653
Shahbazian B A *et al* 1981 Communication of the Joint Institute for Nuclear Research, Dubna DI-81-113