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General relations among observables in a large class of neutral-current processes mediated by one, two, or three Z bosons

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Abstract. Without assuming any symmetry among μ, e and τ leptons, we consider several neutral current processes involving 73 physical parameters and deduce general relations among them implied by different classes of gauge models with one, two and three neutral Z bosons. In the single-Z boson model there are 60 general relations while the two (three)-boson model leads to 48 (36) relations. If only μ-e universality is imposed, the physical parameters reduce to 41 and the single (two)-Z boson model yields 31 (22) relations, while only 13 relations exist for the three Z boson model. Imposing μ-e-τ universality decreases the number of physical parameters to 18, while the number of relations in the single- and two-Z boson models reduces to 11 and 5, respectively. In these relations are contained those obtained by Hung and Sakurai and Parida and Rajasekaran in the corresponding cases. In addition we obtain some new inequalities among the observable parameters which are to be satisfied by all gauge models.

Keywords. Neutral currents; gauge models; general relations; μ-e-τ universality.

1. Introduction

Recently there has been considerable interest in obtaining general relations among observable parameters in various neutral-current processes from general phenomological considerations not tied down to any particular gauge model (Wolfenstein 1974, 1975; Hung and Sakurai 1977, 1979; Dass and Ram Babu 1979; Parida and Rajasekaran 1979; Bajaj and Rajasekaran 1979, 1980, 1981). While all other earlier derivations were restricted to single-Z-boson hypothesis and rather less number of neutral-current sectors, a more elaborate list of relations implied by single-Z and two-Z boson hypotheses including parity violating parameters in nuclear force has been obtained by Parida and Rajasekaran (1979). In this paper† we extend this analysis to a large number of neutral-current processes involving μ, e and τ leptons, their associated neutrinos and parity violation in nucleon-nucleon interaction. We obtain general relations among the observable parameters in three different cases, (i) without assuming any symmetry among μ, e and τ (ii) with μ-e universality alone and (iii) with μ-e-τ universality. Besides possessing potentialities for testing different class of gauge models (Hung and Sakurai 1977; Dass and Ram Babu 1979; Parida and Rajasekaran 1979) these general relations also have other important

†A part of the work described in the present paper was reported at the V High Energy Physics Symposium held at Cochin (Hazra and Parida, 1980).

uses as discussed in § 4. In § 2.2 we also obtain new inequalities which are to be obeyed by all gauge models including the standard model (Weinberg 1967; Salam 1968).

2. Neutral-current processes and derivation of constraint equations

In this section we do not assume any universality among μ, e, and τ leptons and consider 28 number of different processes involving 73 physical parameters. Most of these processes were left out from earlier analyses (Wolfenstein 1974, 1975; Hung and Sakurai 1977, 1979; Dass and Ram Babu 1978; Parida and Rajasekaran 1979). Wherever possible we follow notations of Parida and Rajasekaran (1979) and conveniently modify them to save space.

2.1. *Effective Lagrangians and observables for neutral-current processes*

The observable physical parameters and the neutral-current processes in which they occur are presented by various matrix elements in table 1. For example, the process $\nu_e \nu_e \to \nu_e \nu_e$ is denoted by the matrix element (aa) and the effective Lagrangian for this process with the observable parameter L is denoted* as

$$\mathscr{L}_{\nu_e \nu_e}(L) = -\frac{G}{\sqrt{2}} L \bar{\nu}_e \gamma_\lambda (1+\gamma_5) \nu_e \bar{\nu}_e \gamma_\lambda (1+\gamma_5) \nu_e. \tag{1}$$

It may be noted that this parameter L is the analogue of C_ν^2 (Hung and Sakurai 1977, 1979). Similarly for the process $\nu_e \nu_\mu \to \nu_e \nu_\mu$ the effective Lagrangian is defined as

$$\mathscr{L}_{\nu_e \nu_\mu}(\tilde{L}) = -\frac{G}{\sqrt{2}} \tilde{L} \bar{\nu}_e \gamma_\lambda (1+\gamma_5) \nu_e \bar{\nu}_\mu \gamma_\lambda (1+\gamma_5) \nu_\mu. \tag{2}$$

The effective Lagrangians for the processes

$$\nu_e \nu_\tau \to \nu_e \nu_\tau, \quad \nu_\mu \nu_\mu \to \nu_\mu \nu_\mu, \quad \nu_\mu \nu_\tau \to \nu_\mu \nu_\tau, \quad \nu_\tau \nu_\tau \to \nu_\tau \nu_\tau$$

corresponding to the matrix elements (ac), (bb), (bc) and (cc) can be defined by

$$\mathscr{L}_{\nu_e \nu_\tau}(\tilde{L}'), \ \mathscr{L}_{\nu_\mu \nu_\mu}(L'), \ \mathscr{L}_{\nu_\mu \nu_\tau}(\tilde{L}''), \text{ and } \mathscr{L}_{\nu_\tau \nu_\tau}(L'')$$

respectively. The process $\nu_e N \to \nu_e N$ is denoted by the matrix element (a, g) with the physical parameters α, β, γ and δ occurring in the Lagrangian $L_{\nu_e N}(\alpha, \beta, \gamma, \delta)$ (Parida and Rajasekaran 1979). Similarly the processes $\nu_\mu N \to \nu_\mu N$ and $\nu_\tau N \to \nu_\tau N$ are represented by the matrix elements (bg) and (cg), respectively, and the effective

*The parameters like L, L′, $\tilde{\text{L}}$ etc. were not treated as observables by Parida and Rajasekaran (1979), but the parameter L was included in the single-Z-boson factorisation relations of Hung and Sakurai (1977). This parameter has been evaluated in a recent analysis (Hung and Sakurai 1979).

Table 1. Enumeration of neutral current processes and observable parameters.

	(a) ν_e	(b) ν_μ	(c) ν_τ	(d) $e\pm$	(e) $\mu\pm$	(f) $\tau\pm$	(g) N
(a) ν_e	L	$\tilde{L}$	$\tilde{L}'$	g_V^e g_A^e	$g_A'^e$ $g_A'^e$	$g_V''^e$ $g_A''^e$	α, β γ, δ
(b) ν_μ		L′	$\tilde{L}''$	g_V^μ g_A^μ	$g_V'^\mu$ $g_A'^\mu$	$g_V''^\mu$ $g_A''^\mu$	α', β' γ', δ'
(c) ν_τ			L″	g_V^τ g_A^τ	$g_V'^\tau$ $g_A'^\tau$	$g_V''^\tau$ $g_A''^\tau$	α'', β'' γ'', δ''
(d) $e\pm$				k_{VV} k_{AA} k_{VA}	k'_{VV}, k'_{AA} k'_{VA}, k'_{AV}	k''_{VV}, k''_{AA} k''_{VA}, k''_{AV}	$\tilde{\alpha}, \tilde{\beta}$, $\tilde{\gamma}, \tilde{\delta}$
(e) $\mu^\pm$					$\tilde{k}_{VV}$ $\tilde{k}_{AA}$ $\tilde{k}_{VA}$	$\tilde{k}'_{VV}, \tilde{k}'_{AA}$ $\tilde{k}'_{VA}, \tilde{k}'_{4V}$	$\tilde{\alpha}', \tilde{\beta}'$ $\tilde{\gamma}', \tilde{\delta}'$
(f) $\tau\pm$						$\tilde{k}''_{VV}$ $\tilde{k}''_{AA}$ $\tilde{k}''_{VA}$	$\tilde{\alpha}'', \tilde{\beta}''$ $\tilde{\gamma}'', \tilde{\delta}''$
(g) N							ξ, η ζ, ρ

Lagrangians for these processes with the physical parameters α', β', γ', δ' and α'', β'', γ'', δ'' are denoted by $\mathscr{L}_{\nu_\mu N}(\alpha', \beta', \gamma', \delta')$ and $\mathscr{L}_{\nu_\tau N}(\alpha'', \beta'', \gamma'', \delta'')$ respectively. The effective Lagrangian for the parity violation in atoms and in $e-N$ scattering corresponding to the element (dg) involves four measurable parameters, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\delta}$ and is denoted by $\mathscr{L}_{eN}(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}, \tilde{\delta})$ (Parida and Rajasekaran 1979). Similarly the effective Lagrangians with relevant physical parameters for the processes $\mu^\pm N \to \mu^\pm N$ and $\tau^\pm N \to \tau^\pm N$ corresponding to the matrix elements (eg) and (fg) are represented by $\mathscr{L}_{\mu N}(\tilde{\alpha}', \tilde{\beta}', \gamma', \tilde{\delta}')$ and $\mathscr{L}_{\tau N}(\tilde{\alpha}'', \tilde{\beta}'', \tilde{\gamma}'', \tilde{\delta}'')$ respectively. The process $\nu_e\, e \to \nu_e\, e$ corresponds to the matrix element (ad) with the physical parameters g_v^e and g_A^e and the Lagrangian is

$$\mathscr{L}_{\nu_e e}(g_V^e, g_A^e) = -\frac{G}{\sqrt{2}} \bar{\nu}_e \gamma_\lambda (1+\gamma_5) \nu_e\, \bar{e} \gamma_\lambda (g_V^e + g_A^e \gamma_5)\, e. \tag{3}$$

Similarly the processes $\nu_e\, \mu \to \nu_e\, \mu$, $\nu_e\, \tau \to \nu_e\, \tau$, $\nu_\mu\, e \to \nu_\mu\, e$, $\nu_\mu \mu \to \nu_\mu\, \mu$, $\nu_\mu\, \tau \to \nu_\mu\, \tau$, $\nu_\tau\, e \to \nu_\tau\, e$, $\nu_\tau\, \mu \to \nu_\tau\, \mu$ and $\nu_\tau\, \tau \to \nu_\tau\, \tau$ corresponding to the matrix elements *(ae)*, *(af)*, *(bd)*, *(be)*, *(bf)*, *(cd)*, *(ce)* and *(cf)* can be denoted by the effective Lagrangians

$$\mathscr{L}_{\nu_e \mu}(g_v'^{e}, g_A'^{e}),\ \mathscr{L}_{\nu_e \tau}(g_v''^{e}, g_A''^{e}),\ \mathscr{L}_{\nu_\mu e}(g_v^{\mu}, g_A^{\mu}),\ \mathscr{L}_{\nu_\mu \mu}(g_v'^{\mu}, g_A'^{\mu}),$$

$$\mathscr{L}_{\nu_\mu \tau}(g_v''^{\mu}, g_A''^{\mu}),\ \mathscr{L}_{\nu_\tau e}(g_v^{\tau}, g_A^{\tau}),\ \mathscr{L}_{\nu_\tau \mu}(g_v'^{\tau}, g_A'^{\tau}) \text{ and}$$

$$\mathscr{L}_{\nu_\tau \tau}(g_v''^{\tau}, g_A''^{\tau})$$

respectively. The processes $e^\pm e^\pm \to e^\pm e^\pm$ are represented by the matrix element *(dd)* and the effective Lagrangian involving the physical parameters k_{VV}, k_{AA} and k_{VA} is denoted by

$$\mathscr{L}_{ee}(k_{VV}, k_{VA}, k_{AA}) = -\frac{G}{\sqrt{2}}[k_{VV}\, \bar{e}\, \gamma_\lambda\, e\, \bar{e}\, \gamma_\lambda\, e +$$

$$2\, k_{VA}\, \bar{e}\, \gamma_\lambda\, e\, \bar{e}\, \gamma_\lambda\, \gamma_5\, e + k_{AA}\, \bar{e}\, \gamma_\lambda\, \gamma_5\, e\, \bar{e}\, \gamma_\lambda\, \gamma_5\, e]. \tag{4}$$

There are four physical parameters for the process $e^\pm e^\pm \to \mu^\pm \mu^\pm$ corresponding to the matrix element *(de)* with the effective Lagrangian

$$\mathscr{L}_{e\mu}(k'_{VV}, k'_{AA}, k'_{VA}, k'_{AV}) = -\frac{G}{\sqrt{2}}[k'_{VV} \bar{e}\, \gamma_\lambda\, e \bar{\mu}\, \gamma_\lambda\, \mu$$

$$+ k'_{AA} \bar{e}\, \gamma_\lambda \gamma_5\, e \bar{\mu} \gamma_\lambda\, \gamma_5\, \mu + k'_{VA} \bar{e}\, \gamma_\lambda\, e\, \bar{\mu}\, \gamma_\lambda \gamma_5\, \mu$$

$$+ k'_{AV} \bar{e}\, \gamma_\lambda\, \gamma_5\, e \bar{\mu}\, \gamma_\lambda\, \mu] \tag{5}$$

Similarly the processes $e^\pm\, e^\pm \to \tau^\pm\, \tau^\pm$, $\mu^\pm\, \mu^\pm \to \mu^\pm\, \mu^\pm$, $\mu^\pm\, \mu^\pm \to \tau^\pm\, \tau^\pm$, and $\tau^\pm \tau^\pm \to \tau^\pm \tau^\pm$ corresponding to the matrix elements *(df)*, *(ee)*, *(ef)* and *(ff)* are represented by the effective Lagrangians

$$\mathscr{L}_{e\tau}(k''_{VV}, k''_{AA}, k''_{VA}, k''_{AV}),\ \ \mathscr{L}_{\mu\mu}(\tilde{k}_{VV}, \tilde{k}_{AA}, \tilde{k}_{VA}),$$

$$\mathscr{L}_{\mu\tau}(\tilde{k}'_{VV}, \tilde{k}'_{AA}, \tilde{k}'_{VA}, \tilde{k}'_{AV}) \text{ and } \mathscr{L}_{\tau\tau}(\tilde{k}''_{VV}, \tilde{k}''_{AA}, \tilde{k}''_{VA})$$

respectively. We represent the effective Lagrangian for the parity violation in *NN* interaction involving the parameters ξ, η, ζ and ρ (Parida and Rajasekaran 1979) by $\mathscr{L}_{NN}(\xi, \eta, \zeta, \rho)$. Counting the number of processes as distinguished by effective Lagrangians we have 28 of them involving 73 different observable parameters.

