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Pion-nucleus optical potential: The pion-absorption contribution

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Abstract. The real and imaginary parts of pion-nucleus optical potential arising from pion absorption channel have been computed. A two-nucleon model of pion absorption which includes π and ρ rescattering and S-wave interaction has been used. The effects of short-range nucleon-nucleon correlations, Pauli blocking and form-factors have been included. The threshold values of imaginary absorption potential are reasonably close to density-squared terms of phenomenological potentials. The real part of P-wave potential is attractive and that of S-wave potential is weakly attractive at lower pion energies and changes sign as pion energy is increased. The calculation shows that the real part of absorption is significantly affected by short-range correlations and Pauli-blocking.

Keywords. Pion optical potential; pion absorption.

1. Introduction

Recent experiments (Schiffer 1980) have shown that pion absorption reactions, constitute a dominant channel of pion-induced reactions. Particularly at low pion energies ($T_\pi \lesssim 100$ MeV) pion-absorption cross-sections are larger than other reaction cross-sections. Also, pion-absorption cross-sections increase more rapidly than other reaction cross-sections with the increase in the target mass number. As a result, pion-absorption channel is expected to play a significant role in pion elastic scattering and optical potentials.

The importance of pion-absorption channel was first recognised by Ericsons (Ericson and Ericson 1966) who introduced extra complex terms, proportional to the square of nuclear density (ρ) in pion-nucleus optical potentials which were used to analyse level-shifts and widths of pionic atoms. The ρ^2-dependence of these terms is due to the two-nucleon model of pion absorption. The imaginary part of the coefficients of these ρ^2-dependent terms, generally written as Bo and Co for S-wave and P-wave absorption respectively, have been estimated by considering specific models for pion absorption on a pair of nucleons (Hachenberg *et al* 1977; Dover *et al* 1971; Bertsch and Riska 1978; Shimizu and Faessler 1978) or by using pion-absorption data on deuteron (Thomas and Landau 1980; Yoo and Landau 1981). This method of introducing ρ^2-dependent terms, in pion optical potentials has been extended to non-zero pion kinetic energies by making Bo and Co energy-dependent (Landau and Thomas 1978; Ko and Riska 1978; Chai and Riska 1979; Liu and Shakin 1979). All these calculations determine imaginary parts of Bo and Co. The real parts are usually assumed to be repulsive, having same strength as the corresponding imaginary parts (Stricker *et al* 1979). Although arguments have been offered

to justify this assumption (Hüfner 1975), real parts are generally not calculated (see, however Miller and Noble 1980).

In the present work we have calculated both real and imaginary parts of the optical potential due to pion absorption ("absorption potential") by using a two-nucleon absorption model which includes π-and ρ-rescattering terms and S-wave rescattering. Two-nucleon mode is probably the most important mode for pion-absorption due to the following reasons. One-nucleon mode contributes very little (Miller 1978) since it requires high momentum components ($\sim$ 500 MeV/c) of single-nucleon wave functions which have a very small amplitude. Multi-nucleon modes (involving more than two nucleons) are possible but these are not expected to give significant contribution since the probability of finding more nucleons in a small volume, where pion-absorption takes place, is small. Furthermore, since the pion energy is now shared amongst more nucleons, average nucleon energy becomes comparable with the Fermi energy which increases Pauli blocking and consequently reduces the multi-nucleon contribution.

The pion-absorption and emission operators are short-range operators and hence one would expect that the short-range behaviour of two-nucleon wave function would play an important role in the absorption potential calculation. Here we have investigated the effect of short-range correlations on the absorption potential by introducing Jastrow-type correlation functions. The effects of Pauli-blocking (in intermediate two-nucleon states) and absorption-vertex form-factors have also been included in the present work. One of the interesting results of our calculation is the behaviour of the real part of absorption potential. The real part of Co is attractive (not repulsive) and that of Bo is small and attractive at the threshold, becoming repulsive at higher pion energies. The real part is generally more sensitive to short-range effects, such as correlations and form-factors than is the imaginary part.

The paper is organised as follows. In § 2 the absorption potential is expressed in terms of the two-nucleon density and pion-absorption operators. §§ 3, 4 are devoted to pion-absorption operators and Pauli-blocking, respectively. The final expression for the optical potential is then obtained in § 5 and the results of the calculation are discussed in § 6.

