Pramāṇa, Vol. 18, No. 5, May 1982, pp. 439-449. © Printed in India.

On the mass spectroscopy of charmed multiquark hadrons

G BHAMATHI*, K PREMA** and A RAMACHANDRAN†
*Department of Theoretical Physics, University of Madras, Madras 600025, India and Department of Physics, University of Alberta, Edmonton, Alberta, Canada
**Department of Physics, Seethalakshmi Ramaswami College, Tiruchirapalli 620002, India
†Department of Physics, Vivekananda College, Madras 600 004, India

MS received 22 May 1981; revised 9 February 1982

Abstract. The possible existence of charmed multiquark hadrons are investigated using phenomenological MIT bag model and the SU(4) flavour symmetry. The masses of 6*q*, 9*q* and 15*q* systems having the same quantum numbers as the physically interesting ordinary nuclei, hypernuclei and supernuclei are estimated. We find that several new states with distinct signatures are predicted.

Keywords. M.I.T. bag model; multiquark states; charm; SU(4) flavour; multibaryon states.

1. Introduction

Recently, there have been numerous theoretical predictions of hadronic states composed of exotic quark configurations other than the usual $q\bar{q}$ for mesons and q^3 for baryons. The motivation for such studies came in several contexts, the earliest among them being the dual unitarization (Chew 1976; Rosenzwig 1976; Igi and Yazaki 1979). The immediate cause for these studies was the observed narrow widths of baryonium states which could be interpreted as manifestations of the existence of multiquark states. Several workers in the recent past have tried to explain, using the bag model and string model approaches (Igi 1978), the various narrow width mesons coupled strongly to $B\bar{B}$ channels, the baryon-meson resonances and the dibaryon and multibaryon states as evidence for the existence of multiquark states which go beyond the usual quark models. Thus the observed narrow baryonium states are being interpreted as $q^2\bar{q}^2$ states rather than as $q^3\bar{q}^3$ states. Extension of these ideas has led to the possible existence of narrow $q^4\bar{q}$ states‡) similar to meson-baryon resonances (Sorba *et al* 1978) and q^6 to q^N states as single hadron states with baryon number B ranging from 2 to $N/3$. A systematic study (Jaffe 1977; Aerts *et al* 1978) of these states has so far indicated a very rich spectrum of such exotic, cryptoexotic and other extraneous states.

In the case of multiquark states, having the same quantum numbers as dibaryon systems the only possible bound state has been found to be the six-quark dihyperon $(\Lambda_0\,\Lambda_0)$ state by Jaffe (1977) using a MIT bag model approach. Further studies of Aerts *et al* (1978) using three flavours of quarks have brought out the possibility of

‡A possible candidate for $q^4\bar{q}$ resonance is reported in Cern Courier Jan/Feb. 1980 p. 450.

narrow resonances in certain other dibaryon channels. One of the interesting consequences of the $6q$ model is the possible existence of hadron states in certain channels with values of iso-spin and other quantum numbers which cannot be coupled to mere dibaryon systems. These will, therefore, have definite signatures which distinguish them from the normal dibaryon states.

The recent discovery of charmed particles and the existence of charm quantum number provides us with the possibility of looking for multiquark states in channels where the charm quantum number is non-zero. Sorba and Hogaasen (1978) have suggested that the same dynamics that leads to a bound state may lead to many new bound bibaryons when the number of flavours exceeds three. In an earlier paper (Bhamathi *et al* 1980), we had reported our results on the mass spectroscopy of charmed six-quark states within the framework of MIT bag model. In this paper we give a brief description of the bag model and its extension to systems containing six or more quarks. In § 2 the basics of the bag model are set up. In § 3 we indicate the group theoretic analysis of the multiquark system in the spherical cavity approximation of the MIT bag model. In § 4 we discuss the extension to SU(4) flavour symmetry and in § 5 we present a critical evaluation of the predictions of the model.

2. The bag Hamiltonian

The original MIT bag model was set up by Chodos *et al* (1974) and applied to estimate the masses of the ground states of the stable hadrons by de Grand *et al* (1975). In this model the quarks are confined in a region of space satisfying the free Dirac equation and satisfying certain boundary conditions on the bag surface. The quarks interact through exchange of coloured vector gluons described by the Yang-Mills gauge theory. The resulting field equations and the boundary conditions lead to the existence of only colour singlet states. The lowest Dirac eigen mode of a spherical cavity of radius R is populated with quarks of appropriate colour, flavour and spin in order to form the S-wave hadrons.