2.2. *Derivation of the constraint equations*

The method adopted here is the same as that of Parida and Rajasekaran (1979). Let us suppose that the neutral-current interaction is due to a number of current $\times$ current terms added together. Then with the hypothesis corresponding to the existence of n neutral currents, the general form of interaction Lagrangian is

$$\mathscr{L} = -\frac{G}{\sqrt{2}} \sum_{i=1}^{n} J_\lambda^{(i)} J_\lambda^{(i)}, \tag{6}$$

where the current can be parametrized as

$$\begin{aligned} J_\lambda^{(i)} = [C_0^{(i)}\, \bar{\nu}_e\, \gamma_\lambda (1+\gamma_5)\, \nu_e + C_0'^{(i)}\, \bar{\nu}_\mu\, \gamma_\lambda (1+\gamma_5)\, \nu_\mu \\ + C_0''^{(i)}\, \bar{\nu}_\tau\, \gamma_\lambda (1+\gamma_5)\, \nu_\tau + \bar{e}\, \gamma_\lambda (C_V^{(i)} + C_A^{(i)} \gamma_5)\, e + \\ + \bar{\mu}\, \gamma_\lambda (C_V'^{(i)} + C_A'^{(i)} \gamma_5)\, \mu + \bar{\tau}\, \gamma_\lambda (C_V''^{(i)} + C_A''^{(i)} \gamma_5)\, \tau \\ + \tfrac{1}{2} C_\alpha^{(i)} (\bar{u}\, \gamma_\lambda\, u - \bar{d}\, \gamma_\lambda\, d) + \tfrac{1}{2} C_\beta^{i} (\bar{u}\, \gamma_\lambda\, \gamma_5\, u - \bar{d}\, \gamma_\lambda\, \gamma_5\, d) \\ + \tfrac{1}{2} C_\gamma^{(i)} (\bar{u}\, \gamma_\lambda\, u + \bar{d}\, \gamma_\lambda\, d) + \tfrac{1}{2} C_\delta^{(i)} (\bar{u}\, \gamma_\lambda\, \gamma_5\, u + \bar{d}\, \gamma_\lambda\, \gamma_5\, d)], \end{aligned} \tag{7}$$

containing 13 current parameters $C_0^{(i)}$ ...., $C_\delta^{(i)}$ for each neutral current $J^{(i)}$. The 73 observable parameters can be related to the current parameters by comparing equation (6) with the effective Lagrangian for each process (Parida and Rajasekaran 1979). Denoting each current parameter by the corresponding n component vector, the constraint equations can be stated as follows,

$\nu_e\, \nu_e \to \nu_e\, \nu_e$

$$L = \mathbf{C}_0 \cdot \mathbf{C}_0, \tag{8}$$

$\nu_e\, \nu_\mu \to \nu_e\, \nu_\mu$

$$\tilde{L} = 2\mathbf{C}_0 \cdot \mathbf{C}_0', \tag{9}$$

$\nu_e \nu_\tau \to \nu_e \nu_\tau$

$$\tilde{L}' = 2\mathbf{C}_0 \cdot \mathbf{C}_0'', \tag{10}$$

$\nu_e\, e \to \nu_e\, e$

$$g_V^e = 2\mathbf{C}_0 \cdot \mathbf{C}_v, \tag{11a}$$

$$g_A^e = 2\mathbf{C}_0 \cdot \mathbf{C}_A, \tag{11b}$$

$\nu_e\, \mu \to \nu_e\, \mu$

$$g_v'^e = 2\mathbf{C}_0 \cdot \mathbf{C}_A', \tag{12a}$$

$$g_A'^e = 2\mathbf{C}_0 \cdot \mathbf{C}_A', \tag{12b}$$

$\nu_e\, \tau \to \nu_e\, \tau$

$$g_v''^e = 2\mathbf{C}_0 \cdot \mathbf{C}_v'', \tag{13a}$$

$$g_A''^e = 2\mathbf{C}_0 \cdot \mathbf{C}_A'', \tag{13b}$$

$\nu_e\, N \to \nu_e\, N$

$$\alpha = 2\mathbf{C}_0 \cdot \mathbf{C}_\alpha, \tag{14a}$$

$$\beta = 2\mathbf{C}_0 \cdot \mathbf{C}_\beta, \tag{14b}$$

$$\gamma = 2\mathbf{C}_0 \cdot \mathbf{C}_\gamma, \tag{14c}$$

$$\delta = 2\mathbf{C}_0 \cdot \mathbf{C}_\delta, \tag{14d}$$

$\nu_\mu\, \nu_\mu \to \nu_\mu\, \nu_\mu$

$$L' = \mathbf{C}_0' \cdot \mathbf{C}_0' \tag{15}$$

$\nu_\mu\, \nu_\tau \to \nu_\mu\, \nu_\tau$

$$\tilde{L}'' = 2\mathbf{C}_0' \cdot \mathbf{C}_0'', \tag{16}$$

$\nu_\mu\, e \to \nu_\mu\, e$

$$g_V^\mu = 2\mathbf{C}_0' \cdot \mathbf{C}_V, \tag{17a}$$

$$g_A^\mu = 2\mathbf{C}_0' \cdot \mathbf{C}_A, \tag{17b}$$

$\nu_\mu\, \mu \to \nu_\mu\, \mu$

$$g_V'^\mu = 2\mathbf{C}_0' \cdot \mathbf{C}_V', \tag{18a}$$

$$g_A'^\mu = 2\mathbf{C}_0' \cdot \mathbf{C}_A', \tag{18b}$$

$\nu_\mu\, \tau \to \nu_\mu\, \tau$

$$g_V''^{\mu} = 2\mathbf{C}_0' \cdot \mathbf{C}_V'', \quad (19a)$$

$$g_A''^{\mu} = 2\mathbf{C}_0' \cdot \mathbf{C}_A'' \quad (19b)$$

$\nu_\mu\, N \to \nu_\mu\, N$

$$\alpha' = 2\mathbf{C}_0' \cdot \mathbf{C}_\alpha, \quad (20a)$$

$$\beta' = 2\mathbf{C}_0 \cdot \mathbf{C}_\beta, \quad (20b)$$

$$\gamma' = 2\mathbf{C}_0' \cdot \mathbf{C}_\gamma, \quad (20c)$$

$$\delta' = 2\mathbf{C}_0' \cdot \mathbf{C}_\delta, \quad (20d)$$

$\nu_\tau\, \tau \to \nu_\tau\, \tau$

$$L'' = \mathbf{C}_0'' \cdot \mathbf{C}_0'', \quad (21)$$

$\nu_\tau\, e \to \nu_\tau\, e$

$$g_V^{\tau} = 2\mathbf{C}_0'' \cdot \mathbf{C}_V, \quad (22a)$$

$$g_A^{\tau} = 2\mathbf{C}_0'' \cdot \mathbf{C}_A, \quad (22b)$$

$\nu_\tau\, \mu \to \nu_\tau\, \mu$

$$g_V'^{\tau} = 2\mathbf{C}_0'' \cdot \mathbf{C}_V', \quad (23a)$$

$$g_A'^{\tau} = 2\mathbf{C}_0'' \cdot \mathbf{C}_A', \quad (23b)$$

$\nu_\tau\, \tau \to \nu_\tau\, \tau$

$$g_V''^{\tau} = 2\mathbf{C}_0'' \cdot \mathbf{C}_V'', \quad (24a)$$

$$g_A''^{\tau} = 2\mathbf{C}_0'' \cdot \mathbf{C}_A'', \quad (24b)$$

$\nu_\tau\, N \to \nu_\tau\, N$

$$\alpha'' = 2\mathbf{C}_0'' \cdot \mathbf{C}_\alpha, \quad (25a)$$

$$\beta'' = 2\mathbf{C}_0'' \cdot \mathbf{C}_\beta, \quad (25b)$$

$$\gamma'' = 2\mathbf{C}_0'' \cdot \mathbf{C}_\gamma \quad (25c)$$

$$\delta'' = 2\mathbf{C}_0'' \cdot \mathbf{C}_\delta'', \quad (25d)$$

$e^\pm\, e^\pm \to e^\pm\, e^\pm$

$$k_{AA} = \mathbf{C}_A \cdot \mathbf{C}_A, \quad (26a)$$

$$k_{VV} = \mathbf{C}_V \cdot \mathbf{C}_V, \quad (26b)$$

$$k_{VA} = \mathbf{C}_V \cdot \mathbf{C}_A$$

$e^\pm\, e^\pm \to \mu^\pm\, \mu^\pm$, (26c)

$$k_{VV}' = 2\mathbf{C}_V \cdot \mathbf{C}_V', \quad (27a)$$

$$k_{AA}' = 2\mathbf{C}_A \cdot \mathbf{C}_A', \quad (27b)$$

$$k_{VA}' = 2\mathbf{C}_V \cdot \mathbf{C}_A', \quad (27c)$$

$$k_{AV}' = 2\mathbf{C}_A \cdot \mathbf{C}_V' \quad (27d)$$

$e^\pm\, e^\pm \to \tau^\pm\, \tau^\pm$

$$k_{VV}'' = 2\mathbf{C}_V \cdot \mathbf{C}_V'', \quad (28a)$$

$$k_{AA}'' = 2\mathbf{C}_A \cdot \mathbf{C}_A'', \quad (28b)$$

$$k_{VA}'' = 2\mathbf{C}_V \cdot \mathbf{C}_A'', \quad (28c)$$

$$k_{AV}'' = 2\mathbf{C}_A \cdot \mathbf{C}_V'', \quad (28d)$$

$e\, N \to e\, N$

$$\tilde{\alpha} = 2\mathbf{C}_A \cdot \mathbf{C}_\alpha, \quad (29a)$$

$$\tilde{\beta} = 2\mathbf{C}_V \cdot \mathbf{C}_\beta, \quad (29b)$$

$$\tilde{\gamma} = 2\mathbf{C}_A \cdot \mathbf{C}_\gamma, \quad (29c)$$

$$\tilde{\delta} = 2\mathbf{C}_V \cdot \mathbf{C}_\delta$$

$\mu^\pm\, \mu^\pm \to \mu^\pm\, \mu^\pm$, (29d)

$$\tilde{k}_{VV} = \mathbf{C}_V' \cdot \mathbf{C}_V', \quad (30a)$$

$$\tilde{k}_{AA} = \mathbf{C}_A' \cdot \mathbf{C}_A', \quad (30b)$$

$$\tilde{k}_{VA} = \mathbf{C}_V' \cdot \mathbf{C}_A' \quad (30c)$$

$\mu^\pm\, \mu^\pm \to \tau^\pm\, \tau^\pm$

$$\tilde{k}_{VV}' = 2\mathbf{C}_V' \cdot \mathbf{C}_V'', \quad (31a)$$

$$\tilde{k}_{AA}' = 2\mathbf{C}_A' \cdot \mathbf{C}_A, \quad (31b)$$

$$\tilde{k}_{VA}' = 2\mathbf{C}_V' \cdot \mathbf{C}_A'', \quad (31c)$$

$$\tilde{k}_{AV}' = 2\mathbf{C}_A' \cdot \mathbf{C}_V'' \quad (31d)$$

$\mu\, N \to \mu\, N$

$$\tilde{\alpha}' = 2\mathbf{C}_A' \cdot \mathbf{C}_\alpha, \quad (32a)$$

$$\tilde{\beta}' = 2\mathbf{C}_V' \cdot \mathbf{C}_\beta, \quad (32b)$$

$$\tilde{\gamma}' = 2\mathbf{C}_A' \cdot \mathbf{C}_\gamma, \quad (32c)$$

$$\tilde{\delta}' = 2\mathbf{C}_V' \cdot \mathbf{C}_\delta, \quad (32d)$$

$\tau^\pm\, \tau^\pm \to \tau^\pm\, \tau^\pm$

$$\tilde{k}_{VV}'' = \mathbf{C}_V'' \cdot \mathbf{C}_V'', \quad (33a)$$

$$\tilde{k}_{AA}'' = \mathbf{C}_A'' \cdot \mathbf{C}_A'', \quad (33b)$$

$$\tilde{k}_{VA}'' = \mathbf{C}_V'' \cdot \mathbf{C}_A'', \quad (33c)$$

$\tau\, N \to \tau\, N$

$$\tilde{\alpha}'' = 2\mathbf{C}_A'' \cdot \mathbf{C}_\alpha, \quad (34a)$$

$$\tilde{\beta}'' = 2\mathbf{C}_V'' \cdot \mathbf{C}_\beta, \quad (34b)$$

$$\tilde{\gamma}'' = 2\mathbf{C}_A'' \cdot \mathbf{C}_\gamma, \quad (34c)$$

$$\tilde{\delta}'' = 2\mathbf{C}_V'' \cdot \mathbf{C}_\delta, \quad (34d)$$

$NN \to NN$,

$$\xi = 2\mathbf{C}_\alpha \cdot \mathbf{C}_\beta, \quad (35a)$$

$$\eta = 2\mathbf{C}_\gamma \cdot \mathbf{C}_\delta, \quad (35b)$$

$$\zeta = 2\mathbf{C}_\alpha \cdot \mathbf{C}_\delta, \quad (35c)$$

$$\rho = 2\mathbf{C}_\beta \cdot \mathbf{C}_\gamma, \quad (35d)$$

Using Schwartz inequality in (24), (29)–(31) and (33) we get

$$\tilde{k}_{VV} \geq 0, \quad \tilde{k}_{AA} \geqslant 0, \quad \tilde{k}_{VV}\tilde{k}_{AA} \geq \tilde{k}^2_{VA}, \tag{36a}$$

$$\tilde{k}''_{VV} \geqslant 0, \quad \tilde{k}''_{AA} \geqslant 0, \quad \tilde{k}''_{VV}\tilde{k}''_{AA} \geqslant \tilde{k}''^2_{VA}, \tag{36b}$$

and the inequalities (35) of Parida and Rajasekaran (1979). Further using (21), (29), (30), (24), (30) and (33) we obtain

$$k'^2_{VV}\, k'^2_{AA} \leqslant 16\, k_{VV}\, k_{AA}\, \tilde{k}_{VV}\, \tilde{k}_{AA}, \tag{37a}$$

$$k'^2_{VA}\, k'^2_{AV} \leqslant 16\, k_{VV}\, k_{AA}\, \tilde{k}_{VV}\, \tilde{k}_{AA}, \tag{37b}$$

$$k''^2_{VV}\, k''^2_{AA} \leqslant 16\, k_{VV}\, k_{AA}\, \tilde{k}''_{VV}\, \tilde{k}''_{AA}, \tag{37c}$$

$$k''^2_{VA}\, k''^2_{AV} \leqslant 16\, k_{VV}\, k_{AA}\, \tilde{k}''_{VV}\, \tilde{k}''_{AA}, \tag{37d}$$

$$\tilde{k}'^2_{VV}\, \tilde{k}'^2_{AA} \leqslant 16\, \tilde{k}_{VV}\, \tilde{k}_{AA}\, \tilde{k}''_{VV}\, \tilde{k}''_{AA}, \tag{37e}$$

$$\tilde{k}'^2_{VA}\, \tilde{k}'^2_{AV} \leqslant 16\, \tilde{k}_{VV}\, \tilde{k}_{AA}\, \tilde{k}''_{VV}\, \tilde{k}''_{AA}. \tag{37f}$$

It may be noted that although some of the relations of the type (36) were obtained by Parida and Rajasekaran (1979), the inequalities (37a)–(37f) are completely new. All these inequalities are quite general and are valid for all classes of gauge models hypothesizing any number of neutral Z-bosons.