2. Formalism

The matrix element of the absorption potential in momentum-space can be written as (Mizutani and Koltun 1977)

$$\langle \mathbf{k}' \mid U_{\text{abs}} \mid \mathbf{k} \rangle = \left\langle \mathbf{k}' \, \Phi_0 \left| R_{10} \frac{1}{E - H_0 + i\eta} R_{01} \right| \mathbf{k} \, \Phi_0 \right\rangle, \tag{1}$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the pion momenta, Φ_0 the nuclear ground state wavefunction, H_0 the nuclear Hamiltonian and R_{01} and R_{10} the pion-absorption and emission operators, respectively. In (1) we have suppressed the pion isospin coordinate. Assuming the two-nucleon mechanism for pion-absorption and neglecting the interaction of two nucleons which participate in the absorption process with the rest of the nucleons, the absorption potential becomes

$$\langle \mathbf{k}' \mid U_{\text{abs}} \mid \mathbf{k} \rangle = U_{\text{abs}}(\mathbf{k}, \mathbf{k}')$$

$$= \frac{(A-1)}{A(2\pi)^{12}} \int \mathrm{d}^3R\, \mathrm{d}^3R'\, \mathrm{d}^3r\, \mathrm{d}^3r'\, \mathrm{d}^3P\, \mathrm{d}^3P'\, \mathrm{d}^3Q\, \mathrm{d}^3Q'\, \mathrm{d}^3Q''\, \delta(\mathbf{P}'-\mathbf{P}+\mathbf{q}_\pi)$$

$$\times \psi^*\left(\mathbf{R}'+\frac{\mathbf{r}'}{2}, \mathbf{R}'-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2}, \mathbf{R}-\frac{r}{2}\right)\exp\left[i(\mathbf{P}\cdot\mathbf{R}-\mathbf{P}'\cdot\mathbf{R}'+\mathbf{Q}\cdot\mathbf{r}-\mathbf{Q}'\cdot\mathbf{r}')\right]$$

$$\times \left\langle R_{10}(\mathbf{Q}', \mathbf{k}'; \mathbf{Q}'') \frac{1}{\omega_{k_0} - \frac{Q''^2}{m} - \frac{P''^2}{4m} + i\eta} R_{01}(\mathbf{Q}, \mathbf{k}; \mathbf{Q}'') \right\rangle_{\substack{\text{spin} \\ \text{isospin}}} \quad (2)$$

Here $\mathbf{q}_\pi = \mathbf{k}' - \mathbf{k}$, $\mathbf{P}''$ and $\mathbf{Q}''$ are the centre-of-mass and relative momenta of nucleons in the intermediate state ($\mathbf{P}'' = \mathbf{P} + \mathbf{k} = \mathbf{P}' + \mathbf{k}'$), ω_{k_0} is the pion energy (restmass plus kinetic), m is the nucleon mass, A is the number of nucleons in the nucleus and $\langle\ \rangle_{\substack{\text{spin} \\ \text{isospin}}}$ stands for taking appropriate averages over nucleon spin-isospin coordinates. The two-nucleon wave functions are expressed in relative (r) and centre-of-mass (R) coordinates in (2) and we have neglected the nucleon-nucleon interaction in the intermediate propagator. Equation (2) is rather complex and some simplifications are necessary. We have made two basic approximations for this purpose. The first approximation consists of neglecting the dependence of the propagator on centre-of-mass momentum. This is reasonable since $\omega_{k_0} > 140$ MeV and an average value of $P^2/4m \sim \epsilon_F$, the nucleon Fermi energy (~ 35 MeV). Hence we have replaced the propagator by $\dfrac{1}{E_{\text{av}} - Q''^2/m + i\eta}$ (The choice of E_{av}, will be discussed later in § 4). With this several integrals can be evaluated and we get

$$U_{\text{abs}}(\mathbf{k}, \mathbf{k}') = \frac{(A-1)}{A(2\pi)^9} \int \mathrm{d}^3 R\, \mathrm{d}^3 r\, \mathrm{d}^3 r'\, \mathrm{d}^3 Q\, \mathrm{d}^3 Q'\, \mathrm{d}^3 Q''$$

$$\times \exp i(\mathbf{q}_\pi \cdot \mathbf{R} + \mathbf{Q}\cdot\mathbf{r} - \mathbf{Q}'\cdot\mathbf{r}')\, \psi^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2}, \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2}, \mathbf{R}-\frac{\mathbf{r}}{2}\right)$$

$$\times \left\langle R_{10}(\mathbf{Q}', \mathbf{k}'; \mathbf{Q}'') \frac{1}{E_{\text{av}} - \frac{Q''^2}{m} + i\eta} R_{01}(\mathbf{Q}, \mathbf{k}; \mathbf{Q}'') \right\rangle_{\substack{\text{spin} \\ \text{isospin}}}. \quad (3)$$

The second approximation consists of using the local density approximation to simplify the product of two-nucleon wavefunctions,

$$\psi^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2}, \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2}, \mathbf{R}-\frac{\mathbf{r}}{2}\right)^\dagger$$

†Note that the product $\psi^*\psi$ above is the two-nucleon density matrix. In the expression here we have suppressed the nucleon spin-isospin labels.

