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Bag like potential and quarkonium

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Abstract. We discuss the properties of charmonium in the frame work of a bag like potential and show that the upsilon and charmonium spectra can be fitted by a common potential.

Keywords. Bag like potential; quarkonium, charmonium spectra; upsilon spectra.

1. Introduction

There have been numerous fits of the charmonium and upsilon spectra with various potentials (Appelquist and Politzer 1975; de Rejula and Glashow 1975; Eichten 1978; Quigg and Rosner 1979) with varying degree of success, the common version being $-(4/3)\,\alpha_s/r + ar$ the first term supposed to be due to one gluon exchange while second is the confining potential. There have been improved versions like $-16\pi/27 \dfrac{1}{r \log (r/r_0)}$ which takes the asymptotic freedom into account, and the potential which in medium range of distances behaves like $\log (r/r_0)$. There also have been bag models of hadrons (Chaddos 1974; Lee 1979) which envision them as systems of quarks confined by an infinite potential barrier to the interior of a spherical cavity of radius r_0 where they are assumed to be free. In the present paper we propose to soften the surface of the bag by replacing the infinite barrier at $r = r_0$ by a finite one of height C/r_0. When the quark separation is larger than r_0 the system is stabilized by a linearly rising potential, thus confining the quarks and at very short distances asymptotic freedom operates. Two important requirements are:

(i) the velocity of quarks inside the bound state is small, in order to substantiate the non-relativistic approximation.
(ii) no additive term is allowed in the formula relating the masses of a bound state to its non-relativistic energy eigen values.

In the present paper we study the properties of charmonium and upsilon families and also calculate the constants of vector mesons.

2. Model

The mass of the nth radial excitation of the s-wave $q\bar{q}$ bound state is given by (Martin 1980)

$$M(n, m_q) = 2\, m_q + E_n, \tag{1}$$

where m_q is the quark mass and E_n is the energy eigenvalue of Schrödinger equation for the n^3S_1 states. The potential that we use is

$$V(r) = (r - r_0)\,[B\,(r - r_0) + C/r]. \tag{2}$$

For solving the Schrödinger equation the radial equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{l(l+1)}{r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{2\mu}{\hbar^2}\{V(r) - E\}\right] R = 0,$$

is brought into the dimensionless form

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} - \frac{l(l+1)}{\rho} + \frac{2}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho} + \frac{2}{\rho} - \epsilon\right] R = 0,$$

where

$$\epsilon = \frac{2\mu r}{\rho^2\,\hbar^2}(C - E)$$

and

$$\rho = \frac{\mu}{\hbar^2} r^2\,[B\,(r - r_0)^2 - Cr_0].$$

Assuming that r_0 is proportional to the Compton wave length of the bound state we can write

$$r_0^{\psi'} = \frac{M_\psi}{M_{\psi'}} r_0^{\psi} \quad ; \quad r_0^{\psi''} = \frac{M_{\psi'}}{M_{\psi''}} r_0^{\psi'}$$

$$r_0^{\Upsilon'} = \frac{M_{\uparrow}}{M_{\uparrow'}} r_0^{\Upsilon} \text{ and so on.}$$

Thus we are able to find the values of r_0, r_1, r_2, r_3, etc. in each case. Using these values of r_0 and r, we fit the values of other parameters. They are found to be

$B = 1832{\cdot}312$ GeV/fm^2 and $C = -\,372{\cdot}8$ GeV for charmonium system.

$B = 1223{\cdot}22$ GeV/fm^2 and $C = -\,499{\cdot}89$ GeV for upsilonium.

These values are found to give acceptable fit for both the spectra as given in table 1.

3. Decay constants

Poggio and Schnitzer (1972) have shown that most of the next to leading corrections in α_s and quark velocity to the leptonic decay of a quarkonium state in a static linear

Table 1. Mass spectra of charmonium and upsilonium.

Charmonium μ=reduced mass of c-quark = 1·5/2 GeV

States	Energy (GeV) Theory	Experiment
1S	3·095	3·095
2S	3·69	3·684
3S	4·035	4·08
4S	4·36	4·414
Upsilonium μ-reduced mass of b-quark = 4·73/2 GeV		
1S	9·46	9·46
2S	10·02	10·02
3S	10·346	10·35
4S	10·554	10·57

confinement potential can be included in the Van Royen-Weisskopf formula. Using their results, only in truly non-relativistic system we can reliably calculate

$$f^2\,({}^3S_1\,(\bar{\mu}\mu) \rightarrow e^+\,e^-) = |\phi\,(0)|^2 \times 4 \times M^{-1}_{\mu\mu}$$
$$= \frac{3\,M_\mu\,\Gamma_{e^+e^-}}{2\pi\,\alpha^2}, \tag{3}$$

where $\phi(0)$ is the Schrödinger wave function at the origin. When the states become more and more relativistic, perturbation theory quickly becomes unreliable and breaks down as is probably the case for the light mesons π and K. A relativistic treatment of the bound state problem is only known in principle or for special cases *i.e.* the Bethe salpeter equation in ladder approximation. We understand charmonium and upsilonium in terms of the same non-relativistic potential picture. We find both phenomenologically and experimentally (Flugge 1979).

$$\frac{f_v^2\,(\Upsilon \rightarrow e^+e^-)}{M_\Upsilon} = \frac{f_v^2\,(\psi \rightarrow e^+e^-)}{M_\phi}. \tag{4}$$

For vector mesons we write the decay constants as

$$f_v \sim \frac{30\,(\mathrm{MeV})^{1/2}}{(M_b + M_q)^{1/2}\left(\frac{1}{M_b} + \frac{1}{M_q}\right)}, \tag{5}$$

or $f_v = d_\mu \cdot\ M^{-1/2}$ where $d = 30\,(\mathrm{MeV})^{1/2}$.
d is given by Krasemann (1980). Using (5) the decay constants of B-mesons are reported in table 2.

Table 2. Decay constants of upsilon family.

Masses of light quarks are taken as:

$m_{u,d} = 35$ MeV and $m_s = 330$ MeV

Vector meson	Decay constant(MeV)
f_{B_u}	134
f_{B_s}	193·28
f_{B_c}	413·35
f_{Υ}	675

4. Conclusion

We find that this fit is in remarkable agreement with experiment and has the advantage of containing the qualities of both the potential model and the bag model. The model suggested here envisages both the asymptotic freedom and quark confinement.

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