Using the constraint equations obtained in this section we derive general relations among physical parameters for different cases in the next three sections to follow.

3. Single-Z-boson factorisation relations

3.1 *Without any symmetry among μ, e and τ*

In the case of all single-Z-boson models there are only 13 current parameters C_0, C'_0, C''_0, C_V, C'_V, C''_V, C_A, C'_A, C''_A, C_α, C_β, C_γ, C_δ. Eliminating these from 73 eqs. (8)-(35d) yields total 60 factorisation relations. Twentyone of these relations are the same as eqs. (36a)-(37k) of Parida and Rajasekaran (1979) and the rest 39 relations are as stated below.

$$\frac{\tilde{k}_{VV}}{\tilde{k}_{VA}} = \frac{\tilde{k}_{VA}}{\tilde{k}_{AA}}, \quad (38a) \qquad \frac{\tilde{k}_{VV}}{\tilde{k}_{VA}} = \frac{g'^{\mu}_V}{g'^{\mu}_A}, \quad (38b) \qquad \xi = \frac{2\alpha'\beta'\tilde{k}_{VA}}{g'^{\mu}_V\, g'^{\mu}_A}, \quad (38c)$$

$$\eta = \frac{2\gamma'\delta'\tilde{k}_{VA}}{g'^{\mu}_V\, g'^{\mu}_A}, \quad (38d) \qquad \zeta = \frac{2\alpha'\delta'\tilde{k}_{VA}}{g'^{\mu}_V\, g'^{\mu}_A}, \quad (38e) \qquad \rho = \frac{2\gamma'\beta'\tilde{k}_{VA}}{g'^{\mu}_V\, g'^{\mu}_A}, \quad (38f)$$

$$\frac{\tilde{k}'_{VV}}{\tilde{k}'_{VA}} = \frac{\tilde{k}'_{VA}}{\tilde{k}'_{AA}}, \quad (39a) \qquad \frac{\tilde{k}'_{VV}}{\tilde{k}'_{VA}} = \frac{g_V''^{\tau}}{g_A''^{\tau}}, \quad (39b) \qquad \tilde{\alpha}'' = \frac{\alpha'' k''_{VA}}{g_V^{\tau}}, \quad (39c)$$

$$\tilde{\beta}'' = \frac{\beta'' k''_{AV}}{g_A^{\tau}}, \quad (39d) \qquad \tilde{\gamma}'' = \frac{\gamma'' k''_{VA}}{g_V^{\tau}}, \quad (39e) \qquad \tilde{\delta}'' = \frac{\delta'' k''_{AV}}{g_A^{\tau}}, \quad (39f)$$

$$\xi = \frac{2\alpha''\beta''\tilde{k}'_{VA}}{g_V''^{\tau}\, g_A''^{\tau}}, \quad (39g) \qquad \eta = \frac{2\gamma''\delta''\tilde{k}'_{VA}}{g_V''^{\tau}\, g_A''^{\tau}}, \quad (39h) \qquad \zeta = \frac{2\alpha''\delta''\tilde{k}'_{VA}}{g_V''^{\tau}\, g_A''^{\tau}}, \quad (39i)$$

$$\rho = \frac{2\gamma''\beta''\tilde{k}'_{VA}}{g_V''^{\tau}\, g_A''^{\tau}}, \quad (39j) \qquad \frac{\tilde{k}_{VA}}{\tilde{k}_{AA}} = \frac{g_V'^{e}}{g_A'^{e}}, \quad (40a) \qquad \frac{k''_{VV}}{k''_{VA}} = \frac{k''_{AV}}{k''_{AA}}, \quad (40b)$$

$$\frac{k''_{VV}}{k''_{AV}} = \frac{k'_{VV}}{k'_{AV}}, \quad (40c) \qquad \frac{k''_{VV}}{k''_{AV}} = \frac{g_V^{\tau}}{g_A^{\tau}}, \quad (40d) \qquad \frac{\tilde{k}'_{VA}}{\tilde{k}'_{AA}} = \frac{g_V''^{e}}{g_A''^{e}}, \quad (40e)$$

$$\frac{\alpha'}{g_V^{\mu}} = \frac{\alpha''}{g_V^{\tau}}, \quad (40f) \qquad \frac{\beta'}{g_V^{\mu}} = \frac{\beta''}{g_V^{\tau}}, \quad (40g) \qquad \frac{\gamma'}{g_V^{\mu}} = \frac{\gamma''}{g_V^{\tau}}, \quad (40h)$$

$$\frac{\delta'}{g_V^{\mu}} = \frac{\delta''}{g_V^{\tau}}, \quad (40i) \qquad \frac{\tilde{k}'_{VA}}{\tilde{k}'_{AA}} = \frac{g_V''^{\mu}}{g_A''^{\mu}}, \quad (41a) \qquad \frac{\tilde{k}'_{VV}}{\tilde{k}'_{VA}} = \frac{\tilde{k}'_{AV}}{\tilde{k}'_{AA}}, \quad (41b)$$

$$\frac{\tilde{k}'_{VV}}{\tilde{k}'_{AV}} = \frac{k'_{VV}}{k'_{VA}}, \quad (41c) \qquad \frac{\tilde{k}'_{VV}}{\tilde{k}'_{AV}} = \frac{g_V'^{\tau}}{g_A'^{\tau}}, \quad (41d) \qquad \frac{\alpha'}{g_V'^{\mu}} = \frac{\alpha''}{g_V'^{\tau}}, \quad (41e)$$

$$\frac{\beta'}{g_V'^{\mu}} = \frac{\beta''}{g_V'^{\tau}}, \quad (41f) \qquad \frac{\gamma'}{g_V'^{\mu}} = \frac{\gamma''}{g_V'^{\tau}}, \quad (41g) \qquad \frac{\delta'}{g_V'^{\mu}} = \frac{\delta''}{g_V'^{\tau}}, \quad (41h)$$

$$\frac{4L}{\tilde{L}} = \frac{\tilde{L}}{L'}, \quad (42a) \qquad \frac{\tilde{L}}{L'} = \frac{2g_V'^{e}}{g_V'^{\mu}}, \quad (42b) \qquad \frac{4L}{\tilde{L}'} = \frac{\tilde{L}'}{L''}, \quad (42c)$$

$$\frac{\tilde{L}'}{L''} = \frac{2g_V''^{e}}{g_V''^{\tau}}, \quad (42d) \qquad \frac{4L'}{\tilde{L}''} = \frac{\tilde{L}''}{L''}, \quad (42e) \qquad \frac{\tilde{L}''}{L''} = \frac{2g_V''^{\mu}}{g_V''^{\tau}}. \quad (42f)$$

3.2 *With μ-e universality alone*

With μ-e universality there are three constraints on the current parameters: $\mathbf{C}_0 = \mathbf{C}'_0$, $\mathbf{C}_V = \mathbf{C}'_V$, $\mathbf{C}_A = \mathbf{C}'_A$, which reduce the current parameters to 10 in number. The physical parameters are subject to 32 constraints: $2L = \tilde{L} = 2L'$, $\tilde{L}' = \tilde{L}''$, $g_V^e = g_V^{\mu} = g_V'^{e} = g_V'^{\mu}$, $g_V''^{e} = g_V''^{\mu}$, $g_A''^{e} = g_A''^{\mu}$, $g_A^{\mu} = g_A^{e} = g_A'^{e} = g_A'^{\mu}$, $g_V^{\tau} = g_V'^{\tau}$, $g_A^{\tau} = g_A'^{\tau}$, $\alpha = \alpha'$, $\beta = \beta'$, $\gamma = \gamma'$, $\delta = \delta'$, $k'_{VV} = 2k_{VV} = 2\tilde{k}_{VV}$, $k'_{AA} = 2k_{AA} = 2\tilde{k}_{AA}$, $k'_{AV} = k'_{VA} = 2k_{VA} = 2\tilde{k}_{VA}$, $k''_{VV} = \tilde{k}'_{VV}$, $k''_{AA} = \tilde{k}'_{AA}$, $k''_{VA} = \tilde{k}'_{VA}$, $k''_{AV} = \tilde{k}'_{AV}$, $\tilde{\alpha} = \tilde{\alpha}'$,

$\tilde{\beta} = \tilde{\beta}'$, $\tilde{\gamma} = \tilde{\gamma}'$ and $\tilde{\delta} = \tilde{\delta}'$ which reduce 73 physical parameters to 41. Eliminating 10 current parameters from 41 constraint equations: (11a), (11b), (26a)–(26c), (14a)–(14d), (29a)–(29d), (22a), (22b), (28a)–(28d), (25a)–(25d), (34a)–(34d), (13a), (13b), (24a), (24b), (33a)–(33c), (35a)–(35d), (8) and (21) we get 31 relations. Ten of these relations are the same as (36a)–(36j) of Parida and Rajasekaran (1979). Out of the rest 21 relations 15 are the same as (39a)–(39j), (40b), (40d), (40e), (42c) and (42d) of § 3.1 above and the remaining 6 are the following.

$$\frac{k''_{VV}}{k''_{AV}} = \frac{k_{VV}}{k_{AA}}, \quad (43a) \qquad \frac{\alpha}{g^e_V} = \frac{\alpha''}{g^\tau_V}, \quad (43b) \qquad \frac{\beta}{g^e_V} = \frac{\beta''}{g^\tau_V}, \quad (43c)$$

$$\frac{\gamma}{g^e_V} = \frac{\gamma''}{g^\tau_V}, \quad (43d) \qquad \frac{\delta}{g^e_V} = \frac{\delta''}{g^\tau_V}, \quad (43e) \qquad \frac{\tilde{L}'}{L} = \frac{2g^\tau_V}{g^e_V}, \quad (43f)$$

3.3 *With μ-e-τ universality*

With μ-e-τ universality there are three additional constraints on current parameters $\mathbf{C}'_0 = \mathbf{C}''_0$, $\mathbf{C}'_V = \mathbf{C}''_V$, $\mathbf{C}'_A = \mathbf{C}''_A$ which reduce the 13 current parameters to 7. Besides the 32 constraints of μ-e universality as mentioned in § 3.2, the physical parameters obey 23 additional constraints: $L' = L''$, $\tilde{L} = \tilde{L}''$, $g'^{\mu}_V = g''^{e}_V = g^\tau_V = g'^\tau_V$, $g'^{\mu}_A = g''^{e}_A = g^\tau_A = g'^\tau_A$, $\alpha' = \alpha''$, $\beta' = \beta''$, $\gamma' = \gamma''$, $\delta' = \delta''$, $\tilde{\alpha}' = \tilde{\alpha}''$, $\tilde{\beta}' = \tilde{\beta}''$, $\tilde{\gamma}' = \tilde{\gamma}''$, $\tilde{\delta}' = \tilde{\delta}''$, $2\tilde{k}_{VV} = 2\tilde{k}''_{VV} = k''_{VV}$, $2\tilde{k}_{AA} = 2\tilde{k}''_{AA} = k''_{AA}$, $2\tilde{k}_{VA} = 2\tilde{k}''_{VA} = k''_{VA}$ $= k''_{AV}$ which reduce the physical parameters to 18 only. Equating k_{VV}, k_{VA} and k_{VA} with h_{VV}, h_{VA} and h_{AA} (Parida and Rajasekaran 1979), respectively and eliminating 7 current parameters from 18 constraint equations, one obtains 11 factorisation relations. Ten of these are the same as the relations (14a)–(14j) of Parida and Rajasekaran (1979) and the other relation is

$$L = g^e_V g^e_A / 4h_{VA}. \quad (44)$$

The relation (44) involving the square of the $\nu\nu Z$ coupling was not obtained by Parida and Rajasekaran (1979); but a factorisation relation for this parameter has been recently obtained by Hung and Sakurai (1979) in a different form.