The hadron energy in this model can be written as

$$E(R) = E_0 + E_v + E_Q + E_g, \tag{1}$$

where E_0, the zero-point energy of confined quarks is represented by $-Z_0/R$, E_v the volume energy associated with the confining pressure B is given by $4\pi/3\ R^3\ B$, E_Q, the quark kinetic energy is given by $(1/R) \sum_{i=1}^{n} \alpha_i\ (m_i\ R)$ with $\alpha_i\ (m_i\ R) = R\ \omega\ (m_i\ R)$ and $\omega\ (m_i\ R)$ is the frequency of the lowest eigen mode and E_g, the interaction energy due to a single gluon exchange between the quarks. This can be split into two parts, the colour electric and colour magnetic energies E_E and E_M respectively given by

$$E_E = \frac{\alpha_c}{R}\left[\frac{2}{3}\sum_i E_{ii}\ (R) + \sum_{i>j} E_{ij}\ (R)\frac{\lambda_i^c \cdot \lambda_j^c}{2}\right] \tag{2}$$

where

$$E_{ij}(R) = R \int_0^R \frac{dr}{r^3} \rho_i(r)\, \rho_j(r), \tag{3}$$

$$E_M = -3\,\alpha_c \sum_a \sum_{i>j} (\vec{\sigma_i}\lambda_j^a)\cdot(\vec{\sigma_j}\lambda_j^a) \frac{\mu(m_i R)\,\mu(m_j R)}{R^3}$$

$$\times\, I(m_i R, m_j R), \tag{4}$$

$$E_M = -\frac{\alpha_c}{R}\sum_a \sum_{i>j} M_{ij}(R)\, (\vec{\sigma}\lambda^a)_i \cdot (\vec{\sigma}\lambda^a)_j, \tag{5}$$

and $\alpha_c = \frac{g^2}{4\pi}$,

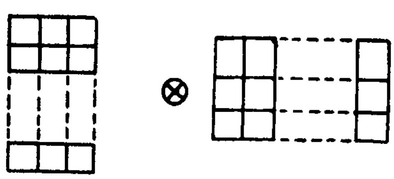
where g is the gauge coupling constant, λ_i^a and σ_i are the colour and spin matrices of the ith quark, $\mu(m_i R)$ is the quark magnetic moment. The expression for the functions $\mu(m_i R)$, $I(m_i R, m_j R)$ and $\rho_i(R)$ have been given by de Grand *et al* (1975) and (3) and (4) define the functions $E_{ij}(R)$ and $M_{ij}(R)$. The masses of the S-wave hadrons containing n quarks can be obtained by the standard procedure of constructing the appropriate fully antisymmetrised wave function in the colour flavour and spin space, diagonalising the Hamiltonian in this basis and minimising with respect to the bag radius R for each state.

3. Antisymmetrisation of the wave function

The quarks in the bag may be taken to transform as an irreducible representation of the overall symmetry group SU (T, FCJ) given by

$$\mathrm{SU}(T, FCJ) \supset \mathrm{SU}(f, F) \otimes \mathrm{SU}(3, C) \otimes \mathrm{SU}(2, J)$$

with $T = f \times 3 \times 2$ where f, C and J stand for flavour, colour and spin. The restriction of overall antisymmetry of an N quark state due to Pauli principle and the fact that only colour singlet states are allowed permits us to consider only states corresponding to Young patterns which in the flavour spin space are conjugate to the Young patterns which are colour singlets. Diagrammatically this may be represented by



Thus we find that for baryon numbers $B = 2$, 3 and 5 we have irreducible representations of dimensions 2520, 14112 and $\overline{14112}$ when the number of flavours is four. In order to evaluate E_E and E_M in the ground state of the various hadrons it is necessary to consider further decompositions of the flavour spin symmetry group SU $(2f, FJ)$ since we know that only the quantum numbers C, Y, I and J are conserved. The following decompositions were found useful in computing the contributions to E_E and E_M.

$$SU(8, FJ) \supset U(1, c) \otimes SU(2, J_c) \otimes SU(6, F'J_{ns})$$

$$SU(6, F'J_{ns}) \supset U(1, Y) \otimes SU(2, J_s) \otimes SU(4, I_n J_n)$$

$$SU(8, FJ) \supset SU(4, F) \otimes SU(2, J)$$

and

$$SU(6, F'J) \supset SU(3, F') \otimes SU(2, J)$$

The latter two decompositions determine the flavour multiplets to which the hadrons belong. In table 1 we exhibit the two different decompositions of the irreducible representation 2520 of the six quark system as an example.

