Pramāṇa, Vol. 18, No. 3, March 1982, pp. 271–278. © Printed in India.

Incoherent scattering functions of 145 keV gamma rays by K-shell electrons in Y, Ag and Au

A RAGHAVA RAO, S V S RAMANA REDDY, K PREMCHAND, K L NARASIMHAM, K PARTHASARADHI* and V LAKSHMI-NARAYANA†

Laboratories for Nuclear Research, Andhra University, Waltair 530 003, India
*Cyclotron Institute, Texas A & M University, Texas, USA
†Physics Department, National University of Singapore, Singapore

MS received 8 June 1981; revised 27 January 1982

Abstract. The values of incoherent scattering functions are determined experimentally for 145 keV gamma rays in elements Au, Ag and Y at scattering angles 40°, 70° and 100°, using a x-ray gamma coincidence technique. The corresponding theoretical values are obtained from the tabulations of Hubbell *et al*, and computed from the models of Jauch and Rohrlich and Shimizu *et al*. A comparison between the theoretical and experimental results showed that the non-relativistic approach adopted in the theory of Shimizu *et al* is inapplicable to the present cases. A gross agreement is noticed between the present experimental results and the other theoretical values.

Keywords. Incoherent scattering function; relative photopeak efficiency; x-ray-gamma ray coincidence.

1. Introduction

Scattering of gamma rays by electrons with an associated change in the energy of the deflected gamma ray is known as Compton scattering or incoherent scattering. Assuming free and stationary character for scattering electrons, this process can be described as an elastic collision and results in an unique energy for the scattered photon for any given angle of scattering. Free electron scattering cross-sections are described by the Klein-Nishina Law. The basic assumptions underlying this approach are valid for scattering of high energy gamma rays by outer shell electrons and by electrons in light atoms. For scattering by bound electrons in innermost shells of heavy atoms and for low energy photons these assumptions fail, resulting in modifications of both the energy kinematics and cross-section probabilities. Binding effects are estimated by a factor called incoherent scattering function. It is defined as the probability for an atom to get excited or ionised when one of its electrons receives certain amount of recoil momentum. The differential cross-section for incoherent scattering by electrons, bound in an atom of atomic number Z can be written as

$$\frac{d\sigma_{a\ \text{incoh}}}{d\Omega} = \frac{d\sigma_e}{d\Omega} \cdot Z \cdot S(q, z), \tag{1}$$

where $d\sigma_e/d\Omega$ represents Klein-Nishina cross-section per electron and $S(q, z)$ is the

incoherent scattering function. It is numerically equal to the bound to free electron scattering cross-section ratio.

The incoherent scattering function is usually expressed as $S(v)$ or $S(x, z)$, where $v = (0{\cdot}333\ qaz^{3/2}/\hbar)$ and $x = \sin(\theta/2)/\lambda$. Here q is the momentum transferred to the electron, a is the Bohr radius and θ is the scattering angle. Calculations of $S(v)$ or $S(x, z)$ are made, using different atomic models. Bewilogua (1931) and Wheeler and Lamb (1939, 1956) carried out calculations of $S(v)$ based on Thomos Fermi Model for $v > 0{\cdot}04$ and for $v \leqslant 0{\cdot}04$ respectively. $S(x, z)$ is not a function of a single variable and hence a separate calculation is needed for each element. Cromer and Mann (1967) and Cromer (1969) carried out non-relativistic calculations of $S(x, z)$ based on Hartree-Fock-Slater model for $Z = 2$ to 100 and x from 0·005 to 80 Å^{-1}. Shimizu *et al* (1965) calculated the incoherent scattering function S_k exclusively for K-shell electron scattering, using non-relativistic hydrogen like wave functions. The general behaviour of the incoherent scattering function, computed using various atomic models, is that it becomes zero in the limit of vanishing energy or scattering angle, becomes unity in the limit of infinite momentum transfer, and for other values of q and z, $S(q, z)$ varies between zero and one. However this trend is not in agreement with the relativistic theory of incoherent scattering by Jauch and Rohrlich (1955), where they observed that the bound-to-free electron scattering cross-section ratio exceeds one at backward angles (*i.e.* at high momentum transfers). In the development of their theory they assumed the electron to be free but not stationary. They developed a formula for the differential incoherent scattering cross-section. Motz and Missoni (1961) slightly modified their formula which is used in the computations of the theoretical values of the present work.