This approximation is justified by the fact that the range of absorption operators is much smaller than the extent of nuclear wave functions. Since the absorption operators depend on relative coordinates, we have $r \ll R$ and $r' \ll R$. Under these conditions one can replace $\psi^* \psi$ by corresponding nuclear-matter two-nucleon density-matrix. Hence, (see appendix for details)†.

$$\psi^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2}, \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2}, \mathbf{R}-\frac{\mathbf{r}}{2}\right)$$

$$\simeq \rho^2(R) f(\mathbf{r}) f(\mathbf{r}') \left\{ \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right) \delta_{\substack{m_{\sigma_1} m'_{\sigma_1} \\ m_{\sigma_2} m'_{\sigma_2}}} \delta_{\substack{m_{\tau_1} m'_{\tau_1} \\ m_{\tau_2} m'_{\tau_2}}} \right.$$

$$\left. - \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right) \delta_{\substack{m_{\sigma_1} m'_{\sigma_2} \\ m_{\sigma_2} m'_{\sigma_1}}} \delta_{\substack{m_{\tau_1} m'_{\tau_2} \\ m_{\tau_2} m'_{\tau_1}}} \right\}, \tag{4}$$

where $f(r)$ is the Jastrow-type correlation function†† (Jastrow 1955),

$$\hat{j}, (x) = \frac{3}{x}\left(\frac{\sin x}{x^2} - \frac{\text{ocs } x}{x}\right),$$

and ρ is the single-nucleon density, normalized to A. Using (4) the absorption potential becomes,

$$U_{\text{abs}}(\mathbf{k}, \mathbf{k}') = \frac{(A-1)}{A(2\pi)^9} \hat{\rho}^2(\mathbf{q}_\pi) \int \mathrm{d}^3 r\, \mathrm{d}^3 r'\, \mathrm{d}^3 Q\, \mathrm{d}^2 Q'\, \mathrm{d}^3 Q'' f(r) f(r')$$

$$\times \exp\left[i(\mathbf{Q}\cdot\mathbf{r} - \mathbf{Q}'\cdot\mathbf{r}')\right] \left\{ \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right)\right\}$$

$$\sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_2} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \middle| \frac{R_{10} R_{01}}{E_{\text{av}} - \dfrac{Q''^2}{m} + i\eta} \middle| \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \right\rangle$$

$$- \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right) \sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_2} m_{\sigma_1} \\ m_{\tau_2} m_{\tau_1} \end{matrix} \middle| \frac{R_{10} R_{01}}{E_{\text{av}} - \dfrac{Q''^2}{m} + i\eta} \middle| \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \right\rangle, \tag{5}$$

where $\hat{\rho}^2(q_\pi)$ is the Fourier transform of $\rho^2(R)$.

†The two terms in the curly bracket in (4) essentially correspond to direct and exchange contributions.

††$f^2(r)$ is the usual two-nucleon correlation function.

We have used three different types of correlation functions ($f(r)$) in our calculation. One of them corresponds to no correlation ($f(r) = 1$). The other two correspond to soft-core correlation ($f(r) = 1 - \exp\{r^2 / 0{\cdot}16\}$, r in fm) and hard-core correlation ($f(r) = 0$ $r < 0{\cdot}4$ fm, $f(r) = 1 - \exp\{-(r - 0{\cdot}4)^2/0{\cdot}04\}$ $r < 0{\cdot}4$ fm). It would be better to use the realistic correlation function computed in variational calculations (*e.g.* Pandharipande and Wiringa 1979) but we believe that hard core and realistic correlations would yield essentially the same results for absorption potential.

3. Absorption operators

The pion-absorption and emission operators (R_{01} and E_{10} respectively) that are required for the absorption potential calculation, have been chosen to include S-wave rescattering and π- and ρ- rescattering. Brack *et al* (1977) have shown that using these rescattering terms one can explain $\pi^+ d \to pp$ reaction data in the 3-3 resonance region. This probably implies that these rescattering terms describe the two-nucleon absorption process rather well.