4. Evaluation of E_E and E_M

The colour magnetic and colour electric interaction terms can be rewritten as

$$\begin{aligned} E_M = {} & M_{nc}(X_{nn} + X_{nc} + X_{cc}) + M_{sc}(X_{ss} + X_{sc} + X_{cc}) \\ & + M_{ns}(X_{nn} + X_{ns} + X_{ss}) + (M_{nn} - M_{ns} - M_{nc}) X_{nn} \\ & + (M_{ss} - M_{ns} - M_{sc}) X_{ss} + (M_{cc} - M_{nc} - M_{sc}) X_{cc}, \end{aligned} \tag{6}$$

and

$$\begin{aligned} E_E = {} & \frac{\alpha_c}{4R} \Big\{ (E_{ns} + E_{nc} + E_{sc})(8/3\, N) \\ & + [E_{nn} - (E_{ns} + E_{nc} + E_{sc})] \Big(8/3\, N_n + \sum_{n_1 > n_2} \lambda_1^c \cdot \lambda_2^c \Big) + \end{aligned}$$

Table 1. Decomposition

IR $[\mu]$	$SU(8, FJ) \supset U(1, c) \otimes SU(2, J_c) \otimes SU(6, F' J_{ns})$
2520	$(+6)(0, 1) + (+5)(\frac{1}{2}, 6) + (+4)[(1, 21) + (0, 15)]$ $+ (+3)[(3/2, 56) + (\frac{1}{2}, 70)] + (+2)[(0, 105) + (1, 210)]$ $+ (+1)[(\frac{1}{2}, 420)] + (0)(0, 490)$
2520	$SU_8 \supset SU(2, J) \otimes SU(4, F)$
	$(0, 10 + \overline{10} + 84 + 126) + (1, 6 + 50 + 64 + 70 + 140)$ $+ (2, 64 + 126) + (3, 50)$

$$+ [E_{ss} - (E_{ns} + E_{nc} + E_{sc})] \left(8/3\, N_s + \sum_{s_1 > s_2} \lambda_1^c \cdot \lambda_2^c\right)$$

$$\times [E_{cc} - (E_{ns} + E_{nc} + E_{ss})] \left(8/3\, N_c + \sum_{c_1 > c_2} \lambda_1^c \cdot \lambda_2^c\right)$$

$$+ E_{ns} \sum_{i>j} \lambda_i^c \cdot \lambda_j^c + E_{nc} \sum_{i>j} \lambda_i^c \cdot \lambda_j^c + E_{sc} \sum_{i>j} \lambda_i^c \cdot \lambda_j^c \Big\} \quad (7)$$

where $X_{\alpha\beta}$ stand for the operators $- \sum_{\alpha>\beta} (\lambda^a \sigma)_\alpha \cdot (\lambda^a \sigma)_\beta$ and the indices α and β refer to the types of the pair of quarks interacting *i.e.* $\alpha, \beta = n, s, c$. To evaluate the expectation values of these terms one has to make use of the permutation symmetry relation $P^F_{ij} P^J_{ij} P^C_{ij} = -1$ which permits the conversion of the colourspin operators to flavour spin operators. For example the use of the above permutation symmetry relations leads to the following relations

$$X_{nn} = 3/4\, N_n^2 - N_n - C_4^{I_n J_n} + 4/3\, J_n^2 + 4\, I_n^2,$$

$$X_{ns(c)} = N_{ns(c)} (N_{ns(c)} - 10) + 4/3\, J^2_{ns(c)} + 4C_3^{n(sc)},$$

$$X_{sc} = 3/4\, N_{sc}^2 - N_{sc} - C_4^{sc} + 4/3\, J_{sc}^2 + 4\, I^2,$$

where N, J and I stand for the number, spin and isospin operators and C_4 and C_3 are the Casimir operators of the appropriate SU (4) and SU (3) subgroups. Similar expressions can be deduced for the operators occurring in E_E.

5. Results and discussion

The mass estimates of the stable hadrons in the bag model depends on four parameters, namely the bag pressure B, the gauge coupling constant $\alpha_c = g^2/4\pi$, Z_0 the constant which parametrises the zero-point energy and the mass of the strange quark m_s. These parameters were obtained by de Grand *et al* (1975), in fitting the masses of the hadrons p, Δ, ω and Ω^-. In extending the model to SU (4) symmetry the mass of the charmed quark is needed and it was determined by Babu Joseph *et al* (1978) that $m_c = 1{\cdot}5$ GeV fits the data on the charmed mesons. In principle the bag radius R has to be determined for each state by minimising the ground state energy with respect to R. However in practice it has been found that an average value of R for the entire SU (8, FJ) multiplet gives a good fit to all the stable baryons. The functions $\alpha_i(R)$, $M_{ij}(R)$ and $E_{ij}(R)$ depending on the bag radius R and the masses of the quarks were evaluated numerically.