The incoherent scattering functions exclusively for K-shell electrons can be experimentally determined using a coincidence method. It is well-known that in Compton scattering, the electron is either ionised or excited. If a K-vacancy occurs as a result of this process, the subsequent filling of it produces the characteristic K-x-ray. Thus the scattered photon is in coincidence with this x-ray and a coincidence gating therefore selects K-electron scattering events exclusively. In view of the direct and specific information furnished by these studies extensive investigations (Brini *et al* 1960; Sujkowski and Missoni 1961; Motz and Missioni 1961; Varma and Eswaran 1962; Dilazzaro and Missoni 1966; Shimizu *et al* 1965; Ramalingareddy *et al* 1966, 1967, 1968; Pingot 1968, 1969; East and Lewis 1969; Krishnareddy *et al* 1970, 1971, 1974; Murty *et al* 1973; Spitale and Bloom 1974, 1977) were carried out earlier. In all these studies the qualitative trends were observed. However there are considerable divergencies and discrepancies in the values of the incoherent scattering functions derived from these studies. A common observation was that no single theory could predict the trends observed in the experiments. An extensive programme of work has been undertaken in the Laboratories for Nuclear Research, Andhra University, Waltair and as a part of this programme some measurements with a coincidence system were carried out in elements, Y, Ag and Au using 145 keV gamma rays.

2. Experimental details

The general view of the geometrical arrangement used in the present investigations is shown in figure 1. The source supplied in a sealed metallic capsule is introduced

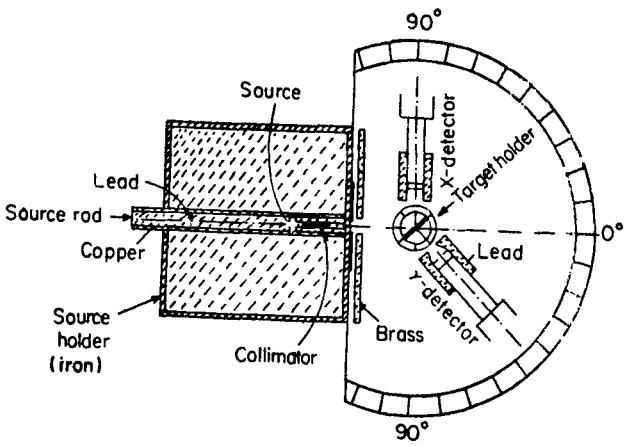

Figure 1. A general view of the geometrical arrangement of the NaI(Tl) coincidence system.

in a hole, drilled in a lead filled copper tube, smoothly running into the axial brass tube of a lead filled cylinder. Provision is made at the other end of the axial tube to collimate the gamma rays by the introduction of various collimating tubes defining different solid angles at the target. The heavy source holder rests in a groove of a heavy duty iron stand. The face on the target side of the source holder is covered with 5 mm thick brass sheet with a 1″ hole in the centre to absorb the lead x-rays produced in the surface layers of the source holder. The target assembly is fixed at the centre of a table marked in degrees with a glass plate on its top.

The two NaI(Tl) detectors are held in position on either side of the scatterer by two aluminium stands. The x detector is fixed on one side of the scatterer and the angular position of the gamma detector could be varied by placing the stand at different angles. The NaI(Tl) crystals used for gamma ray and x-ray detection are of dimensions $1\frac{3}{4}$″ diameter × 2″ thick and $1\frac{3}{4}$″ diameter × 2 mm thick respectively. These crystals are mounted on two photo multipliers of type RCA 8575. To minimize false coincidence events the crystals are surrounded by conical lead shields. The characteristics of each one of the detectors are studied independently. The resolution of the gamma detector is found to be 15% at 145 keV and that of the x-detector is found to be 22% to 30 keV. Figure 2 shows the block diagram of the coincidence system. The output from the anode of each of the photomultiplier tubes is used to generate the time logic pulses by constant fraction technique. The fast channel is formed with two constant fraction discriminators (ORTEC Model 473A), and a time to pulse height converter and single channel analyser (ORTEC Model 467). The output from time to pulse height converter forms one of the inputs to the slow coincidence unit. The other two inputs to the slow coincidence unit are derived from the x- and gamma ray channels as shown in figure 2. The gated spectrum is recorded on the ND 512 multichannel analyser.