3.1 *S-wave rescattering*

The S-wave rescattering vertex has been calculated by using a phenomenological interaction Hamiltonian (Koltun and Reitan 1966) (see figure 1a)

$$H_I = 4\pi \frac{\lambda_1}{m_\pi} \bar{\psi} \vec{\phi} \cdot \vec{\phi} \psi + 4\pi \frac{\lambda_2}{m_\pi^2} \bar{\psi} \boldsymbol{\tau} \cdot (\vec{\phi} \times \vec{\pi}) \psi, \tag{7}$$

where ψ's are nucleon field operators. $\vec{\phi}$'s are pion field operators, $\vec{\pi}$ is the momentum operator conjugate to $\vec{\phi}$, $\boldsymbol{\tau}$ is the nucleon isospin operator and m_π is the pion mass. The coupling constants λ_1 and λ_2 are determined from the S-wave pion-nucleon scattering lengths (0·003 and 0·05 respectively). With this the S-wave contribution to absorption operator for π^+ meson becomes

$$R_{01}^s(\pi^+) = i \frac{4\pi f}{m_\pi^2 \sqrt{2\omega_k}} \frac{\boldsymbol{\sigma}^2 \cdot \mathbf{q}}{q^2 + m_\pi^2 - \omega_q^2} \left[2\lambda_1 \tau_+^2 - \frac{i\lambda_2}{m_\pi} (\omega_k + \omega_q) (\boldsymbol{\tau}^1 \times \boldsymbol{\tau}^2)_+ \right]$$

$$+ i \frac{4\pi f}{m_\pi^2 \sqrt{2\omega_k}} \frac{\boldsymbol{\sigma}' \cdot \mathbf{q}}{q^2 + m_\pi^2 - \omega_q^2} \left[2\lambda_1 \tau'_+ - \frac{i\lambda_2}{m_\pi} (\omega_k + \omega_q) (\boldsymbol{\tau}^2 \times \boldsymbol{\tau}^1)_+ \right]. \tag{8}$$

Here f is the pseudoscalar coupling constant ($f^2/4\pi = 0{\cdot}08$), $\mathbf{q}$ and ω_q are respectively the momentum and energy carried by the virtual pion, and the nucleon isospin operators τ_+^1, etc. are defined by $\tau_+^1 = (\tau_x^1 + i\tau_y^1)/\sqrt{2}$. The two terms in (8) are due to S-wave rescattering taking place on nucleon 1 and 2, respectively. Momentum conservation implies that $\mathbf{q} = \mathbf{k} \cdot /2 \pm (\mathbf{Q} - \mathbf{Q}'')$ where $\mathbf{k}$ is the incident pion momentum and $\mathbf{Q}$ and $\mathbf{Q}''$ are the two-nucleon momenta, respectively ($+(-)$ sign

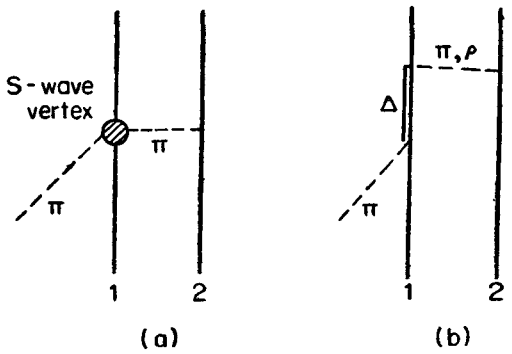

Figure 1. The rescattering diagrams included in absorption potential calculation.

for S-wave rescattering taking place on nucleon 1 (2)). Following Brack *et al* (1977) we have chosen $\omega_q = 0$. It is convenient to express R^s_{01} in configuration space. Thus

$$R^s_{01}(\pi^+) = \frac{f}{\sqrt{2\omega_k}} \Big\{ (2\lambda_1 \tau^2_+ - i\hat{\lambda}_2 (\boldsymbol{\tau}^1 \times \boldsymbol{\tau}^2)_+)$$

$$\times \int \mathrm{d}^3\rho\, \boldsymbol{\sigma}^2 \cdot \hat{\rho} \exp\left[i\left(\frac{\mathbf{k}}{2} + \mathbf{Q} - \mathbf{Q}''\right)\cdot \boldsymbol{\rho} \right]$$

$$\times X(m_\pi \rho) + (2\lambda_1 \tau^1_+ - i\hat{\lambda}_2 (\tau^2 \times \tau^1)_+)$$

$$\times \int \mathrm{d}^3\rho\, \boldsymbol{\sigma}^1 \cdot \hat{\rho} \exp\left[i\left(\frac{\mathbf{k}}{2} - \mathbf{Q} + \mathbf{Q}''\right)\cdot \boldsymbol{\rho} \right] X(m_\pi) \Big\}, \tag{9a}$$

where $$X(m_\pi \rho) = \left(1 + \frac{1}{m_\pi \rho}\right) \frac{\exp(-m_\pi \rho)}{m_\pi \rho} \quad \text{and} \quad \hat{\lambda}_2 = \frac{\omega_k \lambda_2}{m_\pi}$$