Assuming the above set of parameters the mass of the six, nine and fifteen quark systems having the same quantum numbers, as the physically interesting ordinary nuclei, hypernuclei and supernuclei were estimated. The mass operator is diagonal with respect to the quantum numbers C, Y, I and J and in some cases mixing between different flavour multiplets having the same quantum numbers occurs. Therefore an exact calculation of the masses of these states requires a complete knowledge of the SU (4) fractional parentage coefficients. Since these are not readily available and are too lengthy to calculate we have estimated the masses of these states in each of the irreducible representation in which they lie separately and have presented the limits of the masses. We now discuss the results presented in table 2. It may be noted that most of the results in the $C = 0$ sector agree well with the earlier results of Aerts *et al* (1978). However, in some cases differences arise due to a slight change in the values of the coefficients α_i, M_{ij} and E_{ij} which have been recalculated and due to the inclusion of the terms involving the charmed quark.

Table 2. Mass estimates of selected $C = 0$, $B = 2$ and 3 states.

Y	I	J	mass in GeV	Particle channel and threshold in GeV	SU(3) IR
$B = 2$					
2	3	0	2·297	$NN\pi\pi$ (2·16), $\Delta\Delta$ (2·472)	28
	2	1	2·201	$NN\pi$(2·2), ΔN(2·176)	35
1	5/2	1	2·275	$\Sigma N\pi$(2·273)	35
		0	2·332	$\Lambda N\pi\pi$(2·335)	28
	3/2	2	2·332	ΣN(2·133)	27
		1	2·258	$\Lambda N\pi$(2·195)	35
		0	2·208	ΣN(2·133)	27
0	2	2	2·396	$\Sigma\Sigma$(2·386)	27
		1	2·433	$\Xi N\pi$(2·398), $\Sigma\Lambda\pi$(2·448)	35
		0	2·452$^{(a)}$	$\Xi N\pi\pi$(2·548)	28
		0	2·311$^{(a)}$	$\Sigma\Sigma$(2·386)	27
	1	1	2·375$^{(a)}$	$\Xi N\pi$(2·398)	35
	0	0	2·179$^{(a)}$	$\Lambda\Lambda$(2·23)	1
			2·244$^{(c)}$	$\Lambda\Lambda$(2·23), ΞN(2·258)	27
−1	3/2	1	2·496	$\Xi\Lambda\pi$(2·573)	35
			2·432	$\Xi\Sigma$(2·511)	$\overline{10}$*
		0	2·669$^{(d)}$	$\Xi\Lambda\pi\pi$(2·713)	$\overline{28}$
			2·436	$\Xi\Sigma$(2·511)	27
	1/2	1	2·563$^{(d)}$	$\Xi\Lambda\pi$ (2·583)	35
		0	2·436	$\Xi\Lambda$(2·433)	27
−2	1	0	2·832$^{(a)}$	$\Xi\Xi\pi\pi$ (2·916)	28
	0	1	2·744$^{(a)}$	$\Xi\Xi\pi$(2·776)	35
			2·659$^{(a)}$	$\Xi\Xi$(2·636)	10
−3	1/2	1	2.966	$\Lambda\Xi$(2·990)	35
		0	3·023	$\Omega\Xi\pi$(3·130)	28
−4	0	0	3·293	$\Omega\Omega$ (3·344)	28
$B = 3$					
2	2	3/2	3·055	ΣNN(3·073)	64
		1/2	3·019	ΣNN(3·073)	35
	1	5/2	2.999	ΛNN(2·995)	27
		3/2	2·986	ΛNN(2·995)	27
		1/2	2·977	ΛNN(2·995)	27
	0	1/2	2·966	ΛNN(2·995)	10*

(a), (b), (c) and (d) represent uncertainties in mass by 55, 75 ~ 80, 20 and 115 MeV respectively.