The foils used are of Au (161·15 mg/cm²), Ag (105·8 mg/cm²) and Y (134·2 mg/cm²). The source used is ^{141}Ce of 530 mC strength supplied by the isotope division of BARC, Bombay, India.

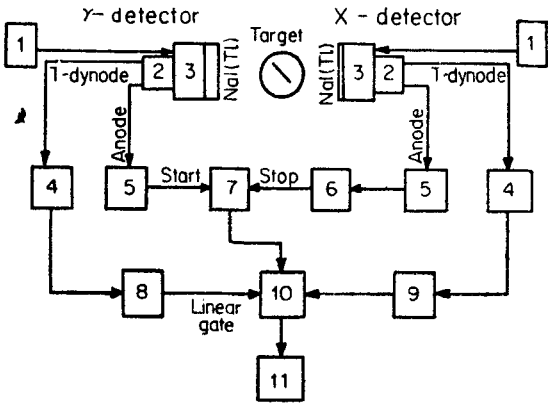

Figure 2. Block diagram of the NaI(Tl) coincidence system: 1. High voltage unit, 2. emitter follower, 3. RCA 8575, 4. linear amplifier—PA563, 5. constant fraction discriminator—473A, 6. nano sec. delay—DU672, 7. TPNC/SCA—467, 8. stretcher—PS665, 9. timing SCA—SC 617, 10. slow coincidence—LG 666, 11. MCA—ND 1100.

The bound-to-free electron scattering cross-section ratio is determined by using the formula given below.

$$\frac{d\sigma_K}{d\sigma_F} = \left[\frac{N_c/\epsilon_{\gamma K}\ \nu_{ab}\ (\gamma k)}{N_F/\epsilon_\gamma\ \nu_{ab}\ (\gamma)}\right] \times \left[\frac{1}{\omega_x\ \epsilon_x\ \nu_{ab}\ (x)}\right] \times \left[\frac{1}{E_c}\right] \times \left[\frac{1}{\nu_K}\right] \times \left[\frac{d_{Al}}{d'} \cdot \frac{A}{A_{Al}} \cdot \frac{13}{2}\right], \tag{2}$$

where N_c is true coincidence rate, ϵ_{γ_k} is the efficiency of the gamma detector for the scattered spectrum produced by K-shell electrons, ω_x is the solid angle of the x-ray detector and ϵ_x is the photo peak efficiency of the x-detector and E_c is the coincidence efficiency of the system. d' and d_{Al} are the thicknesses of the experimental foil and aluminium scatterer respectively expressed in mg/cm^2. A and A_{Al} are the atomic weights of the target element and aluminium respectively. ν_K is the K-shell fluorescent yield. $\nu_{ab}(\gamma k)$ is the absorption factor of the scattered gamma ray due to K-shell electron and $\nu_{ab}(x)$ is the absorption factor for x-rays. N_F is the count rate recorded under the scattered spectrum by electrons in the aluminium foil. ϵ_r and $\nu_{ab}(\gamma)$ are the photo peak efficiency and absorption factor respectively of the gamma detector for photons scattered by aluminium target. Here it is assumed that the electrons in aluminium are free.

It can be seen from equation (2) that it is conveniently arranged as a product of five factors. The last factor involves numerical constants and data on the scatterer which can be evaluated exactly. ν_k, the fluorescent yield is obtained from the tabulated values of Bambynek *et al* (1972).