Similarly the emission operator R^s_{10} is

$$R^s_{10}(\pi^+) = \frac{f}{\sqrt{2\omega_{k'}}} \Big\{ (2\lambda_1 \tau^2_- - \hat{\lambda}_2\, i\, (\boldsymbol{\tau}^1 \times \boldsymbol{\tau}^2)_-)$$

$$\times \int \mathrm{d}^3_{\rho'}\, \boldsymbol{\sigma}^2 \cdot \hat{\rho}' \exp\left[i\left(\frac{\mathbf{k}'}{2} + \mathbf{Q}' - \mathbf{Q}''\right)\cdot \boldsymbol{\rho}' \right]$$

$$\times X(m_\pi \rho') + (2\lambda_1 \tau^2 - i\hat{\lambda}_2 (\tau^2 \times \tau^1)_-)$$

$$\times \int \mathrm{d}^3\rho'\, \boldsymbol{\sigma}^1 \cdot \hat{\rho}' \exp\left[i\left(\frac{\mathbf{k}'}{2} - \mathbf{Q}' + \mathbf{Q}''\right)\cdot \boldsymbol{\rho}' \right] \chi(m_\pi \rho') \Big\}. \tag{9b}$$

3.2 *Pion and ρ-meson rescattering*

One of the dominant processes for p-wave pion-absorption is the excitation of a Δ-resonance in the intermediate state followed by pion or ρ-meson exchange (figure 1b). For pion rescattering, absorption and emission operators in configuration space are (Brack *et al* (1977)

$$R_{01}^{\pi}(\pi^+) = \frac{ff^{*2}}{12\pi\sqrt{2\omega_k}} \frac{1}{\omega_{k0} - \omega_R + \frac{i\Gamma}{2}}$$

$$\times \left\{ \mathbf{T}^{1\dagger} \cdot \boldsymbol{\tau}^2 \, T_+^1 \int \mathrm{d}^3\rho \exp\left[i\left(\frac{\mathbf{k}}{2} + \mathbf{Q} - \mathbf{Q}''\right) \cdot \boldsymbol{\rho} \right] \right.$$

$$\times \left[\mathbf{S}^{1\dagger} \cdot \boldsymbol{\sigma}^2 \, Y(m_\pi \rho) + S_{12}^*(\hat{\rho}) \, Z(m_\pi \rho) \right] \mathbf{S}^1 \cdot \mathbf{k} + \mathbf{T}^{2\dagger} \cdot \boldsymbol{\tau}^1 \, T_+^2$$

$$\times \int \mathrm{d}^3\rho \exp\left[i\left(\frac{\mathbf{k}}{2} - \mathbf{Q} + \mathbf{Q}''\right) \cdot \boldsymbol{\rho} \right]$$

$$\left. \times \left[\mathbf{S}^{2\dagger} \cdot \boldsymbol{\sigma}^1 \, Y(m_\pi \rho) + S_{21}^*(\hat{\rho}) \, Z(m_\pi \rho) \right] \mathbf{S}^2 \cdot \mathbf{k} \right\}, \tag{10a}$$

and

$$R_{10}^{\pi}(\pi^+) = \frac{ff^{*2}}{12\pi\sqrt{2\omega_{k'}}} \frac{1}{\omega_{k0} - \omega_R - \frac{i\Gamma}{2}}$$

$$\times \left\{ T_-^{1\dagger} \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2 \int \mathrm{d}^3\rho' \exp\left[i\left(\frac{\mathbf{k}'}{2} + \mathbf{Q}' - \mathbf{Q}''\right) \cdot \boldsymbol{\rho}' \right] \right.$$

$$\times \mathbf{S}^{1\dagger} \cdot \mathbf{k}'_\pi \left[\mathbf{S}^1 \cdot \boldsymbol{\sigma}^2 \, Y(m_\pi \rho') + S_{12}^{*\dagger}(\hat{\rho}') Z(m_\pi \rho') \right] + T_-^{2\dagger} \, \mathbf{T}^2 \cdot \boldsymbol{\tau}^1$$