4. General relations for two-Z-boson models

4.1. *Without any symmetry among μ, e and τ*

Each current parameter in this case is a two-dimensional vector. But, in the constraint equations, these vectors occur as scalar products. The total number of independent scalar variables for a system of 13 two-dimensional vectors is only 25 (13 magnitudes of the vectors and 12 relative angles). Following the procedure outlined in Appendix B of the paper by Parida and Rajasekaran and eliminating these 25 variables from 73 equations in § 2.2 yields 48 general relations among the physical parameters as given below by (45a)-(56d).

$$\xi = \pm [2\,\tilde{\alpha}\,\tilde{\beta}\,L - \tilde{\alpha}\,\beta\,g_V^e - \alpha\,\tilde{\beta}\,g_A^e + 2\,\alpha\,\beta\,k_{VA}]/D_A, \tag{45a}$$

$$\eta = \pm [2\,\tilde{\gamma}\,\tilde{\delta}\,L - \tilde{\gamma}\,\delta\,g_V^e - \gamma\,\tilde{\delta}\,g_A^e + 2\,\gamma\,\delta\,k_{VA}]/D_A, \tag{45b}$$

$$\zeta = \pm [2\,\tilde{\alpha}\,\tilde{\delta}\,L - \tilde{\alpha}\,\delta\,g_V^e - \alpha\,\tilde{\delta}\,g_A^e + 2\,\alpha\,\delta\,k_{VA}]/D_A, \tag{45c}$$

$$\rho = \pm [2\,\tilde{\beta}\,\tilde{\gamma}\,L - \tilde{\gamma}\,\beta\,g_V^e - \gamma\,\tilde{\beta}\,g_A^e + 2\,\beta\,\gamma\,k_{VA}]/D_A, \tag{45d}$$

$$\xi = \pm [2\,\alpha\,\beta\,\tilde{k}''_{VA} - \tilde{\alpha}''\beta\,g_V''^e - \alpha\,\tilde{\beta}''\,g_A''^e + 2\,\tilde{\alpha}''\,\tilde{\beta}''\,L]/D_B, \tag{46a}$$

$$\eta = \pm [2\,\gamma\,\delta\,\tilde{k}''_{VA} - \tilde{\gamma}''\delta\,g_V''^e - \gamma\,\tilde{\delta}''\,g_A''^e + 2\,\tilde{\gamma}''\,\tilde{\delta}''\,L]/D_B, \tag{46b}$$

$$\zeta = \pm [2\,\alpha\,\delta\,\tilde{k}''_{VA} - \tilde{\alpha}''\,\delta\,g_V''^e - \alpha\,\tilde{\delta}''\,g_A''^e + 2\,\tilde{\alpha}''\,\tilde{\delta}''\,L]/D_B, \tag{46c}$$

$$\rho = \pm [2\,\beta\,\gamma\,\tilde{k}''_{VA} - \tilde{\beta}''\,\gamma\,g_V''^e - \beta\,\tilde{\gamma}''\,g_A''^e + 2\,\tilde{\beta}''\,\tilde{\gamma}''\,L]/D_B, \tag{46d}$$

$$\xi = \pm [2\tilde{\alpha}\,\tilde{\beta}\,L - \tilde{\alpha}\,\beta''\,g_V^T - \alpha''\,\tilde{\beta}\,g_A^T + 2\,\alpha''\,\beta''\,k_{VA}]/D_C, \tag{47a}$$

$$\eta = \pm [2\,\tilde{\gamma}\,\tilde{\delta}\,L - \tilde{\gamma}\,\delta''\,g_V^T - \gamma''\,\tilde{\delta}\,g_A^T + 2\,\gamma''\,\delta''\,k_{VA}]/D_C, \tag{47b}$$

$$\zeta = \pm [2\,\tilde{\alpha}\,\tilde{\delta}\,L - \tilde{\alpha}\,\delta''\,g_V^T - \alpha''\,\tilde{\delta}\,g_A^T + 2\alpha''\,\delta''\,k_{VA}]/D_C, \tag{47c}$$

$$\rho = \pm [2\tilde{\beta}\,\tilde{\gamma}\,L - \tilde{\gamma}\,\beta''\,g_V^T - \gamma''\,\tilde{\beta}\,g_A^T + 2\,\beta''\,\gamma''\,k_{VA}]/D_C, \tag{47d}$$

$$\xi = \pm [2\,\alpha''\,\beta''\,\tilde{k}''_{VA} - \tilde{\alpha}''\,\beta''\,g_V''^T - \alpha''\,\tilde{\beta}''\,g_A''^T + 2\,\tilde{\alpha}''\,\tilde{\beta}''\,L'']/D_d, \tag{48a}$$

$$\eta = \pm [2\gamma''\,\delta''\tilde{k}''_{VA} - \tilde{\gamma}''\,\delta''\,g_V''^T - \gamma''\,\tilde{\delta}''\,g_A''^T + 2\,\tilde{\gamma}''\,\tilde{\delta}''\,L'']/D_d, \tag{48b}$$

$$\zeta = \pm [2\,\alpha''\,\delta''\,\tilde{k}''_{VA} - \tilde{\alpha}''\,\delta''\,g_A''^T - \alpha''\,\tilde{\delta}''\,g_A''^T + 2\,\tilde{\alpha}''\,\tilde{\delta}''\,L'']/D_d, \tag{48c}$$

$$\rho = \pm [2\,\beta''\,\gamma''\,\tilde{k}''_{VA} - \tilde{\beta}''\,\gamma''\,g_A''^T - \beta''\,\tilde{\gamma}''\,g_V''^T + 2\tilde{\beta}''\,\tilde{\gamma}''\,L'']/D_d, \tag{48d}$$

$$\tilde{L}' = \frac{[2\,\tilde{k}''_{VV}\,(g_V^e\,g_V''^e\,L'' + g_V^T\,g_V''^T\,L) - k''_{VV}\,\{(g_V''^e)^2\,L'' + (g_V''^T)^2\,L\}]}{[\tilde{k}''_{VV}\,(g_V^T\,g_V''^e + g_V''^T\,g_V^e) - k''_{VV}\,g_V''^T\,g_V''^e]}, \tag{49a}$$

$$\tilde{L}' = \frac{[2\,\tilde{k}''_{AA}\,(g_A^e\,g_A''^e\,L'' + g_A^T\,g_A''^T\,L) - k''_{AA}\,\{(g_A''^e)^2\,L'' + (g_A''^T)^2\,L\}]}{[\tilde{k}''_{AA}\,(g_A^T\,g_A''^e + g_A''^T\,g_A^e) - k''_{AA}\,g_A''^T\,g_A''^e]}, \tag{49b}$$

$$\xi = \pm [2\tilde{\alpha}'\,\tilde{\beta}'\,L - \tilde{\alpha}'\,\beta'\,g_V'^e - \alpha\,\tilde{\beta}'\,g_A'^e + 2\,\alpha\,\beta\,\tilde{k}_{VA}]/D_e, \tag{50a}$$

$$\eta = \pm [2\,\tilde{\gamma}'\,\tilde{\delta}'\,L - \tilde{\gamma}'\,\delta\,g_V'^e - \gamma\,\tilde{\delta}'\,g_A'^e + 2\,\gamma\,\delta\,\tilde{k}_{VA}]/D_e, \tag{50b}$$

$$\zeta = \pm [2\,\tilde{\alpha}'\,\tilde{\delta}'\,L - \tilde{\alpha}'\,\delta\,g_V'^e - \alpha\,\tilde{\delta}'\,g_A'^e + 2\,\alpha\,\delta\,\tilde{k}_{VA}]/D_e, \tag{50c}$$

$$\rho = \pm [2\,\tilde{\beta}'\,\tilde{\gamma}'\,L - \tilde{\beta}'\,\gamma\,g_A'^e - \beta\,\tilde{\gamma}'\,g_V'^e + 2\,\beta\,\gamma\,\tilde{k}_{VA}]/D_e, \tag{50d}$$

$$\xi = \pm [2\tilde{\alpha}\,\tilde{\beta} L' - \tilde{\alpha}\,\beta' g_V^\mu - \alpha'\,\tilde{\beta} g_A^\mu + 2\,\alpha'\,\beta'\,k_{VA}]/D_f, \tag{51a}$$

$$\eta = \pm [2\tilde{\gamma}\,\tilde{\delta} L' - \tilde{\gamma}\,\delta' g_V^\mu - \gamma'\,\tilde{\delta} g_A^\mu + 2\,\gamma'\,\delta'\,k_{VA}]/D_f, \tag{51b}$$

$$\zeta = \pm [2\tilde{\alpha}\,\tilde{\delta} L' - \tilde{\alpha}\,\delta' g_V^\mu - \alpha'\,\tilde{\delta} g_A^\mu + 2\alpha'\,\delta'\,k_{VA}]/D_f, \tag{51c}$$

$$\rho = \pm [2\,\tilde{\beta}\,\tilde{\gamma}\,L' - \tilde{\beta}\,\gamma' g_A^\mu - \beta'\,\tilde{\gamma}\,g_V^\mu + 2\,\beta'\,\gamma'\,k_{VA}]/D_f, \tag{51d}$$

$$\xi = \pm [4\,L^2]\,[2\,\tilde{\alpha}\,\tilde{\beta}\,\tilde{k}_{VA} - \tilde{\alpha}\,\tilde{\beta}'\,k'_{VA} - \tilde{\alpha}'\,\tilde{\beta}\,k'_{AV} + 2\tilde{\alpha}'\,\tilde{\beta}'\,k_{VA}]/\tilde{D}', \tag{52a}$$

$$\eta = \pm\,4\,L^2\,[2\,\tilde{\gamma}\,\tilde{\delta}\,\tilde{k}_{VA} - \tilde{\gamma}\,\tilde{\delta}'\,k'_{VA} - \tilde{\gamma}'\,\tilde{\delta}\,k'_{VA} + 2\,\tilde{\gamma}'\,\tilde{\delta}'\,k_{VA}]/\tilde{D}', \tag{52b}$$

$$\zeta = \pm\,4\,L^2\,[2\,\tilde{\alpha}\,\tilde{\delta}\,\tilde{k}_{VA} - \tilde{\alpha}\,\tilde{\delta}'\,\tilde{k}'_{VA} - \tilde{\alpha}'\,\tilde{\delta}\,\tilde{k}'_{VA} + 2\,\tilde{\alpha}'\,\tilde{\delta}'\,k_{VA}]/\tilde{D}', \tag{52c}$$

$$\rho = \pm\,4\,L^2\,[2\,\tilde{\beta}\,\tilde{\gamma}\,\tilde{k}_{VA} - \tilde{\beta}'\,\tilde{\gamma}\,\tilde{k}'_{VA} - \tilde{\beta}\,\tilde{\gamma}'\,\tilde{k}'_{VA} + 2\,\tilde{\beta}'\,\tilde{\gamma}'\,k_{VA}]/\tilde{D}', \tag{52d}$$

$$\xi = \pm [2\,\tilde{\alpha}'\,\tilde{\beta}'\,L' - \alpha'\,\tilde{\beta}'\,g_A'^\mu - \tilde{\alpha}'\,\beta'\,g_V'^\mu + 2\,\alpha'\,\beta'\,\tilde{k}_{VA}]/D_g, \tag{53a}$$

$$\eta = \pm [2\,\tilde{\gamma}'\,\tilde{\delta}'\,L' - \gamma'\,\tilde{\delta}'\,g_A'^\mu - \tilde{\gamma}'\,\delta'\,g_V'^\mu + 2\,\gamma'\,\delta'\,\tilde{k}_{VA}]/D_g, \tag{53b}$$

$$\zeta = \pm [2\,\tilde{\alpha}'\,\tilde{\delta}'\,L' - \alpha'\,\tilde{\delta}'\,g_A'^\mu - \tilde{\alpha}'\,\delta'\,g_V'^\mu + 2\,\alpha'\,\delta'\,\tilde{k}_{VA}]/D_g, \tag{53c}$$

$$\rho = \pm [2\,\tilde{\beta}'\,\tilde{\gamma}'\,L' - \beta'\,\tilde{\gamma}'\,g_V'^\mu - \tilde{\beta}'\,\gamma'\,g_A'^\mu + 2\,\beta'\,\gamma'\,\tilde{k}_{VA}]/D_g, \tag{53d}$$

$$\tilde{L} = \frac{[2k_{VV}\,(g_V'^e\,g_V^e\,L' + g_V'^\mu\,g_V^\mu\,L) - k'_{VV}\,\{(g_V^\mu)^2\,L + (g_V^e)^2\,L'\}]}{[k_{VV}\,(g_V'^e\,g_V^\mu + g_V'^\mu\,g_V^e) - k'_{VV}\,g_V^\mu\,g_V^e]}, \tag{54a}$$

$$\tilde{L} = \frac{[2k_{AA}\,(g_A'^e\,g_A^e\,L' + g_A'^\mu\,g_A^\mu\,L) - k'_{AA}\,\{(g_A^\mu)^2\,L + (g_A^e)^2\,L'\}]}{[k_{AA}\,(g_A'^e\,g_A^\mu + g_A'^\mu\,g_A^e) - k'_{AA}\,g_A^\mu\,g_A^e]}, \tag{54b}$$

$$\tilde{L}'' = \frac{[2\tilde{k}_{VV}(g_V'^\tau\,g_V''^\tau\,L' + g_V'^\mu\,g_V''^\mu\,L'') - \tilde{k}'_{VV}\{(g_V'^\mu)^2\,L'' + (g_V'^\tau)^2\,L'\}]}{[\tilde{k}_{VV}\,(g_V'^\tau\,g_V''^\mu + g_V'^\mu\,g_V''^\tau) - \tilde{k}'_{VV}\,g_V'^\mu\,g_V'^\tau]}, \tag{54c}$$

$$\tilde{L}'' = \frac{[2\tilde{k}_{AA}\,(g_A'^\tau\,g_A''^\tau\,L' + g_A'^\mu\,g_A''^\mu\,L'') - \tilde{k}'_{AA}\,\{(g_A'^\mu)^2\,L'' + (g_A'^\tau)^2\,L'\}]}{[\tilde{k}_{AA}\,(g_A'^\tau\,g_A''^\mu + g_A'^\mu\,g_A''^\tau) - \tilde{k}'_{AA}\,g_A'^\mu\,g_A'^\tau]}, \tag{54d}$$

$$\tilde{L}'' = \frac{[2\tilde{k}_{VV}\,(g_A'^\mu\,g_V''^\mu\,L'' + g_A'^\tau\,g_V''^\tau\,L') - \tilde{k}'_{AV}\,\{(g_V'^\mu)^2\,L'' + (g_V'^\tau)^2\,L'\}]}{[\tilde{k}_{VV}\,(g_V''^\mu\,g_A'^\tau + g_A'^\mu\,g_V''^\tau) - k'_{AV}\,g_V'^\mu\,g_V'^\tau]}, \tag{54e}$$

$$\tilde{L}'' = \frac{[2\tilde{k}_{AA}\,(g_V'^\tau\,g_A''^\tau\,L' + g_V'^\mu\,g_A''^\mu\,L'') - \tilde{k}'_{VA}\,\{(g_A'^\mu)^2\,L'' + (g_A'^\tau)^2\,L'\}]}{[\tilde{k}_{AA}\,(g_V'^\tau\,g_A''^\mu + g_V'^\mu\,g_A''^\tau) - \tilde{k}'_{VA}\,g_A'^\mu\,g_A'^\tau]} \tag{54f}$$

$$\xi = \pm [2\,\alpha'\,\beta'\,\tilde{k}''_{VA} - \alpha''\,\beta'\,g_V''^\mu - \alpha'\,\tilde{\beta}''\,g_A''^\mu + 2\,\tilde{\alpha}''\,\tilde{\beta}''\,L']/D_h, \tag{55a}$$

$$\eta = \pm [2\,\gamma'\,\delta'\,\tilde{k}''_{VA} - \tilde{\gamma}''\,\delta'\,g_V''^\mu - \gamma'\,\tilde{\delta}''\,g_A''^\mu + 2\,\tilde{\gamma}''\,\tilde{\delta}''\,L']/D_h, \tag{55b}$$

