Pramāṇa, Vol. 19, No. 1, July 1982, pp. 99–102. © Printed in India.

Some comments on quark masses and baryon magnetic moments

K BABU JOSEPH and M N SREEDHARAN NAIR
Department of Physics, University of Cochin, Cochin 682 022, India

MS received 4 March 1982

Abstract. The suggestion made by Lipkin regarding the quark masses and magnetic moments of baryons is examined in the context of the variable pressure bag model. We find that the exact agreement obtained by Lipkin between theory and experiment in the case of $\mu(\Lambda)$ cannot be considered accidental, contrary to the scepticism expressed by Minami.

Keywords. Quark model; variable pressure bag model; quark masses; baryon magnetic moments.

Lipkin (1978a) has recently made a proposal regarding the quark masses in relation to hadron mass splittings which reproduced the magnetic moment of Λ. Using the mass difference $M_\Lambda - M_N$ to define the SU(3) breaking and setting this difference equal to the quark mass difference,

$$m_s - m_u = M_\Lambda - M_N, \tag{1}$$

he has predicted the magnetic moment of the strange hyperon Λ in almost exact agreement with its precisely measured value, namely,

$$\mu(\Lambda) = -0{\cdot}61 \text{ nm} \tag{2}$$

The prediction was obtained from an extension (Lipkin 1978b) of the non-relativistic constituent quark model of De Rújula *et al* (1975) using the well-known additivity assumption and assuming SU(3) and SU(6) breaking effects.

It is interesting as well as instructive to examine the consequences of the Lipkin relation (1) in the context of a relativistic colour quark model, particularly, in view of the criticism raised by Minami (1979). One might think of the phenomenological MIT bag model (De Grand *et al* 1975) as especially suited for the purpose, as it permits explicit evaluation of absolute magnitudes of static hadronic properties like magnetic moments. However, this model is known to be a miserable failure in reproducing the observed baryon magnetic moments. But a variant of this model called the variable pressure bag model recently developed by the present authors (Babu Joseph and Sreedharan Nair 1981) has proved to be an excellent tool to deal with the problem. Using this model we got our earlier prediction of $\mu(\Lambda)$ as $-0{\cdot}68$ nm (Babu Joseph and Sreedharan Nair 1981). Now we recall the suggestion made by Lipkin (1978a) as to how the magnetic moment of Λ can be reproduced by invoking relation (1).

In the bag model, instead of the bare quark mass m, one should use the effective quark mass $\omega\ (m\ R)$. Thus setting

$$\omega\ (m_s\ R) - \omega\ (m_n\ R) = M_\Lambda - M_N,$$

or $$\omega_s - \omega_n = M_\Lambda - M_N, \tag{3}$$

we find $\omega_s = 0{\cdot}472$ GeV.

This corresponds to a bare quark mass $m_s = 0{\cdot}357$ GeV. It gives $\mu_s = -0{\cdot}62$ nm instead of $-0{\cdot}68$ nm. Accordingly, $\mu(\Lambda)$ becomes $-0{\cdot}62$ nm, where the agreement with experiment is now within 2%. The above procedure for fixing m_s also brings about changes in the magnetic moment predictions for other baryons directed towards the respective experimental values. Lipkin has obtained another quark model prediction of the Λ magnetic moment in exact agreement with experiment using a different input for SU(3) breaking in quark masses, namely,

$$m_s/m_u = (M_{\Sigma^*} - M_\Sigma)/(M_\Lambda - M_N). \tag{4}$$

Before examining the implication of such a relation in the context of the bag model we would correct the above equation (Lipkin 1978a) to

$$m_u/m_s = (M_{\Sigma^*} - M_\Sigma)/(M_\Lambda - M_N). \tag{5}$$

In a colour-quark model, the mass splittings $M_{\Sigma^*} - M_\Sigma$ and $M_\Lambda - M_N$ arise from " colour magnetic " interactions of quarks. It follows that the above mass splittings are proportional to the quark magnetic moments μ_s and μ_u respectively, which in turn must be inversely proportional to the respective quark masses. Hence (5) must be the correct relation for the quark mass ratio.