3. Determination of the factors

3.1 *The first factor*

$$\frac{N_C/\epsilon_r(k)\ \nu_{ab}(rk)}{N_F/\epsilon_r\nu_{ab}(r)}$$

From the recorded coincidence spectrum with a given scatterer, the chance and false coincidences are subtracted to yield the true coincidence count rate. The chance rate is evaluated channel by channel covering the entire spectrum with the individual singles rates and resolving time of the coincidence circuit using the relation, N-chance $= 2\tau N_x N_\gamma$. The false events are estimated by conducting a coincidence experiment with an equivalent aluminium foil in the scatterer position. The false events in the experiments may arise due to any effect introducing an event in the window selected the x-ray channel and anywhere in the scattered photon spectrum recorded in the multichannel analyser. In the experiment it is presumed that all the effects that contribute to false events are taken care of by replacing the scatterer by an equivalent aluminium foil. The coincidence spectra recorded with the scatterers and aluminium foils are first corrected for efficiency and absorption in the scatterers. In the case of spectrum with the experimental scatterer it is observed that the spectrum extends over a wide range of energies. For efficiency and absorption correction therefore the spectrum is stripped into a number of 20 keV blocks and the pooled average in each block is corrected for efficiency and absorption. The efficiencies in different energy channels are inferred from the relative photo peak efficiency curve of the detector as a function of energy. To effect the absorption correction, the individual count rates in the blocks are multiplied by $e^{\mu x}$ where μ represents the absorption coefficient assumed from the tables of Storm and Israel (1970) and x is the mean thickness of the scatterer. Similar procedure is followed for the estimation of false coincidence. The final value of the numerator in the first factor is obtained by subtracting the pooled chance and false counts from the pooled gross counts. The denominator in the first factor is estimated from the singles spectrum recorded in the gamma detector with thin aluminium target. In this case the spectrum distribution being relatively narrow the counts under the photo peak of the background subtracted spectrum are corrected for efficiency and absorption in the scatterer.

3.2 *The second factor*

$$\left[\frac{1}{\omega_X \epsilon_X \nu_{ab}(X)}\right]$$

This factor represents the solid angle subtended by the x-ray detector, the photo peak efficiency and absorption factor for x-rays. Absorption correction in the scatterer is carried out in the usual way by assuming exponential law, absorption coefficients (μ) and mean thicknesses of the scatterers. The other terms $\omega_X \epsilon_X$ are estimated collectively by conducting an auxiliary experiment to determine the absolute efficiency of the x-detector. For this purpose a ^{133}Ba source is placed in the target position and the absolute efficiencies are extracted for various energies from singles and coincidence rates. A plot of the absolute efficiency as a function of energy with this radioactive source is used in the estimation of absolute efficiencies at the energies of present interest.

The coincidence efficiency E_c of the system is taken as one and is also verified by conducting an auxiliary experiment with ^{133}Ba source in the target position.

Substituting the various factors evaluated as described above, in (2) the final values of $d\sigma_K/d\sigma_F$ are obtained in all cases in the present investigations.

4. Results and discussion

The experimental results obtained with different foils and different scattering angles at 145 keV are summarized in table 1. It can be seen from the table that the experimental value of the ratio varies from 0·56 in Au at 40° to 1·55 in Y at 100°. The former result corresponds to a reduction over the free electron value while the latter represents an enhancement. For a given momentum transfer (fixed scattering angle) the ratio decreases with increase in atomic number. For a given element for low scattering angles the ratio is smaller than one while for larger scattering angles the ratio exceeds one. These general trends are observed in most of the earlier investigations also.

The present experimental values however cannot be compared with other experimental results inasmuchas no data exist at 145 keV in the elements investigated here. Spitale and Bloom (1974, 1977) were the only investigators who carried out measurements at 145 keV in elements Ho, Sn and Fe. Their results in Sn can be compared with the present results in Ag. They however did not furnish numerical values of $d\sigma_K/d\sigma_F$. The values inferred from their graphs indicated substantial decrease in the ratio for 40°. For angles of scattering 70° and 100° however their values were close to unity. Taken together with the present experimental errors there is a gross agreement with the result of Spitale and Bloom (1974, 1977).

In the present experiments, the results with Au scatterer are of special interest, in view of the fact that the degraded photon spectrum in no case extends upto the full energy expected from kinematics of free electron scattering. The binding effects are therefore expected to be severe. As the binding energy of the K electron decreases towards lower Z, the situation is expected to be less severe for Y scatters, the case of Ag scatters being intermediary.

The present experimental values of incoherent scattering functions may be compared with the theoretical values furnished in table 1. The table includes three sets

Table 1. Comparison of experimental and theoretical values of incoherent scattering functions for 145 keV Gamma rays

Scattering angle in degrees		Au	Ag	Y
40°	Expt.	0·56±0·07	0·7±0·1	0·96±0·29
	Theoretical:			
	Shimizu	0·072	0·144	0·188
	Jauch and Rohrlich	0·907	0·97	0·979
	Hubbell *et al*	0·869	0·922	0·937
70°	Expt.	0·93±0·07	0·93±0·17	1·33±0·4
	Theoretical:			
	Shimizu	0·121	0·266	0·366
	Jauch and Rohrlich	1·172	0·997	0·967
	Hubbell *et al*	0·939	0·960	0·969
100°	Expt.	1·22±0·08	1·19±0·2	1·55±0·47
	Theoretical:			
	Shimizu	0·159	0·38	0·539
	Jauch and Rohrlich	2·163	1·229	1·15
	Hubbell *et al*	0·967	0·982	0·988