5.1 *The* $C = 0$, $B = 2$ *and* 3 *states*

Most of the multiquark hadronic states with these quantum numbers will couple strongly to the ordinary dibaryonic and tribaryonic states. These have been discussed in fair detail by Aerts *et al* (1978) and the reason we have chosen to exhibit a few of the selected states in table 2 is to bring to notice certain special features of the states which may help in identifying these states in a less ambiguous manner than those considered heretofore. We may also mention here that our estimates of the masses of the multiquark states in this sector agree reasonably well with those of Aerts *et al* (1978). However there are some cases for which the mass estimates get differences for reasons mentioned earlier and this leads to interesting predictions. Apart from the $^3S_1$ and $^1S_0$ resonances predicted in the NN system at 2·111 GeV and 2·123 GeV respectively, resonances in the $^1D_2$ and the $^3D_3 - {}^3G_3$ NN channels are predicted. Recent experimental evidence from polarised nucleon-nucleon scattering shows some evidence for the existence of high spin resonances. However, it has not been established unambiguously whether these states may be explained with the conventional strong interaction forces between the nucleons. Similar statements hold true for the resonances predicted by the bag model in the $Y = 1$ channel. Therefore we study the resonances which in particular are coupled more strongly to multiparticle final states by virtue of their quantum numbers or by the overall symmetry of the state. For example the $I = 3$, $J = 0$ state in the $Y = 2$ channel which can be coupled to $\Delta\Delta$ should exhibit itself as a resonance in the $NN\pi\pi$ channel. In the case of the $Y = 1$ states the $I = 5/2$ state with $J = 1$ and 0 ought to be seen as a resonance and a bound state just above and below the thresholds in the $\Sigma N\pi$ and $\Lambda N\pi\pi$ channels respectively. The $Y = 0$, $I = 2$ states provide a rich field for the unambiguous establishment of the multiquark states. While the $J = 2$ state is expected to show up as a resonance just above the threshold in the $\Sigma\Sigma$ channel, the spin 1 and 0 states in the limit of exact symmetry should be coupled only to the three and four particle final states available to them since they lie in the SU (3) irreducible representation $\underline{35}$ and $\underline{28}$ respectively. This means that they should show up as resonance and bound state respectively in the $\Xi N\pi$ and $\Xi N\pi\pi$ final states. Similarly the $I = 1$, $J = 1$ state should show up as a bound state (or resonance due to the uncertainty in the energy) in the $\Xi N\pi$ final state. However since SU (3) symmetry is broken these may also be coupled to the $\Sigma\Sigma$, $\Xi N\pi$ and ΞN states as well. However this coupling is expected to be weaker and therefore if a study of the relevant reactions shows strong evidence for a resonance in the multiparticle final state but not so strong or none in the corresponding two particle channel this could be considered as a strong evidence for the multiquark state. Similar conclusions can be drawn for all the $B = 2$ states which lie in the $\underline{35}$ or $\underline{28}$ IR of SU(3) in table 2. The other states included in the table are bound states in the dibaryonic channels but which may be rather difficult to produce in experiments due to the much higher thresholds involved, significant among these being the $\Lambda\Lambda$ bound state which if produced would have a unique signature since it would decay only through weak decay modes. In the case of the $B = 3$ multiquark states we find that bound multiquark states are predicted only in the hypernuclear sector *i.e.* system with one hyperon and two nucleons. The significant difference from the $B = 2$ states is that the binding energies are much smaller of the order of 10–20 MeV only. However it is well-known from the data on light hypernuclei that they are very lightly bound with binding

Table 3. Mass estimates of $B = 2$, $C \neq 0$ states

$C = 1$

Y	I	J	Mass in GeV	Particle channel and threshold in GeV			
1	3/2	2	3·2636	C_1N(3·36)			
		1, 0	3·269[(a)]	C_1N(3·36)			
	1/2	3	3·2565	C_0N(3·20),	C_1N(3·36)		
		2	3·2569	C_0N(3·20),	C_1N(3·36)		
		1, 0	3·2576[(b)]	C_0N(3·20),	C_1N(3·36)		
0	2	1, 0	3·133[(a)]	$C_1 \Sigma$(3·615)			
	1	3	3·0747	$C_0 \Sigma$(3·453),	$C_1 \Lambda$(3·535)		
		2	3·075[(b)]	$C_0 \Sigma$(3·453),	$C_1 \Lambda$(3·535)		
		1	3·1334[(a)]	$C_0 \Sigma$(3·453),	$C_1 \Lambda$(3·535)		
		0	3·0754	$C_0 \Sigma$(3·453),	$C_1 \Lambda$(3·535)		
	0	2	3·1918	$C_0 \Lambda$(3·375)			
		1	3·0712	$C_0 \Lambda$(3·375)			
		0	3·1857	$C_0 \Lambda$(3·375)			
−1	3/2	3	3·647	$C_1 \Xi$ (3·735),	$S \Sigma$(3·75),	$\Lambda \Sigma$(3·65),	TN(3·62)
		2, 1	3·581[(a)]	$C_1 \Xi$ (3·735),	$S \Sigma$(3·75),	$\Lambda \Sigma$(3·65),	TN(3·62)
		0	3·647	$C_1 \Xi$ (3·735),	$S \Sigma$(3·75),	$\Lambda \Sigma$ (3·65),	TN(3·62)
	1/2	1, 0	3·574[(a)]	$C_0 \Xi$ (3·578),	TN(3·62)		
−2	1	2, 0	3·5834	$\Lambda \Xi$ (3·775),	$S \Xi$(3.79),	$T \Sigma$(3.87)	
	0	1	3·5791	$\Lambda \Xi$ (3·775),	$S \Xi$ (3·79),	$T \Lambda$(3.8)	
−3	1/2	2	3·9972	$T \Xi$ (3·998)			
		1, 0	3·7793	$T \Xi$ (3·998)			
			3·8883[(d)]	$T \Xi$ (3·998)			
−4	0	2	4·4524	$T \Omega$(4·5356)			
		1	4·4508	$T \Omega$(4·5356)			
		0	4·4492	$T \Omega$(4·5356)			
$C=2$							
0	2	2, 1, 0	4·596	$C_1 C_1$ (4·84)			
	1	3, 0	4·587	$C_1 C_0$ (4·68),	$X_u N$ (4·49)		
		2, 1	5·585[(b)]	$C_1 C_0$ (4·68),	$X_u N$ (4·49)		
	0	2, 1,	0 4·586	$C_0 C_0$ (4·52),	$X_u N$ (4·49),	$C_1 C_1$ (4·84)	
−1	3/2	1, 0	4·374	$X_s N$ (4·69),	$C_1 \Lambda$ (4·89)		
	1/2	2	4·362	$X_s N$ (4·69),	$C_0 \Lambda$ (4·73)		
		1	4·333[(a)]	$X_s N'$(4·69),	$C_0 \Lambda$ (4·73)		
		0	4·313	$X_s N$ (4·69),	$C_0 \Lambda$ (4·73)		
−2	1	2	4·692[(a)]	$\Lambda\Lambda$ (4·92),	$X_s \Sigma$ (4·94)		
		0	4·51[(c)]	$\Lambda\Lambda$ (4·92),	$X_s \Sigma$ (4·94)		
	0	2, 0	4·705[(c)]	$X_s \Lambda$ (4·87),	TC_0(4·94)		
−3	1/2	0	5·248[(d)]	ST (5·25),	AT(5·15)		
		1	5·0846	ST (5·25),	AT (5·15)		
		2	5·248[(d)]	ST (5·25),	AT(5·15)		
		3	5·4115	ST (5·25),	AT (5·15)		
−4	0	0	5·2648[(d)]	TT (5·36)			
		1	5·2521	TT (5·36)			
		2	5·4653	TT (5·36)			