$$\zeta = \pm\ [2\,\alpha'\,\delta'\,\tilde{k}''_{VA} - \tilde{\alpha}''\,\delta'\,g''^{\mu}_{V} - \alpha'\,\tilde{\delta}''\,g''^{\mu}_{A} + 2\,\tilde{\alpha}''\,\tilde{\delta}''\,L']/D_h, \quad (55c)$$

$$\rho = \pm\ [2\,\beta'\,\gamma'\,\tilde{k}''_{VA} - \tilde{\beta}''\,\gamma'\,g''^{\mu}_{A} - \beta'\,\tilde{\gamma}''\,g''^{\mu}_{V} + 2\,\tilde{\beta}''\,\tilde{\gamma}''\,L']/D_h, \quad (55d)$$

$$\tilde{L}' = \frac{[2\,k_{VV}\,(g^T_V\,g''^T_V\,L + g^e_V\,g''^e_V\,L'') - k''_{VV}\,\{(g^e_V)^2\,L'' + (g^T_V)^2\,L\}]}{[k_{VV}\,(g^T_V\,g''^e_V + g^e_V\,g''^T_V) - k''_{VV}\,g^e_V\,g^T_V]}, \quad (56a)$$

$$\tilde{L}' = \frac{[2\,k_{AA}\,(g^T_A\,g''^T_A\,L + g^e_A\,g''^e_A\,L'') - k''_{AA}\,\{(g^e_A)^2\,L'' + (g^T_A)^2 L\}]}{[k_{AA}\,(g^T_A\,g''^e_A + g^e_A\,g''^T_A) - k''_{AA}\,g^e_A\,g^T_A]}, \quad (56b)$$

$$\tilde{L}' = \frac{[2\,k_{VV}(g^e_A\,g''^e_V L'' + g^T_A\,g''^T_V\,L) - k''_{AV}\,\{(g^e_V)^2\,L'' + (g^T_V)^2\,L\}]}{[k_{VV}\,(g''^e_V\,g^T_A + g^e_A\,g''^T_V) - k''_{AV}\,g^e_V\,g^T_V]}, \quad (56c)$$

$$\tilde{L}' = \frac{[2\,k_{AA}\,(g^T_V\,g''^T_A\,L + g^e_V\,g''^e_A\,L'') - k''_{VA}\,\{(g^e_A)^2\,L'' + (g^T_A)^2\,L\}]}{[k_{AA}\,(g^T_V\,g''^e_A + g^e_V\,g''^T_A) - k''_{VA}\,g^e_A\,g^T_A]}, \quad (56d)$$

where

$$D_A = [4\,L\,k_{VV} - (g^e_V)^2]^{1/2}\,[4\,L\,k_{AA} - (g^e_A)^2]^{1/2} \quad (57a)$$

$$D_B = [4\,L\,\tilde{k}''_{VV} - (g''^e_V)^2]^{1/2}\,[4L^2\,\tilde{k}''_{AA} - (g''^e_A)^2]^{1/2} \quad (57b)$$

$$D_C = [4\,L''\,k_{VV} - (g^T_V)^2]^{1/2}\,[4\,L''\,k_{AA} - (g^T_A)^2]^{1/2} \quad (57c)$$

$$D_d = [4\,L''\,\tilde{k}''_{VV} - (g''^T_V)^2]^{1/2}\,[4\,L''\,\tilde{k}''_{AA} - (g''^T_A)^2]^{1/2} \quad (57d)$$

$$D_e = [4\,L\,\tilde{k}_{VV} - (g'^e_V)^2]^{1/2}\,[4\,L\,\tilde{k}_{AA} - (g'^e_A)^2]^{1/2} \quad (57e)$$

$$D_f = [4\,L'\,k_{VV} - (g^\mu_V)^2]^{1/2}\,[4\,L'\,k_{AA} - (g^\mu_A)^2]^{1/2} \quad (57f)$$

$$\tilde{D}' = [g'^e_A\,g^e_V\,R_A\,R_V - g'^e_A\,g^e_V\,R_A\,R'_V - g^e_A\,g'^e_V\,R'_A\,R_V + g^e_A\,g^e_V\,R'_A\,R'_V] \quad (57g)$$

$$R_A = [4\,L\,k_{AA} - (g^e_A)^2]^{1/2} \quad (57h)$$

$$R'_A = [4\,L\,\tilde{k}_{AA} - (g'^e_A)^2]^{1/2} \quad (57i)$$

$$R_V = [4\,L\,k_{VV} - (g^e_V)^2]^{1/2} \quad (57j)$$

$$R'_V = [4\,L\,\tilde{k}_{VV} - (g'^e_V)^2]^{1/2} \quad (57k)$$

$$D_g = [4\,L'\,\tilde{k}_{AA} - (g'^\mu_A)^2]^{1/2}\,[4\,L'\,\tilde{k}_{VV} - (g'^\mu_V)^2]^{1/2} \quad (57l)$$

4.2 *With μ-e universality*

When only μ-e universality is imposed the current parameters are 10, two-component vectors: $\mathbf{C}_0$, $\mathbf{C}''_0$, $\mathbf{C}_A$, $\mathbf{C}_V$, $\mathbf{C}''_A$, $\mathbf{C}''_V$, $\mathbf{C}_\alpha$, $\mathbf{C}_\beta$, $\mathbf{C}_\gamma$ and $\mathbf{C}_\delta$, the physical observables and so also the number of constraint equations reduce to 41 where the vectors occur as scalar products. The total number of independent parameters in this case is 19 (10 magnitudes and 9 relative angles). Eliminating these 19 parameters from 41 equations yields 22 general relations among the observables given by eqs. (46a)–(46d), (47a)–(47d), (48a)–(48d), (49a)–(49d), (50a), (50b) and (57a)–(57d).

4.3 *With μ-e-τ universality*

When μ-e-τ universality is imposed the current parameters are 7 two-dimensional vectors occurring as scalar products in 18 constraint equations for 18 observable parameters. Denoting $k_{VV} = h_{VV}$, $k_{AA} = h_{AA}$ and $k_{VA} = h_{VA}$ as before and eliminating the current parameters from the constraint equations yields 5 general relations 4 of which have been given by (15a)–(15d) in the paper of Parida and Rajasekaran (1979) and the other relation is

$$L = \frac{[(g_V^e)^2\, h_{AA} + (g_A^e)^2\, h_{VV} - 2g_V^e\, g_A^e\, h_{VA}]}{4[h_{AA}\, h_{VV} - (h_{VA})^2]}\,. \tag{58}$$

It is very important to note that although the relation (58) was obtained in the Appendix B of their paper by Parida and Rajasekaran (1979) this was not considered independent as the authors did not take the observable L as a separate physical parameter.

5. General relations for three-Z-boson models

Here each vector has three components and there are 13 such vectors which occur as scalar products in 73 constraint equations. But the independent scalar parameters in this case are 37 (13 magnitudes plus 24 relative angles). Eliminating these 37 parameters from 73 equations in the manner explained in the Appendix we have the following 36 general relations among the observables, as specified by (59a)–(63u),

$$\xi = [A_1\alpha\beta + A_2\alpha\tilde{\beta} + A_3\alpha\beta' + A_4\alpha'\beta + A_5\tilde{\alpha}\beta + A_6\alpha'\tilde{\beta} + A_7\tilde{\alpha}\tilde{\beta} + A_8\tilde{\alpha}\beta' + A_9\alpha'\beta']\, D_1, \tag{59a}$$

$$\eta = [A_1\gamma\delta + A_2\gamma\tilde{\delta} + A_3\gamma\delta' + A_4\gamma'\delta + A_5\tilde{\gamma}\delta + A_6\gamma'\tilde{\delta} + A_7\tilde{\gamma}\tilde{\delta} + A_8\tilde{\gamma}\delta' + A_9\gamma'\delta']\, D_1, \tag{59b}$$

$$\zeta = [A_1\alpha\delta + A_2\alpha\tilde{\delta} + A_3\alpha\delta' + A_4\alpha'\delta + A_5\tilde{\alpha}\delta + A_6\alpha'\tilde{\delta} + A_7\tilde{\alpha}\tilde{\delta} + A_8\tilde{\alpha}\delta' + A_9\alpha'\delta']\, D_1, \tag{59c}$$

$$\rho = [A_1\gamma\beta + A_2\gamma\tilde{\beta} + A_3\gamma\beta' + A_4\gamma'\beta + A_5\tilde{\gamma}\beta + A_6\gamma'\tilde{\beta} + A_7\tilde{\gamma}\tilde{\beta} + A_8\tilde{\gamma}\beta' + A_9\gamma'\beta']\, D_1, \tag{59d}$$

$$\xi = [Q_1\alpha'\beta' + Q_2\alpha'\beta'' + Q_3\alpha'\tilde{\beta}'' + Q_4\alpha''\beta' + Q_5\alpha''\beta'' + Q_6\alpha''\tilde{\beta}'' + Q_7\tilde{\alpha}''\beta' + Q_8\tilde{\alpha}''\beta'' + Q_9\tilde{\alpha}''\tilde{\beta}'']\, D_7, \tag{60a}$$

$$\eta = [Q_1\gamma'\delta' + Q_2\gamma'\delta'' + Q_3\gamma'\tilde{\delta}'' + Q_4\gamma''\delta' + Q_5\gamma''\delta'' + Q_6\gamma''\tilde{\delta}'' + Q_7\tilde{\gamma}''\delta' + Q_8\tilde{\gamma}''\delta'' + Q_9\tilde{\gamma}''\tilde{\delta}''] D_7, \tag{60b}$$

$$\zeta = [Q_1\alpha'\delta' + Q_2\alpha'\delta'' + Q_3\alpha'\tilde{\delta}'' + Q_4\alpha''\delta' + Q_5\alpha''\delta'' + Q_6\alpha''\tilde{\delta}'' + Q_7\tilde{\alpha}''\delta' + Q_8\tilde{\alpha}''\delta'' + Q_9\tilde{\alpha}''\tilde{\delta}''] D_7, \tag{60c}$$

$$\rho = [Q_1\gamma'\beta' + Q_2\gamma'\beta'' + Q_3\gamma'\tilde{\beta}'' + Q_4\gamma''\beta' + Q_5\gamma''\beta'' + Q_6\gamma''\tilde{\beta}'' + Q_7\tilde{\gamma}''\beta' + Q_8\tilde{\gamma}''\beta'' + Q_9\tilde{\gamma}''\tilde{\beta}''] D_7, \tag{60d}$$

$$\xi = [Y_1\alpha\beta + Y_2\alpha\tilde{\beta}' + Y_3\alpha\beta' + Y_4\alpha'\beta + Y_5\tilde{\alpha}'\beta + Y_6\alpha'\tilde{\beta}' + Y_7\tilde{\alpha}'\tilde{\beta}' + Y_8\tilde{\alpha}'\beta' + Y_9\alpha'\beta'] D_5, \tag{61a}$$

$$\eta = [Y_1\gamma\delta + Y_2\gamma\tilde{\delta}' + Y_3\gamma\delta' + Y_4\gamma'\delta + Y_5\tilde{\gamma}'\delta + Y_6\gamma'\tilde{\delta}' + Y_7\tilde{\gamma}'\tilde{\delta}' + Y_8\tilde{\gamma}'\delta' + Y_9\gamma'\delta'] D_5, \tag{61b}$$

$$\zeta = D_5 [Y_1\alpha\delta + Y_2\alpha\tilde{\delta}' + Y_3\alpha\delta' + Y_4\alpha'\delta + Y_5\tilde{\alpha}'\delta + Y_6\alpha'\tilde{\delta}' + Y_7\tilde{\alpha}'\tilde{\delta}' + Y_8\tilde{\alpha}'\delta' + Y_9\alpha'\delta'], \tag{61c}$$

$$\rho = D_5 [Y_1\gamma\beta + Y_2\gamma\tilde{\beta}' + Y_3\gamma\beta' + Y_4\gamma'\beta + Y_5\tilde{\gamma}'\beta + Y_6\gamma'\tilde{\beta}' + Y_7\tilde{\gamma}'\tilde{\beta}' + Y_8\tilde{\gamma}'\beta' + Y_9\gamma'\beta'], \tag{61d}$$

$$\xi = D_2 [B_1\alpha\beta + B_2\alpha\tilde{\beta}'' + B_3\alpha\beta'' + B_4\alpha''\beta + B_5\tilde{\alpha}''\beta + B_6\alpha''\tilde{\beta}'' + B_7\tilde{\alpha}''\tilde{\beta}'' + B_8\tilde{\alpha}''\beta'' + B_9\alpha''\beta''], \tag{62a}$$

$$\eta = D_2 [B_1\gamma\delta + B_2\gamma\tilde{\delta}'' + B_3\gamma\delta'' + B_4\gamma''\delta + B_5\tilde{\gamma}''\delta + B_6\gamma''\tilde{\delta}'' + B_7\tilde{\gamma}''\tilde{\delta}'' + B_8\tilde{\gamma}''\delta'' + B_9\gamma''\delta''], \tag{62b}$$

$$\zeta = D_2 [B_1\alpha\delta + B_2\alpha\tilde{\delta}'' + B_3\alpha\delta'' + B_4\alpha''\delta + B_5\tilde{\alpha}''\delta + B_6\alpha''\tilde{\delta}'' + B_7\tilde{\alpha}''\tilde{\delta}'' + B_8\tilde{\alpha}''\delta'' + B_9\alpha''\delta''], \tag{62c}$$

$$\rho = D_2 [B_1\gamma\beta + B_2\gamma\tilde{\beta}'' + B_3\gamma\beta'' + B_4\gamma''\beta + B_5\tilde{\gamma}''\beta + B_6\gamma''\tilde{\beta}'' + B_7\tilde{\gamma}''\tilde{\beta}'' + B_8\tilde{\gamma}''\beta'' + B_9\gamma''\beta''], \tag{62d}$$

$$k_{VV} = \frac{1}{2N}[X_1 (g_V^e)^2 + X_2 (g_V^\mu)^2 + X_3 (g_V^\tau)^2 + X_4 (2g_V^e g_V^\mu) + 2X_5 g_V^e g_V^\tau + 2X_6 g_V^\mu g_V^\tau], \tag{63a}$$

$$k_{AA} = \frac{1}{2N}[X_1 (g_A^e)^2 + X_2 (g_A^\mu)^2 + X_3 (g_A^\tau)^2 + 2X_4 g_A^e g_A^\mu + 2X_5 g_A^e g_A^\tau + 2X_6 g_A^\mu g_A^\tau], \tag{63b}$$