The equivalent of relation (5) for the bag model is obtained by replacing m_u and m_s by the effective quark masses ω_u and ω_s:

$$\omega_u/\omega_s = (M_{\Sigma^*} - M_\Sigma)/(M_\Lambda - M_N). \tag{6}$$

Using the observed mass splittings we find $\omega_s = 0{\cdot}448$ GeV, which corresponds to a bare quark mass $m_s = 0{\cdot}328$ GeV. These inputs lead to a bag model prediction: $\mu(\Lambda) = -0{\cdot}65$ nm, which is worse than the prediction resulting from the quark mass difference relation (1). In MIT-type bag models the spin splittings are, in general, related to the colour magnetic interaction terms M_{ij} defined in De Grand *et al* (1975). In the variable pressure bag model (Babu Joseph and Sreedharan Nair 1981) we have

$$(M_{\Sigma^*} - M_\Sigma)/(M_\Lambda - M_N) = M_{ns}/M_{nn}$$

$$= \frac{\mu'\ (m_s\ R)\ \ I\ (m_n\ R,\ m_s\ R)}{\mu'\ (m_n\ R)\ \ I\ (m_n\ R,\ m_n\ R)}. \tag{7}$$

The slowly varying functions $I(m_i R, m_j R)$ appearing on the right side of (7) must be very nearly equal. Hence we may write

$$(M_{\Sigma^*} - M_{\Sigma})/(M_{\Delta} - M_N) = \mu'(m_s R)/\mu'(m_n R)$$

$$= \mu'_s/\mu'_n. \tag{8}$$

Using the bag parameters $R = 8{\cdot}88$ GeV^{-1} and $m_n = 0{\cdot}114$ GeV, which reproduce the proton magnetic moment we get $\mu'_n = 1{\cdot}0048$. This together with the hadron mass splittings gives $\mu'_s = 0{\cdot}661$, which yields for the strange quark moment a value $\mu_s = -0{\cdot}611$ nm. Thus we are led to the prediction

$$\mu(\Lambda) = -0{\cdot}61 \text{ nm},$$

in exact agreement with its precisely measured value (Schachinger *et al* 1978).

The above result provides a striking confirmation of the fact that the mass splittings $M_{\Sigma^*} - M_{\Sigma}$ and $M_{\Delta} - M_N$ arise purely as a result of the colour magnetic interactions of quarks, and the exact agreement obtained by Lipkin between theory and agreement as regards $\mu(\Lambda)$ cannot be considered as accidental contrary to the scepticism expressed by Minami (1979). The latter author used a Lipkin-type relation for the mass difference of the u and d quarks:

$$m_d - m_u = M_n - M_p, \tag{9}$$

and using quark model he found that

$$m_u \simeq 335{\cdot}8 \text{ MeV and } m_d \simeq 337{\cdot}1 \text{ MeV},$$

with $\mu(p) = 2{\cdot}793$ nm as input. This SU (2) breaking effect was then used to predict $\mu(n)$ which turned out to be $-1{\cdot}858$ nm. This is worse than the SU (3) prediction, $-1{\cdot}862$ nm. Further, he finds that the smaller the value of m_u/m_d the larger is the deviation of the calculated value of $\mu(n)$ from the experimental number.

In short, the SU(3) prediction cannot be improved upon with $m_d > m_u$, while the observed $p - n$ mass separation demands m_d to be greater than m_u. Hence it is argued that the success of the Lipkin relation for quark mass difference can perhaps be an accident. We would like to point out that the extension of the said Lipkin relation to the present context is not justified in view of the fact that the $p-n$ splitting here, is caused not by the quark mass difference alone. The energy associated with the electromagnetic field generated by the quarks should definitely have a non-negligible effect. The situation is however different in the $\Lambda - N$ case where the Lipkin relation for the quark mass difference holds. The $\Lambda-N$ mass splitting is caused almost entirely by SU(3) breaking forces that arise from the difference in u and s quark masses, as the colour magnetic interaction effects on Λ and N masses cancel each other and the electromagnetic interaction effects are negligible.

References

Babu Joseph K and Sreedharan Nair M N 1981 *Pramana* **16** 49

De Grand T, Jaffe R L, Johnson K and Kiskis J 1975 *Phys. Rev.* **D12** 2060

De Rújula A, Georgi H and Glashow S L 1975 *Phys. Rev.* **D12** 147

Lipkin H J 1978a *Phys. Rev. Lett.* **41** 1629

Lipkin H J 1978b *Phys. Lett.* **B74** 399

Minami S 1979 *Prog. Theor. Phys.* **62** 1167

Schachinger L *et al* 1978 *Phys. Rev. Lett.* **41** 1348