(a), (b), (c) and (d) represent uncertainties in mass by 50 ~ 60 MeV, 4 MeV, 200 ~ 250 MeV, and 10 to 20 MeV respectively.

energies of the order of one MeV or less. If higher spin bound hypernuclear states are found with much larger binding energies these would also constitute unambiguous evidence for multiquark bag model states. A search for such states can be included in the K^- experiments which look for the conventional Λ and Σ hypernuclear states. It may be remarked here that the experimental results available in the baryonic sector so far are not sufficient to either confirm or rule out the existence of the bag model multiquark states.

5.2 *The* $C \neq 0$, $B = 2$ *and* 3 *states*

In tables 3 and 4 we have presented the mass estimates for the charm non-zero states. It is now known from the application of standard one-boson exchange potential models to the strong interaction of charmed baryons with ordinary baryons (Bhamathi *et al* 1981 and Dover *et al* 1977) that no bound states other than very lightly bound $C_1\,N$ system in the $I = 3/2$ state are expected. Further estimates of the binding energy of light three-body supernuclei show that these are likely to be of the

Table 4. Mass estimates of q^9 system with $B = 3$, and $C = 1$.

Y	I	J	Mass in GeV	Eth in GeV in Tribaryon channels
2	2	3/2	4·409	$C_1\,NN$ (4·30)
		1/2	4·405	$C_1\,NN$ (4·30)
	1	5/2	4·381	$C_0\,NN$ (4·14) $C_1\,NN$ (4·30)
		3/2	4·374[b]	$C_0\,NN$ (4·14) $C_1\,NN$ (4·30)
		1/2	4·364[a]	$C_0\,NN$ (4·14) $C_1\,NN$ (4·30)
	0	5/2	4·361	$C_0\,NN$ (4·14) $C_1\,NN$ (4·30)
		3/2	4·354[b]	$C_0\,NN$ (4·14) $C_1\,NN$ (4·30)
1	5/2	3/2	4·599	$C_1\,\Sigma\,N$ (4·553)
		1/2	4·591	$C_1\,\Sigma\,N$ (4·553)
	3/2	5/2	4·562	$C_0\,\Sigma\,N$ (4·393) $C_0\,\Lambda\,N$ (4·315), $C_1\,\Lambda\,N$ (4·475) $C_1\,\Sigma\,N$ (4·553)
		3/2	4·526[c]	$C_0\,\Sigma\,N$ (4·393), $C_0\,\Lambda\,N$ (4·315), $C_1\,\Lambda\,N$ (4·475) $C_1\,\Sigma\,N$ (4·553)
		1/2	4·585	$C_0\,\Sigma\,N$ (4·393), $C_0\,\Lambda\,N$ (4·315), $C_1\,\Lambda\,N$ (4·475) $C_1\,\Sigma\,N$ (4·533)
	1/2	7/2	4·525	$S\,NN$ (4·45), $A\,NN$ (4·35)
		5/2	4·508[d]	$S\,NN$ (4·45), $A\,NN$ (4·35)
		3/2	4·503	$S\,NN$ (4·45), $A\,NN$ (4·35)
0	2	3/2	4·577	$C_0\,\Sigma\Sigma$ (4·646)
		1/2	4·533	$C_0\,\Sigma\Sigma$ (4·646)
	1	5/2	4·540	$C_0\,\Lambda\Sigma$ (4·568), $C_0\,\Sigma\Sigma$ (4·646), $C_1\,\Lambda\Lambda$ (4·650) $C_1\,\Sigma\Sigma$ (4·806)
		3/2	4·495[d]	$C_0\,\Lambda\Sigma$ (4·568), $C_0\,\Sigma\Sigma$ (4·646), $C_1\,\Lambda\Lambda$ (4·650) $C_1\,\Sigma\Sigma$ (4·806)
		1/2	4·477[d]	$C_0\,\Lambda\,\Sigma$ (4·568), $C_0\,\Sigma\Sigma$ (4·646), $C_1\,\Lambda\Lambda$ (4·650) $C_1\,\Sigma\Sigma$ (4·806)