$$k_{VA} = \frac{1}{2N} [X_1 g_V^e g_A^e + X_2 g_V^\mu g_A^\mu + X_3 g_V^\tau g_A^\tau + X_4 (g_V^e g_A^\mu + g_V^\mu g_A^e) + X_5 (g_V^e g_A^\tau + g_V^\tau g_A^e) + X_6 (g_V^\mu g_A^\tau + g_V^\tau g_A^\mu)], \tag{63c}$$

$$k_{VV} = \frac{1}{N} [X_1 g_V^e g_V'^e + X_2 g_V^\mu g_V'^\mu + X_3 g_V^\tau g_V'^\tau + X_4 (g_V^e g_V'^\mu + g_V^\mu g_V'^e) + X_5 (g_V^\mu g_V'^\tau + g_V^\tau g_V'^\mu) + X_6 (g_V^e g_V'^\tau + g_V^\tau g_V'^e)], \tag{63d}$$

$$k'_{AA} = \frac{1}{N} [X_1 g_A^e g_A'^e + X_2 g_A^\mu g_A'^\mu + X_3 g_A^\tau g_A'^\tau + X_4 (g_A^e g_A'^\mu + g_A^\mu g_A'^e) + X_5 (g_A^\mu g_A'^\tau + g_A^\tau g_A'^\mu) + X_6 (g_A^e g_A'^\tau + g_A^\tau g_A'^e)], \tag{63e}$$

$$k'_{VA} = \frac{1}{N} [X_1 g_V^e g_A'^e + X_2 g_V^\mu g_A'^\mu + X_3 g_V^\tau g_A'^\tau + X_4 (g_V^e g_A'^\mu + g_V^\mu g_A'^e) + X_5 (g_V^e g_A'^\tau + g_V^\tau g_A'^e) + X_6 (g_V^\mu g_A'^\tau + g_V^\tau g_A'^\mu)], \tag{63f}$$

$$\tilde{k}_{VV} = \frac{1}{2N} [X_1 (g_V'^e)^2 + X_2 (g_V'^\mu)^2 + X_3 (g_V'^\tau)^2 + 2X_4 g_V'^e g_V'^\mu + 2X_5 g_V'^e g_V'^\tau + 2X_6 g_V'^\mu g_V'^\tau], \tag{63g}$$

$$\tilde{k}_{AA} = \frac{1}{2N} [X_1 (g_A'^e)^2 + X_2 (g_A'^\mu)^2 + X_3 (g_A'^\tau)^2 + 2X_4 g_A'^e g_A'^\mu + 2X_5 g_A'^e g_A'^\tau + 2X_6 g_A'^\mu g_A'^\tau], \tag{63h}$$

$$\tilde{k}_{VA} = \frac{1}{2N} [X_1 g_V'^e g_A'^e + X_2 g_V'^\mu g_A'^\mu + X_3 g_V'^\tau g_A'^\tau + X_4 (g_V'^e g_A'^\mu + g_V'^\mu g_A'^e) + X_5 (g_V'^e g_A'^\tau + g_V'^\tau g_A'^e) + X_6 (g_V'^\mu g_A'^\tau + g_V'^\tau g_A'^\mu)], \tag{63i}$$

$$\tilde{k}''_{VV} = \frac{1}{2N} [X_1 (g_V''^e)^2 + X_2 (g_V''^\mu)^2 + X_3 (g_V''^\tau)^2 + 2X_4 g_V''^e g_V''^\mu + 2X_5 g_V''^e g_V''^\tau + 2X_6 g_V''^\mu g_V''^\tau], \tag{63j}$$

$$\tilde{k}''_{AA} = \frac{1}{2N} [X_1 (g_A''^e)^2 + X_2 (g_A''^\mu)^2 + X_3 (g_A''^\tau)^2 + 2X_4 g_A''^e g_A''^\mu + 2X_5 g_A''^e g_A''^\tau + 2X_6 g_A''^\mu g_A''^\tau], \tag{63k}$$

$$\tilde{k}''_{VA} = \frac{1}{2N} [X_1 g_V''^e g_A''^e + X_2 g_V''^\mu g_A''^\mu + X_3 g_V''^\tau g_A''^\tau + X_4 (g_V''^e g_A''^\mu + g_V''^\mu g_A''^e) + X_5 (g_V''^e g_A''^\tau + g_V''^\tau g_A''^e) + X_6 (g_V''^\mu g_A''^\tau + g_V''^\tau g_A''^\mu)], \tag{63l}$$

$$k''_{VV} = \frac{1}{N} [X_1 g_V^e g_V''^e + X_2 g_V^\mu g_V''^\mu + X_3 g_V^\tau g_V''^\tau + X_4 (g_V^e g_V''^\mu + g_V^\mu g_V''^e) + X_5 (g_V^\mu g_V''^\tau + g_V^\tau g_V''^\mu) + X_6 (g_V^e g_A''^\tau + g_V^\tau g_V''^e)], \tag{63m}$$

$$k''_{AA} = \frac{1}{N} [X_1 g_A^e g_A''^e + X_2 g_A^\mu g_A''^\mu + X_3 g_A^\tau g_A''^\tau + X_4 (g_A^e g_A''^\mu + g_A^\mu g_A''^e) + X_5 (g_A^\mu g_A''^\tau + g_A^\tau g_A''^\mu) + X_6 (g_A^e g_A''^\tau + g_A^\tau g_A''^e)], \tag{63n}$$

$$k''_{VA} = \frac{1}{N} [X_1 g_V^e g_A''^e + X_2 g_V^\mu g_A''^\mu + X_3 g_V^\tau g_A''^\tau + X_4 (g_V^e g_A''^\mu + g_V^\mu g_A''^e) + X_5 (g_V^e g_A''^\tau + g_V^\tau g_A''^e) + X_6 (g_V^\mu g_A''^\tau + g_V^\tau g_A''^\mu)], \tag{63p}$$

$$k''_{AV} = \frac{1}{N} [X_1 g_V''^e g_A^e + X_2 g_V''^\mu g_A^\mu + X_3 g_V''^\tau g_A^\tau + X_4 (g_A''^e g_A^\mu + g_V''^\mu g_A^e) + X_5 (g_V''^e g_A^\tau + g_V''^\tau g_A^e) + X_6 (g_V''^\mu g_A^\tau + g_V''^\tau g_A^\mu)], \tag{63q}$$

$$\tilde{k}'_{VV} = \frac{1}{N}\,[X_1\, g'^{e}_{V} g''^{e}_{V} + X_2\, g'^{\mu}_{V} g''^{\mu}_{V} + X_3\, g'^{\tau}_{V} g''^{\tau}_{V} + X_4\,(g'^{e}_{V} g''^{\mu}_{V} + g'^{\mu}_{V} g''^{e}_{V}) + X_5\,(g'^{\mu}_{V} g''^{\tau}_{V} + g'^{\tau}_{V} g''^{\mu}_{V}) + X_6\,(g'^{e}_{V} g''^{\tau}_{V} + g'^{\tau}_{V} g''^{e}_{V}), \qquad (63r)$$

$$\tilde{k}'_{AA} = \frac{1}{N}\,[X_1\, g'^{e}_{A} g''^{e}_{A} + X_2\, g'^{\mu}_{A} g''^{\mu}_{A} + X_3\, g'^{\tau}_{A} g''^{\tau}_{A} + X_4\,(g'^{e}_{A} g''^{\mu}_{A} + g'^{\mu}_{A} g''^{e}_{A}) + X_5\,(g'^{\mu}_{A} g''^{\tau}_{A} + g'^{\tau}_{A} g''^{\mu}_{A}) + X_6\,(g'^{e}_{A} g''^{\tau}_{A} + g'^{\tau}_{A} g''^{e}_{A})], \qquad (63s)$$

$$\tilde{k}'_{VA} = \frac{1}{N}\,[X_1\, g'^{e}_{V} g''^{e}_{A} + X_2\, g'^{\mu}_{V} g''^{\mu}_{A} + X_3\, g'^{\tau}_{V} g''^{\tau}_{A} + X_4\,(g'^{e}_{V} g''^{\mu}_{A} + g'^{\mu}_{V} g''^{e}_{A}) + X_5\,(g'^{e}_{V} g''^{\tau}_{A} + g'^{\tau}_{V} g''^{e}_{A}) + X_6\,(g'^{\mu}_{V} g''^{\tau}_{A} + g'^{\tau}_{V} g''^{\mu}_{A})], \qquad (63t)$$

$$\tilde{k}'_{AV} = \frac{1}{N}\,[X_1\, g''^{e}_{V} g'^{e}_{A} + X_2\, g''^{\mu}_{V} g'^{\mu}_{A} + X_3\, g''^{\tau}_{V} g'^{\tau}_{A} + X_4\,(g''^{e}_{V} g'^{\mu}_{A} + g''^{\mu}_{V} g'^{e}_{A}) + X_5\,(g''^{e}_{V} g'^{\tau}_{A} + g''^{\tau}_{V} g'^{e}_{A}) + X_6\,(g''^{\mu}_{V} g'^{\tau}_{A} + g''^{\tau}_{V} g'^{\mu}_{A})], \qquad (63u)$$

where

$A_1 = [4\, k_{VA} L' - g^{\mu}_{V} g^{\mu}_{A}]$, (64a) $A_2 = [\tilde{L}\, g^{\mu}_{A} - 2\, L'\, g^{e}_{A}]$, (64b)

$A_3 = [g^{\mu}_{V}\, g^{e}_{A} - 2\, \tilde{L}\, k_{VA}]$, (64c) $A_4 = [g^{\mu}_{A} g^{e}_{V} - 2\, \tilde{L}\, k_{VA}]$, (64d)

$A_5 = [\tilde{L} g^{\mu}_{V} - 2\, L'\, g^{e}_{V}]$, (64e) $A_6 = [\tilde{L}\, g^{e}_{A} - 2\, L\, g^{\mu}_{A}]$, (64f)

$A_7 = [4\, L\, L' - (\tilde{L})^2]$, (64g) $A_8 = [\tilde{L}\, g^{e}_{V} - 2\, L\, g^{\mu}_{V}]$, (64h)

$A_9 = [4\, L\, k_{VA} - g^{e}_{V} g^{e}_{A}]$, (64i) $X_1 = [4\, L'\, L'' - \tilde{L}'']$, (64j)

$X_2 = [4\, L\, L'' - \tilde{L}']$, (64k) $X_3 = [4\, L\, L' - \tilde{L}]$, (64l)

$X_4 = [\tilde{L}'\, \tilde{L}'' - 2\, L''\, \tilde{L}]$, (64m) $X_5 = [\tilde{L}\, \tilde{L}'' - 2\, L'\, \tilde{L}]$, (64n)

$X_6 = [\tilde{L}\, \tilde{L}' - 2\, L\, \tilde{L}'']$, (64o)

$$N = \frac{1}{k'_{AV}}\,[X_1\, g'^{e}_{V} g^{e}_{A} + X_2\, g'^{\mu}_{V} g^{\mu}_{A} + X_3\, g'^{\tau}_{V} g^{\tau}_{A} + X_4\,(g'^{e}_{V} g^{\mu}_{A} + g'^{\mu}_{V} g^{e}_{A}) + X_5\,(g'^{e}_{V}\, g^{\tau}_{A} + g'^{\tau}_{V}\, g^{e}_{A}) + X_6\,(g'^{\mu}_{V}\, g^{\tau}_{A} + g'^{\tau}_{V}\, g^{\mu}_{A})], \qquad (64p)$$

$$D_1 = N/[X_5\, g^{e}_{V} + X_6\, g^{\mu}_{V} + X_3\, g^{\tau}_{V}]\; [X_5\, g^{e}_{A} + X_6\, g^{\mu}_{A} + X_3\, g^{\tau}_{A}], \qquad (64q)$$

$Q_1 = [4\, L''\, \tilde{k}''_{VA} - g''^{\tau}_{V}\, g''^{\tau}_{A}]$ (65a) $Q_2 = [g''^{\tau}_{V}\, g''^{\mu}_{A} - 2\, \tilde{L}'\, \tilde{k}''_{VA}]$, (65b)

$Q_3 = [2\, L\, L'\, g''^{\mu}_{A} - \tilde{L}''\, g''^{\tau}_{A}]$ (65c) $Q_4 = [g''^{\tau}_{A}\, g''^{\mu}_{V} - 2\, \tilde{L}''\, \tilde{k}''_{VA}]$, (65d)

$Q_5 = [4\, L'\, \tilde{k}''_{VA} - g''^{\mu}_{V}\, g''^{\mu}_{A}]$ (65e) $Q_6 = [2\, L'\, g''^{\tau}_{A} - \tilde{L}''\, g''^{\mu}_{A}]$, (65f)

$Q_7 = [2\, L''\, g''^{\mu}_{V} - \tilde{L}''\, g''^{\tau}_{V}]$ (65g) $Q_8 = [2\, L'\, g''^{\tau}_{V} - \tilde{L}''\, g''^{\mu}_{V}]$ (65h)

$Q_9 = [4\, L'\, L'' - (\tilde{L}'')^2]$ (65i)

$$D_7 = N/[X_1 g_V''^e + X_4 g_V''^\mu + X_5 g_V''^T]\,[X_1 g_A''^e + X_4 g_A''^\mu + X_5 g_A''^T], \tag{65j}$$

$$Y_1 = [4 L' \tilde{k}_{VA} - g_V'^\mu g_A'^\mu], \quad (65k) \qquad Y_2 = [\tilde{L} g_A'^\mu - 2 L' g_A'^e], \tag{65l}$$

$$Y_3 = [g_V'^\mu g_A'^e - 2 \tilde{L}' \tilde{k}_{VA}], \quad (65m) \qquad Y_4 = [g_A'^\mu g_V'^e - 2 \tilde{k}_{VA} \tilde{L}], \tag{65n}$$

$$Y_5 = [\tilde{L} g_V'^\mu - 2 L' g_V'^e], \quad (65o) \qquad Y_6 = [\tilde{L} g_A'^e - 2 L g_A'^\mu], \tag{65p}$$

$$Y_7 = [4 L L' - (\tilde{L})^2], \quad (65q) \qquad Y_8 = [\tilde{L} g_V'^e - 2 L g_V'^\mu], \tag{65r}$$

$$Y_9 = [4 L \tilde{k}_{VA} - g_V'^e g_A'^e], \tag{65s}$$

$$D_5 = N/[X_3 g_V'^T + X_6 g_V'^\mu + X_5 g_V'^e]\,[X_3 g_A'^T + X_6 g_A'^\mu + X_5 g_A'^e], \tag{65t}$$

$$B_1 = [4 L'' \tilde{k}_{VA}'' - g_V''^T g_A''^T], \quad (66a) \qquad B_2 = [2 L'' g_A'^e - \tilde{L}' g_A''^T], \tag{66b}$$

$$B_3 = [g_V''^T g_A{}^e - 2 \tilde{k}_{VA}'' \tilde{L}'], \quad (66c) \qquad B_4 = [g_V''^e g_A''^T - 2 \tilde{L}' \tilde{k}_{VA}''], \tag{66d}$$

$$B_5 = [\tilde{L}' g_V''^T - 2 L'' g_V''^e], \quad (66e) \qquad B_6 = [\tilde{L}' g_A''^e - 2 L g_A''^T], \tag{66f}$$

$$B_7 = [4 L L'' - (\tilde{L}')^2], \quad (66g) \qquad B_8 = [\tilde{L}' g_V''^e - 2 L g_V''^T], \tag{66h}$$

$$B_9 = [4 L \tilde{k}_{VA}'' - g_A''^e g_V''^e], \tag{66i}$$

$$D_2 = N/[X_4 g_V''^e + X_2 g_V''^\mu + X_6 g_V''^T]\,[X_4 g_A''^e + X_2 g_A''^\mu + X_6 g_A''^T], \tag{66j}$$