$$\times \int \mathrm{d}^3\rho' \exp\left[i\left(\frac{\mathbf{k}'}{2} - \mathbf{Q}' + \mathbf{Q}''\right) \cdot \boldsymbol{\rho}' \right] \mathbf{S}^{2+} \mathbf{k}' \left[\mathbf{S}^2 \cdot \boldsymbol{\sigma}^1 \, Y(m_\pi \rho') \right.$$

$$\left. \left. + S_{21}^{*\dagger}(\hat{\rho}') \, Z(m_\pi \rho') \right] \right\}, \tag{10b}$$

where $Y(x) = \dfrac{\exp(-x)}{x}$, $Z(x) = \left(1 + \dfrac{3}{x} + \dfrac{3}{x^2}\right) Y(x)$

the tensor operator $S_{12}^*(\hat{\rho}) = 3\,\mathbf{S}^{1\dagger} \cdot \hat{\rho}\, \boldsymbol{\sigma}^2 \cdot \hat{\rho} - \mathbf{S}^{1\dagger} \cdot \boldsymbol{\sigma}^2$, and f^*, ω_R and Γ are the $\pi N\Delta$ coupling constant, Δ-resonance energy and width, respectively. The

transition spin and isospin operators (S and T) are defined by Sugawara and Von Hippel (1968). We have chosen $f^* = 2f$, $\omega_R = m^* - m + k_0^2/2\ (m + \omega_k)$, $\Gamma = 2f^{*2}\, k^3/12\,\pi\, m_\pi^2$ and $m^* = 1236$ MeV.

Similarly the ρ-rescattering contribution to absorption and emission operators is

$$R_{01}^{\rho}(\pi^+) = \frac{f^* f_\rho f_\rho^*}{12\pi\sqrt{2\omega_k}} \frac{1}{\omega_{k0} - \omega_R + \frac{i\Gamma}{2}} \Big\{ \mathbf{T}^{1\dagger} \cdot \boldsymbol{\tau}^2\, T_+^1 \int \mathrm{d}^3\rho$$

$$\times \exp\left[i\left(\frac{\mathbf{k}}{2} + \mathbf{Q} - \mathbf{Q}''\right)\cdot\boldsymbol{\rho}\right] [2\,\mathbf{S}^{1\dagger}\cdot\boldsymbol{\sigma}^2\, Y(m_\rho\,\rho) - S_{12}^*(\hat{\rho})\, Z(m_\rho\,\rho)]$$

$$\times \mathbf{S}^1\cdot\mathbf{k} + \mathbf{T}^{2\dagger}\,\boldsymbol{\tau}^1\, T_+^2 \int \mathrm{d}^3\rho \exp\left[i\left(\frac{\mathbf{k}}{2} - \mathbf{Q} + \mathbf{Q}''\right)\cdot\boldsymbol{\rho}\right]$$

$$\times [2\,\mathbf{S}^2\cdot\boldsymbol{\rho}\, Y(m_\rho\,\rho) - S_{21}^*(\hat{\rho})\, Z(m_\rho\,\rho)]\,\mathbf{S}^2\cdot\mathbf{k}\Big\}, \tag{11a}$$

and

$$R_{10}^{\rho}(\pi^+) = \frac{f^* f_\rho f_\rho^*}{12\pi\sqrt{2\omega_{k'}}} \frac{1}{\omega_{k0} - \omega_R - \frac{i\Gamma}{2}} \Big\{ T^{1\dagger}\,\mathbf{T}^2\cdot\boldsymbol{\tau}^2 \int \mathrm{d}^3\rho'$$

$$\times \exp\left[i\left(\frac{\mathbf{k}'}{2} + \mathbf{Q}' - \mathbf{Q}''\right)\cdot\boldsymbol{\rho}'\right]$$

$$\times \mathbf{S}^{1\dagger}\cdot\mathbf{k}'\,[2\,\mathbf{S}^1\cdot\boldsymbol{\sigma}^2\, Y(m_\rho\,\rho') - S_{21}^{*\dagger}(\hat{\rho}')\, Z(m_\rho\,\rho')]$$

$$+ T_-^{2\dagger}\,\mathbf{T}^2\cdot\boldsymbol{\tau}^1 \int \mathrm{d}^3\rho' \exp\left[i\left(\frac{\mathbf{k}'}{2} - \mathbf{Q}' + \mathbf{Q}''\right)\cdot\boldsymbol{\rho}'\right]\mathbf{S}^{2\dagger}\cdot\mathbf{k}'\,[2\mathbf{S}^2\cdot\boldsymbol{\sigma}^1\, Y(m_\rho\rho')$$

