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Approximate angular momentum projection from an intrinsic random phase approximation Hartree-Fock state

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Abstract. An approximation procedure is described to calculate the projected energies from an intrinsic RPA HF wave function. The method of moments is used to find the relevant parameters. A model calculation is carried out for illustrative purposes.

Keywords. Approximate angular momentum projection; RPA HF wave function.

1. Introduction

The concept of a deformed intrinsic wave function has played an important role in the theoretical studies of the various properties of nuclei. It is well-known that it is not a simple matter to project out good angular momentum states from a deformed intrinsic state particularly when the number of active nucleons is large. In the 2*s*-1*d* shell nuclei (Ripka 1968) the energies E_J of good angular momentum states projected from an intrinsic deformed Hartree-Fock (HF) state found to be approximately proportional to $J(J+1)$. Because of this, considerable simplification arises in calculating E_J by expressing the many-body Hamiltonian in terms of the square of total angular momentum operator (Skyrme 1957; Wong *et al* 1974).

It has been shown (Gupta and Nazakat Ullah 1971) that the long range part of the two-body interaction makes it necessary to include the RPA correlations in the intrinsic HF wave function. The problem of projecting E_J from such a state becomes even more complicated than what one has when one deals with only an intrinsic HF state. However it was noted that the projected energies E_J from an intrinsic RPA HF states are approximately proportional to J rather than to $J(J+1)$. The purpose of the present note is to obtain relevant operators and expressions to calculate approximately the energies E_J arising from RPA correlated HF state. We present this formulation in the § 2 and apply it to a model calculation in § 3. Concluding remarks are presented in § 4.

2. Formulation

Let $|\Phi_0\rangle$ be the intrinsic HF state and $\eta_a^\dagger$, η_a be the respective operators which create and annihilate a deformed particle-hole state (Gupta and Nazakat Ullah 1971).

They are obtained by solving the usual RPA equation. The intrinsic RPA HF state is then obtained using Sanderson's (1965) method and is given by

$$|R\rangle = N_0 \exp\left(-\tfrac{1}{2}\, C_{\alpha\beta}\, \eta_\alpha^\dagger\, \eta_\beta^\dagger\right) |\Phi_0\rangle, \tag{1}$$

where N_0 is the normalization constant and the coefficients $C_{\alpha\beta}$ are related to RPA amplitudes.

As mentioned in § 1 the projected energies E_J from the state $|R\rangle$ can be approximately written as

$$E_J = E_0 + \tfrac{1}{2}\, \omega\, J. \tag{2}$$

In order to write the many-body Hamiltonian H as a function of J^2, we first need to find an operator which has the eigenvalues J in the good angular momentum states. Let Ω be such an operator. Then

$$\Omega\, |\Psi_J\rangle = J\, |\Psi_J\rangle. \tag{3}$$

We also know that

$$J^2 |\Psi_J\rangle = J(J+1)\, |\Psi_J\rangle. \tag{4}$$

Operating with Ω on both sides of (3) and making use of (4) we can easily show that

$$\Omega = \tfrac{1}{2}\left[-1 + \sqrt{1 + 4J^2}\right]. \tag{5}$$

Replacing E_J, J by the corresponding operators in (2) we find that the relation between H and J^2 is given by

$$H = E_0 + \tfrac{1}{2}\, \omega\, \Omega. \tag{6}$$

We would next like to find expressions for the constants E_0, ω. For this purpose we can follow Skyrme's variational principle (Skyrme 1957) but this will involve the matrix elements of H and the square root function $\sqrt{1 + 4J^2}$. Because of this the calculation of this matrix element will be as involved as the exact projection of a given J state from $|R\rangle$. We therefore follow the method of moments (Ng and Trainer 1974; Nazakat Ullah and Sandhya Devi 1973) which involves simpler matrix elements. Taking the first and second moments of (6) we get the following two relations

$$\langle R|\, H\, |R\rangle = E_0 + \tfrac{1}{2}\, \omega\, \langle R|\, \Omega\, |R\rangle, \tag{7a}$$

$$\langle R|\, H^2\, |R\rangle = E_0^2 + E_0\, \omega\, \langle R|\, \Omega\, |R\rangle + \tfrac{1}{4}\, \omega^2\, \langle R|\, \Omega^2 |\, R\rangle, \tag{7b}$$

to determine E_0, ω.

Solving equations (7) we get

$$\omega = 2\left[\frac{\langle H^2\rangle - \langle H\rangle^2}{\langle \Omega^2\rangle - \langle \Omega\rangle^2}\right]^{1/2}, \tag{8a}$$

$$E_0 = \langle H\rangle - \tfrac{1}{2}\,\omega\,\langle \Omega\rangle, \tag{8b}$$

where the $\langle\ \rangle$ sign denotes the matrix elements with respect to the wave function $R\rangle$.

3. Model calculation

We shall now consider a model calculation (Gupta and Nazakat Ullah 1971) which uses the intrinsic state $|R\rangle$ given by (1). This intrinsic state was applied to the nucleus ^{18}O, with ^{16}O being considered as an inert core, to calculate the $J = 0, 2, 4$ states.

The matrix elements occurring in (8) are calculated using the state $|R\rangle$ and are found to be

$$\langle H^2\rangle - \langle H\rangle^2 = 2{\cdot}724,\ \langle \Omega^2\rangle - \langle \Omega\rangle^2 = 2{\cdot}331.$$

This gives us the following values of ω, E_0:

$$\omega = 2{\cdot}162 \text{ MeV},\ E_0 = -\ 11{\cdot}860 \text{ MeV}.$$

If we carry out an exact projection from the state $|R\rangle$ then we find $\omega = 1{\cdot}523$ MeV and $E_0 = -\ 11{\cdot}118$ MeV. Therefore we see that the approximate values of E_0, ω compare fairly well with their exact values.

4. Concluding remarks

From the discussion of § 2 we find that the operator Ω plays the same role in the present case which the operator J^2 was playing when E_J's were proportional to $J(J+1)$. It is also a simple matter to see that a correlation coefficient ρ_v given by the expression,

$$\rho_v = \frac{\langle (H - \langle H\rangle)(\Omega - \langle \Omega\rangle)\rangle}{[\langle (H - \langle H\rangle)^2\rangle \langle (\Omega - \langle \Omega\rangle)^2\rangle]^{1/2}}$$

can be introduced to see whether the spectrum is proportional to J.

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