5.2 *With only μ-e universality*

The number of current parameters reduce to 10 vectors each with 3 components and the number of constraint equations become 41 corresponding to the number of physical parameters. Elimination of 28 independent scalar parameters (10 magnitudes plus 18 relative angles) yields the following 13 general relations given by (67a) – (68k).

$$\xi = \tilde{D}_2\,[B_1\,\alpha\,\beta + B_2\,\alpha\,\tilde{\beta}'' + B_3\,\alpha\,\beta'' + B_4\,\alpha''\,\beta + B_5\,\tilde{\alpha}''\,\beta + B_6\,\alpha''\,\tilde{\beta}'' + B_7\,\tilde{\alpha}''\,\tilde{\beta}'' + B_8\,\tilde{\alpha}''\,\tilde{\beta}'' + B_9\,\alpha''\,\beta''], \tag{67a}$$

$$\eta = \tilde{D}_2\,[B_1\,\gamma\,\delta + B_2\,\gamma\,\tilde{\delta}'' + B_3\,\gamma\,\delta'' + B_4\,\gamma''\,\delta + B_5\,\tilde{\gamma}''\,\delta + B_6\,\gamma''\,\tilde{\delta}'' + B_7\,\tilde{\gamma}''\,\tilde{\delta}'' + B_8\,\tilde{\gamma}''\,\delta'' + B_9\,\gamma''\,\delta''], \tag{67b}$$

$$\zeta = \tilde{D}_2\,[B_1\,\alpha\,\delta + B_2\,\alpha\,\tilde{\delta}'' + B_3\,\alpha\,\delta'' + B_4\,\alpha''\,\delta + B_5\,\tilde{\alpha}''\,\delta + B_6\,\alpha''\,\tilde{\delta}'' + B_7\,\tilde{\alpha}''\,\tilde{\delta}'' + B_8\,\tilde{\alpha}''\,\delta'' + B_9\,\alpha''\,\delta''], \tag{67c}$$

$$\rho = \tilde{D}_2\,[B_1\,\gamma\,\beta + B_2\,\gamma\,\tilde{\beta}'' + B_3\,\gamma\,\beta'' + B_4\,\gamma''\,\beta + B_5\,\tilde{\gamma}''\,\beta + B_6\,\gamma''\,\tilde{\beta}'' + B_7\,\tilde{\gamma}''\,\tilde{\beta}'' + B_8\,\tilde{\gamma}''\,\beta'' + B_9\,\gamma''\,\beta''], \tag{67d}$$

$$k_{VV} = \frac{1}{2\tilde{N}}\,[2\,\tilde{X}_1\,(g_V^e)^2 + \tilde{X}_3\,(g_V^T)^2 + 2\,\tilde{X}_4\,(g_V^e)^2], \tag{68a}$$

$$k_{AA} = \frac{1}{2\tilde{N}} [2\, \tilde{X}_1 (g_A^e)^2 + \tilde{X}_3 (g_A^T)^2 + 2\, \tilde{X}_4 (g_A^e)^2], \tag{68b}$$

$$\tilde{k}''_{VV} = \frac{1}{2\tilde{N}} [2\, \tilde{X}_1 (g_V''^e)^2 + \tilde{X}_3 (g_V''^T)^2 + 2\, \tilde{X}_4 (g_V''^e)^2], \tag{68c}$$

$$\tilde{k}''_{AA} = \frac{1}{2\tilde{N}} [2\, \tilde{X}_1 (g_A''^e)^2 + \tilde{X}_3 (g_A''^T)^2 + 2\, \tilde{X}_4 (g_A''^e)^2], \tag{68d}$$

$$\tilde{k}''_{VA} = \frac{1}{2\tilde{N}} [2\, \tilde{X}_1 g_V''^e g_A''^e + \tilde{X}_3 g_V''^T g_A''^T + 2\, \tilde{X}_4 g_V''^e g_A''^e], \tag{68e}$$

$$k''_{VV} = \frac{1}{\tilde{N}} [2\, \tilde{X}_1 g_V^e g_V''^e + \tilde{X}_3 g_V^T g_V''^T + 2\, \tilde{X}_4 g_V^e g_V''^e], \tag{68f}$$

$$k''_{AA} = \frac{1}{\tilde{N}} [2\, \tilde{X}_1 g_A^e g_A''^e + \tilde{X}_3 g_A^T g_A''^T + 2\, \tilde{X}_4 g_A^e g_A''^e], \tag{68g}$$

$$k''_{VA} = \frac{1}{\tilde{N}} [2\, \tilde{X}_1 g_V^e g_A''^e + \tilde{X}_3 g_A^T g_A''^T + 2\, \tilde{X}_4 g_V^e g_A''^e], \tag{68h}$$

$$k''_{AV} = \frac{1}{\tilde{N}} [2\, \tilde{X}_1 g_A^e g_V''^e + \tilde{X}_3 g_A^T g_V''^T + 2\, \tilde{X}_4 g_A^e g_V''^e], \tag{68i}$$

where

$$\tilde{N} = \frac{1}{2\, k_{VA}} [2\, \tilde{X}_1 g_V^e g_A^e + \tilde{X}_3 g_V^T g_A^T + 2\, \tilde{X}_4 g_V^e g_A^e], \tag{69a}$$

$$\tilde{D} = \tilde{N}/[\tilde{X}_4 g_V''^e + \tilde{X}_2 g_V''^e + \tilde{X}_6 g_V''^T]\, [\tilde{X}_4 g_A''^e + \tilde{X}_2 g_A''^e + \tilde{X}_6 g_A''^T], \tag{69b}$$

$$\tilde{X}_1 = \tilde{X}_2 = (4LL'' - \tilde{L}''),\ \tilde{X}_3 = (4L^2 - 2L),$$

$$\tilde{X}_4 = [(\tilde{L}')^2 - 4LL''],\ \tilde{X}_5 = \tilde{X}_6 = 0. \tag{69c}$$

5.3 *With μ-e-τ universality*

In this case the current parameters are 7 vectors each with 3 components. Since the vectors occur as scalar products the number of independent scalar parameters reduce to 19 (7 magnitudes of vectors plus 12 relative angles). But since the number of physical parameters are less than the number of current parameters, no relations are possible in this case unless one includes more number of neutral current processes.

4. Summary and discussion

We have obtained general relations for single, two and three-Z-boson hypotheses without assuming any symmetry, with μ-e universality and μ-e-τ universality. The processes considered and physical parameters involved have been summarised in table 1 and the number of relations obtained in different cases has been summarized in table 2. These relations possess the following important and useful potentialities: (1) To serve as tests of gauge models hypothesizing one, two or three neutral Z bosons. (2) To predict values of parameters occurring in certain processes for which experimental measurements are difficult. (3) To provide unambigguous values of observables when combined with phenomenological analyses. To cite evidences in favour of point (1) Hung and Sakurai (1977) have utilized such relations for testing models with single and two-Z bosons. As demonstrations in support of the point (3) these authors (Hung and Sakurai 1979) have determined several neutral current parameters unambiguously with respect to their signs and magnitudes. As a demonstration of the second point, Hung and Sakurai (1979) have calculated the $\nu\nu Z$ coupling constant using experimentally measured values of neutral-current parameters in other sectors. Also recently the values of the parameters contributing to the parity violating nuclear force have been calculated (Hazra and Parida 1981; Parida and Hazra 1982). We hope that the relations obtained in this paper might provide deeper insight into the structure of gauge models and serve many useful purposes in phenomenological analysis.

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Appendix

Determination of general relations for 3 Z-boson model

In the following all vectors have 3 components. We also use the notations

$$\mathbf{S}' = \mathbf{C}_0 \times \mathbf{C}_0', \quad \mathbf{S}'' = \mathbf{C}_0 \times \mathbf{C}_0'', \quad \mathbf{S}''' = \mathbf{C}_0' \times \mathbf{C}_0'' \tag{A-1}$$

$$D = \mathbf{C}_0 \cdot \mathbf{S}''', \quad D_V = \mathbf{C}_V \cdot \mathbf{S}', \quad D_A = \mathbf{C}_A \cdot \mathbf{S}', \quad D_V' = \mathbf{C}_V'' \cdot \mathbf{S}''',$$

$$D_A' = \mathbf{C}_A'' \cdot \mathbf{S}''' \tag{A-2}$$

$$\mathbf{S}^A = \mathbf{C}_0 \times \mathbf{C}_A, \quad \mathbf{S}^V = \mathbf{C}_0 \times \mathbf{C}_V, \quad \mathbf{S}'^A = \mathbf{C}_0' \times \mathbf{C}_A, \quad \mathbf{S}'^V = \mathbf{C}_0' \times \mathbf{C}_V,$$

$$\mathbf{S}''^A = \mathbf{C}_0'' \times \mathbf{C}_A'', \quad \mathbf{S}''^V = \mathbf{C}_0'' \times \mathbf{C}_V'', \quad \mathbf{S}'''^V = \mathbf{C}_0' \times \mathbf{C}_A'',$$

$$\mathbf{S}'''^V = \mathbf{C}^0 \times \mathbf{C}_V''. \tag{A-3}$$

For the x, y, and z components of the vectors which are cross products of two vectors we use the subscripts 1, 2 and 3, respectively *e.g.* $S_1'^A = C_0'^{(2)} C_A^{(3)} - C_0'^{(3)} C_A^{(2)}$. Using six sets of equations (8), (9), (10), (16); (9), (10), (15), (16); (9), (10), (16),

Table 2. List of the number of relations obtained for different cases.

Model	Without μ-e-τ Universality			μ-e Universality only			μ-e-τ Universality		
	No. of physical observables	No. of current parameters	No. of relations	No. of physical observables	No. of current parameters	No. of relations	No. of physical observables	No. of current parameters	No. of relations
1 Z-Boson	73	13	60	41	10	31	18	7	11
2 Z-Boson	73	25	48	41	19	22	18	13	5
3 Z-Boson	73	37	36	41	28	13	18	19	Not possible

(21); (15), (16), (21); (8), (10), (21); (8), (9), (15) and the above notations we get six relations:

$$\mathbf{S}''' \cdot \mathbf{S}''' = (4\,L'\,L'' - \tilde{L}'') = 4\,X_1, \quad \mathbf{S}'' \cdot \mathbf{S}'' = (4\,L\,L'' - \tilde{L}') = 4\,X_2,$$

$$\mathbf{S}'' \cdot \mathbf{S}''' = (\tilde{L}'\,\tilde{L} - 2\,L''\,\tilde{L}) = 4\,X_4, \quad \mathbf{S}' \cdot \mathbf{S}''' = (\tilde{L}\,\tilde{L}'' - 2\,L'\,\tilde{L}') = 4\,X_5,$$

$$\mathbf{S}' \cdot \mathbf{S}' = (4\,L\,L' - \tilde{L}) = 4\,X_3, \quad \mathbf{S}'' \cdot \mathbf{S}' = (\tilde{L}\,\tilde{L}' - 2\,L\,\tilde{L}'') = 4\,X_6 \quad \text{(A-4)}$$

From (11a), (17a) and (22a) we obtain

$$\mathbf{C}_V = \frac{1}{2\,D}\,[g_V^e\,\mathbf{S}''' + g_V^\mu\,\mathbf{S}'' + g_V^\tau\,\mathbf{S}'], \quad \text{(A-5)}$$

$$\mathbf{C}'_V = \frac{1}{2\,D}\,[g_V'^e\,\mathbf{S}''' + g_V'^\mu\,\mathbf{S}'' + g_V'^\tau\,\mathbf{S}']. \quad \text{(A-6)}$$

Using equations (14a), (20a), (29a); (14b), (20b), (29b); (14c), (20c), (29c); (14a), (20d), (29d); one obtains:

$$C_\alpha^{(1)} = \frac{1}{2D_A}\,[\alpha S_1'^A - \alpha' S_1^A + \tilde{\alpha} S_1'], \quad C_\alpha^{(2)} = \frac{1}{2D_A}\,[\alpha S_2'^A - \alpha' S_2^A + \tilde{\alpha} S_2']$$

$$C_\alpha^{(3)} = \frac{1}{2D_A}\,[\alpha S_3'^A - \alpha' S_3^A + \tilde{\alpha} S_3'], \quad \text{(A-7)}$$

$$C_\beta^{(1)} = \frac{1}{2D_V}\,[\beta S_1'^V - \beta' S_1^V - \tilde{\beta} S_1'], \quad C_\beta^{(2)} = \frac{1}{2D_V}\,[\beta S_2'^V - \beta' S_2^V - \tilde{\beta} S_2']$$

$$C_\beta^{(3)} = \frac{1}{2D_V}\,[\beta S_3'^V - \beta' S_3^V - \tilde{\beta} S_3'], \quad \text{(A-8)}$$

$$C_\gamma^{(1)} = \frac{1}{2D_A}\,[\gamma S_1'^A - \gamma' S_1^A + \tilde{\gamma} S_1'], \quad C_\gamma^{(2)} = \frac{1}{2D_A}\,[\gamma S_2'^A + \gamma' S_2^A + \tilde{\gamma} S_2']$$

$$C_\gamma^{(3)} = \frac{1}{2D_A}\,[\gamma S_3'^A - \gamma' S_3^A + \tilde{\gamma} S_3'], \quad \text{(A-9)}$$

$$C_\delta^{(1)} = \frac{1}{2D_V}\,[\delta S_1'^V - \delta' S_1^V + \tilde{\delta} S_1'], \quad C_\delta^{(2)} = \frac{1}{2D_V}\,[\delta S_2'^V - \delta' S_2^V + \tilde{\delta} S_2']$$

$$C_\delta^{(3)} = \frac{1}{2D_V}\,[\delta S_3'^V - \delta' S_3^V + \tilde{\delta} S_3']. \quad \text{(A-10)}$$