$$- S_{21}^{*\dagger}(\hat{\rho}')\, Z(m_\rho\,\rho')]\Big\}, \tag{11b}$$

where f_ρ and f_ρ^* are ρNN and $\rho N\Delta$ coupling constants and m_ρ is the ρ-meson mass ($= 750$ MeV). The ρNN coupling constant is given by $f_\rho^2 = g_\rho^2\, m_\rho^2/4m^2\ (1+K)^2$. The values of g_ρ and K are chosen to be $g_\rho = 0{\cdot}52 \times 4\pi$ and $K = 6{\cdot}6$, which are consistent with $\pi^+ d \to pp$ reaction data (Brack *et al* 1977). The $\rho N\Delta$ coupling constant has been chosen to be $1{\cdot}2\,\sqrt{2}\, f_\rho$.

It is convenient to combine π and ρ rescattering contributions into separate scalar, and tensor terms. Thus

$$R_{01}^{\pi+\rho}(\pi^+) = R_{01}^{\mathrm{Sc}}(\pi^+) + R_{01}^{T}(\pi^+), \tag{12a}$$

where

$$R_{01}^{\mathrm{Sc}}(\pi^+) = \frac{f}{\sqrt{2\omega_k}}\Big\{\mathbf{T}^{1\dagger}\cdot\boldsymbol{\tau}^2\, T' + \mathbf{S}^{1\dagger}\cdot\boldsymbol{\sigma}^2\mathbf{S}^1\cdot\mathbf{k}\int \mathrm{d}^3\,\rho$$

$$+ \exp\left[i\left(\frac{\mathbf{k}}{2} + \mathbf{Q} - \mathbf{Q}''\right)\cdot\boldsymbol{\rho}\right] \times F_0(\rho) + \mathbf{T}^{2\dagger}\cdot\boldsymbol{\tau}^1\, T_+^2\, \mathbf{S}^{2\dagger}\,\boldsymbol{\sigma}^2\,\mathbf{S}^2\,\mathbf{k}$$

$$\int \mathrm{d}^3\,\rho\, \exp\left[i\left(\frac{\mathbf{k}}{2} - \mathbf{Q} + \mathbf{Q}''\right)\cdot\boldsymbol{\rho}\right] F_0\ (\rho)\Big\} \tag{12b}$$

$$R_{01}^{T}(\pi^+) = \frac{f}{\sqrt{2\omega_k}}\Big\{\mathbf{T}^{1\dagger}\cdot\boldsymbol{\tau}^2\, T_+^1 \int \mathrm{d}^3\,\rho \exp\left[i\left(\frac{\mathbf{k}}{2} + \mathbf{Q} - \mathbf{Q}''\right)\right]\cdot\boldsymbol{\rho}\Big]$$

$$\times\, S_{12}^*(\hat{\rho})\,\mathbf{S}^1\cdot\mathbf{k}\, F_2(\rho) + \mathbf{T}^{2\dagger}\cdot\boldsymbol{\tau}^1\,\mathbf{T}_+^2 \int \mathrm{d}^3\,\rho$$

$$+ \exp\left[i\left(\frac{\mathbf{k}}{2} - \mathbf{Q} + \mathbf{Q}''\right)\cdot\boldsymbol{\rho}\right] S_{21}^*(\hat{\rho})\,\mathbf{S}^2\cdot\mathbf{k}\, F_2(\rho)\Big\}, \tag{12c}$$

with

$$\left.\begin{aligned} F_0(\rho) &= \frac{f^{*2}}{12\pi\left(\omega_{k_0} - \omega_R + \dfrac{i\,\Gamma}{Z}\right)}\left(Y(m_\pi\,\rho) + \frac{2f_\rho\, f_\rho^*}{ff^*}\, Y(m_\rho\,\rho)\right), \\ \text{and}\qquad F_2(\rho) &= \frac{f^{*2}}{12\pi\left(\omega_{k_0} - \omega_R + \dfrac{i\,\Gamma}{Z}\right)}\left(Z(m_\pi\,\rho) - \frac{f_\rho\, f_\rho^*}{ff^*} Z(m_\rho\,\rho)\right). \end{aligned}\right\} \tag{12d}$$