of theoretical values. One set is obtained by interpolation of the values tabulated by Hubbel *et al* (1975). These are however the 'whole atom' values. The second set is evaluated using the explicit formulae of Shimizu *et al* (1965). These values are specifically for K shell electrons. But non-relativistic wavefunctions were employed in arriving at the formulae. The third set of theoretical values are evaluated from the formulae furnished by Jauch and Rohrlich (1955). These formulae were arrived on the basis of large momentum transfer to the struck electron, so that it could be considered to be free. But because of the finite momentum of the electron in the K-shell, the stationary criterion in classical scattering does not hold. In the present case, since the primary photon energy is small, the momentum transferred to the electron in the collision is not considerably larger than the momentum of the electron in K-shell. Thus, the formulae are not expected to hold strictly, at least in the case of Au scatterers. Thus the present experimental values are not expected to show complete agreement with any of the theoretical sets.

It may be observed from the table that the present experimental values in all cases, are higher than the theoretical values of Shimizu *et al*, thereby indicating that non-relativistic approach is inapplicable in the present case. The agreement in some cases with the other sets only indicates that predominant contribution to the binding effects arises from K shell electrons or free and non-stationary criteria partially hold. The present experimental study indicates the need for a rigorous relativistic approach in the evaluation of K-shell binding effects.

References

Bambynek W, Crasemann B, Fink R W, Freund H U, Mark H, Swift C D, Price R E and Venugopala Rao P 1972 *Rev. Mod. Phys.* **44** 716

Bewilogua L 1931 *Z. Phys.* **32** 740

Brini D, Puschini E, Grimelini N T and Murty D S R 1960 *Il Nuovo Cimento* **16** 1727

Cromer D T 1969 *J. Chem. Phys.* **50** 4857

Cromer D T and Mann J B 1967 *J. Chem. Phys.* **47** 1892

Dilazzaro M A and Missoni G 1966 *ISS* **66/6**, 7, 8

East L V and Lewis E R 1969 *Physics* **44** 595

Hubbel J H, Veigele Wm J, Briggs E A, Brown R T, Cromer D T and Howerton R J 1975 *J. Phys. Chem. Ref. Data* **4** No. 3

Jauch and Rohrlich F 1955 *The theories of photons and electrons* (Cambridge: Addison Wesley) p. 232

Krishnareddy D V, Narasimhacharyulu E and Murty D S R 1970 *Proc. Indian Acad. Sci.* **A72** 785

Krishnareddy D V, Narasimhacharyulu E and Murty D S R 1971 *Indian J. Pure Appl. Phys.* **9** 305

Krishnareddy D V, Narasimhacharyulu E and Murty D S R 1974 *Physica* **75** 394

Motz J W and Missoni G 1961 *Phys. Rev.* **124** 1458

Murty D S R, Govindareddy V and Narasimhacharyulu E 1973 *J. Phys.* **A6** 265

Pingot O 1968 *Nucl. Phys.* **A119** 667

Pingot O 1969 *Nucl. Phys.* **A133** 334

Ramalingareddy A, Lakshminarayana V and Swamy Jnanananda 1966 *Indian J. Pure Appl. Phys.* **4** 371

Ramalingareddy A, Lakshminarayana V and Swamy Jnanananda 1967 *Proc. Phy. Soc.* **91** 71

Ramalingareddy A, Parthasaradhi K, Lakshminarayana V and Swamy Jnanananda 1968 *Indian J. Pure Appl. Phys.* **4** 371

Shimizu S, Nakayama Y and Mukoyama T 1965 *Phys. Rev.* **A140** 806

Spitale G C and Bloom S D 1974 *UCRL Reports* 51596

Spitale G C and Bloom S D 1977 *Phys. Rev.* **A16** 221–30

Storm E and Israel H T 1970 *Nucl. Data Tables* **A7** 565
Sujkowiski Z and Missoni G 1961 *Phys. Rev.* **124** 1458
Verma, J and Eswaran, M A 1962 *Phy. Rev.* **127** 1197
Wheeler J A and Lamb W E 1939 *Phys. Rev.* **55** 858–62
Wheeler J A and Lamb W E 1956 *Phys. Rev.* **101** 1836