Using (A-7), (A-8), (A-2) in (35a) after some algebraic manipulations one gets

$$\xi = \frac{1}{8\,D_V D_A}\,[A_1\,\alpha\,\beta + A_2\,\alpha\,\tilde{\beta} + A_3\,\alpha\,\beta' + A_4\,\alpha'\,\beta + A_5\,\tilde{\alpha}\,\beta + A_6\,\alpha'\,\tilde{\beta}$$

$$+ A_7\,\tilde{\alpha}\,\tilde{\beta} + A_8\,\tilde{\alpha}\,\beta' + A_9\,\alpha'\,\beta']. \quad \text{(A-11)}$$

Using equations (11b), (17b), (22b), (A-1) and (A-2) we get

$$\mathbf{C}_A = [g^e_A\, \mathrm{S}''' + g^\mu_A\, \mathrm{S}'' + g^T_A\, \mathrm{S}']/2D. \tag{A-12}$$

Using (A-5), (A-2), X_5, X_6 and X_3 we get

$$D_V\, D_A = \frac{1}{64D^2}\, [X_5\, g^e_A + X_6\, g^\mu_A + X_3\, g^T_A]\; [X_5\, g^e_V + X_6\, g^\mu_V + X_3\, g^T_V]. \tag{A-13}$$

Putting (A-12), (A-6) in (27d) and using (A-4) and collecting the coefficients of similar terms we get

$$8D^2 k'_{AV} = [g'^e_V\, g^e_A\, X_1 + g'^\mu_V\, g^\mu_A\, X_2 + g'^T_V\, g^T_A\, X_3 + X_4\, (g'^e_V\, g^\mu_A + g'^\mu_V\, g^e_A) + X_5\, (g'^e_V\, g^T_A + g'^T_V\, g^e_A) + X_6\, (g'^\mu_V\, g^T_A + g'^T_V\, g^\mu_A)], \tag{A-14}$$

whence one gets

$$\frac{1}{8D^2} = k'_{AV}/[X_1\, g'^e_V g^e_A + X_2\, g'^\mu_V\, g^\mu_A + X_3\, g'^T_V\, g^T_A + X_4\, (g'^e_V\, g^\mu_A + g'^\mu_V\, g^e_A) + X_5\, (g'^e_V\, g^T_A + g'^T_V\, g^e_A) + X_6\, (g'^\mu_V\, g^T_A + g'^T_V\, g^\mu_A)]. \tag{A-15}$$

(A-15) is then substituted in (A-13) and the resulting equation is then put in (A-11) which yields (59a). Similarly from (35b)-(35d) the relations (59b)-(59d) can be easily obtained using (A-7)-(A-10) and (A-4). From (13a), (19a), (24a) and (A-1) we get

$$\mathbf{C}''_V = \frac{1}{2D}\, [g''^e_V\, \mathrm{S}''' + g''^\mu_V\, \mathrm{S}'' + g''^T_V\, \mathrm{S}'], \tag{A-16}$$

and from (13b), (19b), (24b) and (A-1) we get

$$\mathbf{C}''_A = \frac{1}{2D}\, [g''^e_A\, \mathrm{S}''' + g''^\mu_A\, \mathrm{S}'' + g''^T_A\, \mathrm{S}']. \tag{A-17}$$

From (12b), (18b), (23b) using (A-1) we obtain

$$\mathbf{C}'_A = \frac{1}{2D}\, [g'^e_A\, \mathrm{S}''' + g'^\mu_A\, \mathrm{S}'' + g'^T_A\, \mathrm{S}']. \tag{A-18}$$

Derivation of relations (60*a*)-(60*d*)

From (20a), (25a), (34a) and (A-1)-(A-3) one gets

$$C^{(1)}_\alpha = \frac{1}{2D'_A}\, [\alpha'\, S''^A_1 - \alpha''\, S'''^A_1 + \tilde{\alpha}''\, S'''_1\,],$$

$$C^{(2)}_\alpha = \frac{1}{2D'_A}\, [\alpha'\, S''^A_2 - \alpha''\, S'''^A_2 + \tilde{\alpha}''\, S'''_2],$$

$$C^{(3)}_\alpha = \frac{1}{2D'_A}\, [\alpha'\, S''^A_3 - \alpha''\, S'''^A_3 + \tilde{\alpha}''\, S'''_3]. \tag{A-19}$$

From (20b), (25b), (34b) and (A-1)-(A-3) one can derive

$$C_\beta^{(1)} = \frac{1}{2D'_V} [\beta' S_1''^V - \beta'' S_1'''^V + \tilde{\beta}'' S_1'''],$$

$$C_\beta^{(2)} = \frac{1}{2D'_V} [\beta' S_2''^V - \beta'' S_2'''^V + \tilde{\beta}'' S_2'''],$$

$$C_\beta^{(3)} = \frac{1}{2D'_V} [\beta' S_3''^V - \beta'' S_3'''^V + \tilde{\beta}'' S_3'']. \quad \text{(A-20)}$$

From (20c), (25c), (34c) and (A-1)-(A-3) we get

$$C_\gamma^{(1)} = \frac{1}{2D'_A} [\gamma' S_1''^A - \gamma'' S_1'''^A + \tilde{\gamma}'' S_1'''],$$

$$C_\gamma^{(2)} = \frac{1}{2D'_A} [\gamma' S_2''^A - \gamma'' S_2'''^A + \tilde{\gamma}'' S_2''']$$

$$C_\gamma^{(3)} = \frac{1}{2D'_A} [\gamma' S_3''^A - \gamma'' S_3''^A + \tilde{\gamma}'' S_3''']. \quad \text{(A-21)}$$

From (20d), (25d), (34d) and (A-1)-(A-3) one obtains

$$C_\delta^{(1)} = \frac{1}{2D'_V} [\delta' S_1''^V - \delta'' S_1'''^V + \tilde{\delta}'' S_1'''],$$

$$C_\delta^{(2)} = \frac{1}{2D'_V} [\delta' S_2''^V - \delta'' S_2'''^V + \tilde{\delta}'' S_2''']$$

$$C_\delta^{(3)} = \frac{1}{2D'_V} [\delta' S_3''^V - \delta'' S_3'''^V + \tilde{\delta}'' S_3''']. \quad \text{(A-22)}$$

Putting (A-19), (A-20) in (35a) and using (33c), (21), (24a), (24b), (19a), (19b), (16), (15) and (A-2) we get

$$\xi = \frac{1}{8D'_V D'_A} [Q_1\alpha'\beta' + Q_2\alpha'\beta'' + Q_3\alpha'\tilde{\beta}'' + Q_4\alpha''\beta' + Q_5\alpha''\beta'' + Q_6\,\alpha''\tilde{\beta}'' + Q_7\,\tilde{\alpha}''\beta' + Q_8\,\tilde{\alpha}''\beta'' + Q_9\,\tilde{\alpha}''\tilde{\beta}''] \quad \text{(A-23)}$$

Using (A-16), (A-17) and (A-4) one gets

$$8D'_V D'_A = \frac{1}{8D^2} [X_1 g_V''^e + X_4 g_V''^\mu + X_5 g_V''^\tau] \, [X_1 g_A''^e + X_4 g_A''^\mu + X_5 g_A''^\tau] \quad \text{(A-24)}$$

Using (A-24) in (A-23) relation (60a) is obtained. Similarly other relations, (60b)-(60d) can be derived.

Derivation of relations (61*a*)-(61*d*)

Using the four sets of equations (14a), (20a), (32a); (14b), (20b), (32b); (14c), (20c), (32c); (14d), (20d), (32d); we obtain expressions for the four sets of parameters ($C_\alpha^{(1)}$, $C_\alpha^{(2)}$, $C_\alpha^{(3)}$); ($C_\beta^{(1)}$, $C_\beta^{(2)}$, $C_\beta^{(3)}$); ($C_\gamma^{(3)}$, $C_\gamma^{(2)}$, $C_\gamma^{(1)}$); and ($C_\delta^{(1)}$, $C_\delta^{(2)}$, $C_\delta^{(3)}$), respectively in the same manner as explained above. Then using the relations (35a)-(35d) and (A-15) and following procedure that led to the derivation of relations (59a)-(59d); the relations (61a)-(61d) can be derived.

Derivation of relations (62*a*)-(62*d*)

For the derivation of these relations we adopt similar procedure using the sets of equations (14a), (25a), (34a); (14b); (25b), (34b); (14c), (25c), (34c); and (14d), (25d), (34d); the equations (35a)-(35d)and (A-15).

Derivation of relations (63*a*)-(63*p*), (63*q*)-(63*u*)

Substituting (A-5) in (26b) and using (11a), (17a), (22a) and (A-2) one gets

$$k_{VV} = \frac{1}{4D^2}\,[(g_V^e)^2\,\mathbf{S}'''\cdot\mathbf{S}''' + (g_V^\mu)^2\,\mathbf{S}''\cdot\mathbf{S}'' + (g_V^\tau)^2\,\mathbf{S}'\cdot\mathbf{S}'$$

$$+ 2\,g_V^e g_V^\mu\,\mathbf{S}'''\cdot\mathbf{S}'' + 2\,g_V^e g_V^\tau\,\mathbf{S}'\cdot\mathbf{S}''' + 2\,g_V^\mu g_V^\tau\,\mathbf{S}'\cdot\mathbf{S}''] \qquad \text{(A-25)}$$

Using (A-4) in (A-25) we obtain

$$k_{VV} = \frac{1}{16D^2}\,[X_1\,(g_V^e)^2 + X_2\,(g_V^\mu)^2 + X_3\,(g_V^\tau)^2 + 2\,X_4\,g_V^e g_V^\mu$$

$$+ 2\,X_5\,g_V^e g_V^\tau + 2\,X_6\,g_V^\mu g_V^\tau] \qquad \text{(A-26)}$$

Substituting (A-15) in (A-25) we get (63a). Similarly the relations (63b), (63c) can be obtained. Using (A-5), (A-6), (11a), (12a), (17a), (18a), (22a), (23a), (A-4) and (A-15) in (27a) the relation (63d) follows. Similarly the relations (63e) and (63f) can be derived. Using (12a), (18a), (23a), (A-4), (A-6) and (A-15) one gets (63g) from (30a). Similarly (63h), (63i) can be deduced easily. Using (13a), (19a), (24a), (A-4), (A-15) and (A-16) is (33a) the relation (63j) is derived. Similarly the relations (63k) and (63l) are obtained. Using (11a), (17a), (22a), (13a), (19a), (24a), (A-4), (A-15), (A-5), (A-16) in (28a), the relation (63m) is obtained. Similarly using (A-5), (A-2), (A-4), (A-15)-(A-17) the relations (63n)-(63q) can be obtained from equations (28-b)-(28d). Using (12a), (13a), (18a), (23a), (24a), (19a), (A-4), (A-6), (A-15), (A-16) one obtains the relation (63r) from equation (31a). Similarly (63s)-(63u) are derived using the equations (31b)-(31d), (A-4), (A-6), (A-15)-(A-18), (12)a-(13b), (18a)-(19b), and (23a)-(24b).

Derivation of relations (67*a*)-(67*d*)

When only μ-e universality is imposed, $\mathbf{C}'_0 = \mathbf{C}_0, \mathbf{C}'_V = \mathbf{C}_V, \mathbf{C}'_A = \mathbf{C}_A$. Consequently

$$g_V'^e = g_V^\mu = g_V'^\mu = g_V^e,\ g_A'^e = g_A^\mu = g_A'^\mu = g_A^e,\ g_V'^\tau = g_V^\tau,\ g_A'^\tau = g_A^\tau,$$

$$g_V''^\mu = g_V''^e,\ g_A''^\mu = g_A''^e,\ \alpha' = \alpha,\ \beta' = \beta,\ \gamma' = \gamma,\ \delta' = \delta,\ \tilde{\alpha}' = \tilde{\alpha},\ \tilde{\beta}' = \tilde{\beta},$$

$$\tilde{\gamma}' = \tilde{\gamma},\ \tilde{\delta}' = \tilde{\delta},\ \tilde{k}'_{VV} = k''_{VV},\ \tilde{k}'_{AA} = k''_{AA},\ \tilde{k}'_{VA} = k''_{VA},\ \tilde{k}'_{AV} = k''_{AV}$$

$$k'_{VV} = 2\tilde{k}_{VV} = k_{VV},\ k'_{VA} = k'_{AV} = 2k_{VA},\ \tilde{L}'' = \tilde{L}',\ L' = L,\ \tilde{L} = 2L.$$

Then right-hand-sides of the relations (60a)-(60d) and (62a)-(62d) vanish. Under μ-e universality,

$$X_1 = (4LL'' - \tilde{L}'),\quad X_2 = (4LL'' - \tilde{L}'),\quad X_3 = (4L^2 - 2L),$$

$$X_4 = \{(\tilde{L}')^2 - 4LL''\},\quad X_5 = X_6 = 0. \tag{A-27}$$

Since under this condition $D_2 = \tilde{D}_2 = \tilde{D}_7$, the right hand sides of (60a), (60d) and (62a)-(62d) yield only four relations which are the same as the relations (67a)-(67d).

Derivation of relations (68*a*)-(68*i*)

Under μ-e universality (63a), (63d) and (63g) reduce to (68a), (63b), (63e) and (63h) reduce to (68b), whereas both sides of (63c), (63f) and (63i) become identically equal yielding no relations. The relations (63j)-(63l) reduce to (68c)-(68e). Thus we have five relations, (68a)-(68e). The other four relations are obtained by noting that the four pairs of relations, (63m), (63r); (63n), (63s); (63p), (63t); and (63q), (63u); reduce to (68f), (68g), (68h) and (68i) respectively. Thus we have nine relations (68a)-(68i).

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