Table 4. (*Contd.*)

Y	I	J	Mass in GeV	Eth in GeV in tribaryon channels
	0	5/2	4·473	$SN\Lambda$(4·625), $SN\Sigma$(4·703), $AN\Lambda$(4·525), $AN\Sigma$(4·603)
		3/2	4·485[c]	$SN\Lambda$(4·625), $SN\Sigma$(4·703), $AN\Lambda$(4·525), $AN\Sigma$(4·603)
		1/2	4·476[a]	$SN\Lambda$(4·625), $SN\Sigma$(4·703), $AN\Lambda$(4·525), $AN\Sigma$(4·603)
−1	3/2	5/2	4·830	$S\Sigma\Sigma$(4·956), $A\Sigma\Sigma$(4·856), $TN\Sigma$(4·813), $S\Sigma\Lambda$(4·878) $A\Sigma\Lambda$(4·778)
		3/2	4·760[b]	$S\Sigma\Sigma$(4·956), $A\Sigma\Sigma$(4·856), $TN\Sigma$(4·813), $S\Sigma\Lambda$(4·878) $A\Sigma\Lambda$(4·778)
		1/2	4·760[b]	$S\Sigma\Sigma$(4·956), $A\Sigma\Sigma$(4·856), $TN\Sigma$(4·813), $S\Sigma\Lambda$(4·878) $A\Sigma\Lambda$(4·778)
	1/2	7/2	4·775	$S\Sigma\Sigma$(4·956), $A\Sigma\Sigma$(4·856), $TN\Sigma$(4·813), $S\Sigma\Lambda$(4·80) $A\Lambda\Lambda$(4·70), $TN\Lambda$(4·735), $A\Sigma\Lambda$(4·778)
		5/2	4·716[d]	$S\Sigma\Sigma$(4·956), $A\Sigma\Sigma$(4·856), TN(4·813), $S\Sigma\Lambda$(4·80)
		3/2	4·644[d]	$A\Lambda\Lambda$(4·70), $TN\Lambda$(4·735) $A\Sigma\Lambda$(4·78)
		1/2	4·635	$S\Sigma\Sigma$(4·956), $A\Sigma\Sigma$(4·856), $TN\Sigma$(4·813), $S\Sigma\Lambda$(4·80) $A\Lambda\Lambda$(4·70), $TN\Lambda$(4·735), $A\Sigma\Lambda$(4·778)
−2	1	5/2	4·916	$T\Xi N$(4·938), $T\Sigma\Sigma$(5·066)
		3/2	4·849	$T\Xi N$(4·938), $T\Sigma\Sigma$(5·066)
		1/2	4·811[d]	$T\Xi N$(4·938), $T\Sigma\Sigma$(5·066)
	0	5/2	4·831	$T\Xi N$(4·938), $T\Lambda\Lambda$(4·910), $T\Sigma\Sigma$(5·066)
		3/2	4·831	$T\Xi N$(4·938), $T\Lambda\Lambda$(4·910), $T\Sigma\Sigma$(5·066)
		1/2	4·845	$T\Xi N$(4·938), $T\Lambda\Lambda$(4·910), $T\Sigma\Sigma$(5·066)
−3	1/2	3/2	5·07	$T\Xi\Lambda$(5·113)
		1/2	5·07	$T\Xi\Lambda$(5·113)

(a), (b), (c) and (d) represent uncertainties in mass by 3 ∼ 4 MeV, 5 ∼ 10 MeV, 9 ∼ 20 MeV, 40 ∼ 70 MeV respectively.

same order of magnitude as the hypernuclei *i.e.* at the most a few MeV. An examination of table 3 shows that several deeply bound multiquark states and resonances are predicted by the bag model. For $C = 1$, bound states are expected in the C_1N and C_0N channels in the $I = 3/2$ as well as $I = 1/2$ states unlike the case of conventional strong interaction forces. In fact deeply bound states are predicted in almost all the $B = 2$ channels. In the $I = 2$ channel with $C = 2$ a virtual bound state of C_1C_1 system decaying into a (X_uN) system is predicted as well as a C_1C_0 virtual bound state with $I = 1$ decaying as a X_uN resonance is predicted. Even more exotic bound state with $C = 2$ and strangeness -2 are predicted. However all these exotic states would be very difficult to produce except perhaps the lowest C_0C_0, C_1N states.

Table 4 shows us that in the case of $B = 3$ multiquark states in the $C = 1$, $Y = 2$ sector (*i.e.* light supernuclei which are the analogues of hypernuclei) no bound states are predicted. In the $Y = 1$ sector there are two resonant state (marked with an *) which will be coupled to several channels. However in the $Y = 0$ to -3 sector there are several bound states predicted again it will be rather difficult to produce most of these states due to their exotic quantum numbers.

Table 5. Mass estimates for q^9 systems with baryon number $B = 3$ and $C = 2$.

Y	I	J	Mass in GeV	Eth in GeV in tribaryon channel
1	3/2	3/2	5·724	$C_1\,C_0\,N$ (5·62), $C_1\,C_1\,N$ (5·780)
	1/2	1/2	5·672	$C_1\,C_0\,N$ (5·62), $C_1\,C_1\,N$ (5·780) $C_0\,C_0\,N$ (5·46)

Table 6. Mass estimates for q^{15} systems, with $B = 5$ and $C = 0$·1

C	Y	I	J	Mass in GeV	Eth in GeV in pertabaryon channel
0	3	3/2	3/2	5·583	$\Sigma\Sigma\,NNN$ (5·206), $\Lambda\Lambda\,NNN$(5·05), $\Xi\,NNNN$ (5·078), $\Sigma\Lambda\,NNN$ (5·128)
		1/2	1/2	5·539	$\Sigma\Sigma\,NNN$ (5·206), $\Lambda\Lambda\,NNN$ (5·05), $\Xi\,NNNN$ (5·078), $\Sigma\,\Lambda\,NNN$ (5·128)
1	3	1/2	3/2	6·584	$C_0\,\Lambda\,NNN$ (6·195), $C_1\,\Lambda\,NNN$ (6·355) $C_0\,\Sigma\,NNN$ (6·273), $C_1\,\Sigma\,NNN$ (6·433)
		1/2	1/2	6·589	$C_0\,\Lambda\,NNN$(6·195), $C_1\,\Lambda\,NNN$ (6·355), $C_0\,\Sigma\,NNN$(6·273), $C_1\,\Lambda\,NNN$ (6·433)

Finally as a matter of curiosity we have exhibited in tables 5 and 6 estimates of the lowest mass $C = 2$, $B = 3$ states and the $C = 0$ and 1, $B = 5$ states. It can be seen that no deeply bound states are expected in these cases.

Thus we conclude that the most likely places to look for the existence or otherwise of multiquark states is in the sectors with $B = 2$ and 3 $C = 0$, 1 or 2 in the specific channels as indicated in the text of the paper.

References

Aerts A Th M, Mulders P J C and de Swart J J 1978 *Phys. Rev.* **D17** 260
Babu Joseph K, Sabir M and Sreedharan Nair M N 1978 *Pramāṇa* **11** 195
Bhamathi G, Prema K and Ramachandran A 1980 *Prog. Theor. Phys.* **64** 330
Bhamathi G and Prema K 1981 *Pramāṇa* **17** 481
Chew G F 1976 LBL Preprint 5391
Chodos A, Jaffe R L, Johnson K and Thorn C B 1974 *Phys. Rev.* **D10** 2599
De Grand T, Jaffe R L, Johnson K and Kiskis J 1975 *Phys. Rev.* **D12** 2060
Dover C B, Kahana C B and Trueman L 1977 *Phys. Rev.* **D16** 799
Igi K 1978 Proceedings of the 19th International Conference on High Energy Physics, Tokyo (contribution A6)
Igi K and Yazaki S 1979 *Prog. Theor. Phys.* **61** 487
Jaffe R L 1977 *Phys. Rev.* **D15** 267; 1977 *Phys. Rev. Lett.* **38** 195
Rosenzweig C 1976 *Phys. Rev. Lett.* **36** 697
Sorba P and Hogaasen H 1978 CERN T H 2531
Sorba P, Hogaasen H and de Crombrugghe M 1978 CERN T H 2537