3.3 *Form factor corrections*

The virtual meson that is exchanged between two nucleons in the absorption process is off-mass shell and therefore it is necessary to introduce form factors for different vertices. This has been done by replacing various coupling constants, f etc., by $f(q^2) = f\,[(\Lambda_\pi^2 - m_\pi^2)\,/\,(\Lambda_\pi^2 - \omega_q^2 + q^2)]$ (similarly for ρ-mesons). Clearly for on-shell pions or for $\Lambda_\pi\ (\Lambda_\rho) \to \infty$ one recovers the results without form factor corrections. The cut-off parameters $\Lambda_\pi\ (\Lambda_\rho)$ are assumed to be same for different vertices. With these form factors it is still possible to express the absorption and emission operators in configuration space by modifying the functions X, Y and Z as given below.

$$X(m_\pi r) \to X(m_\pi r) - \left(\frac{\Lambda_\pi}{m_\pi}\right)^2 X(\Lambda_\pi r) - \frac{1}{2}\left[\left(\frac{\Lambda_\pi}{m_\pi}\right)^2 - 1\right]\exp(-\Lambda_\pi\, r),$$

$$Y(m_\pi r) \to Y(m_\pi r) - \left(\frac{\Lambda_\pi}{m_\pi}\right) Y(\Lambda_\pi r)$$

$$- \frac{1}{2}\frac{\Lambda_\pi}{m_\pi}\left[\left(\frac{\Lambda_\pi}{m_\pi}\right)^2 - 1\right]\exp(-\Lambda_\pi r),$$

$$Z(m_\pi r) \to Z(m_\pi r) - \left(\frac{\Lambda_\pi}{r_\pi}\right)^3 Z(\Lambda_\pi r) - \frac{1}{2}\frac{\Lambda_\pi}{m_\pi}$$

$$\times \left[\left(\frac{\Lambda_\pi}{m_\pi}\right)^2 - 1\right]\left(1 + \frac{1}{\Lambda_\pi r}\right) \exp(-\Lambda_\pi r), \tag{13}$$

and similarly for ρ-exchange terms.

The cut-off parameters Λ_π and Λ_ρ are not well determined, although theoretical estimates suggest that $\Lambda_\pi > 1$ GeV. The calculation of Brack *et al* indicates that $\pi^+ d \to pp$ cross-sections are not very much sensitive to Λ_ρ if $\Lambda_\pi \sim 1$ GeV. We have chosen Λ_π and Λ_ρ to be 1·2 GeV.

Substituting the expressions (9) and (12) for absorption and emission operators in (5) and integrating over $\mathbf{Q}$ and $\mathbf{Q}'$ we get,

$$U_{\text{abs}}(\mathbf{k}\cdot\mathbf{k}') = \frac{(A-1)}{A(2\pi)^3}\hat{\rho}^2(q_\pi)\frac{f^2}{(\omega_k\,\omega_{k'})^{1/2}}\int d^3r\, d^3r'\, d^3Q'' f(r) f(r')$$

$$\frac{\exp[i\mathbf{Q}''(\mathbf{r}-\mathbf{r}')]}{E_{\text{av}} - \dfrac{Q''^2}{m} + i\eta}\left\{\hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right)\cos\left(\frac{\mathbf{k}\cdot\mathbf{r} - \mathbf{k}'\cdot\mathbf{r}'}{2}\right)\right.$$

$$+ \sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \middle| \hat{R}'_{10} R'_{01} \middle| \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \right\rangle$$

$$+ \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right)\cos\left(\frac{\mathbf{k}\cdot\mathbf{r} + \mathbf{k}'\cdot\mathbf{r}'}{2}\right)$$

$$+ \sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \middle| \hat{R}^1_{10} \hat{R}^2_{01} \middle| \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \right\rangle - \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right)$$

$$\times \cos\left(\frac{\mathbf{k}\cdot\mathbf{r} - \mathbf{k}'\cdot\mathbf{r}'}{2}\right) \sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_2} m_{\sigma_1} \\ m_{\tau_2} m_{\tau_1} \end{matrix} \middle| \hat{R}^1_{10} \hat{R}^1_{01} \middle| \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \right\rangle$$

$$- \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right)\cos\left(\frac{\mathbf{k}\cdot\mathbf{r} + \mathbf{k}'\cdot\mathbf{r}'}{2}\right)$$

$$\left.\times \sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_2} m_{\sigma_1} \\ m_{\tau_2} m_{\tau_1} \end{matrix} \middle| \hat{R}^1_{10} \hat{R}^2_{01} \middle| \begin{matrix} m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2} \end{matrix} \right\rangle\right\}. \tag{14}$$

where
$$\hat{R}^1_{01} = (2\lambda_1 \tau^2_+ - \hat{\lambda}_2\, i\,(\boldsymbol{\tau}^1 \times \boldsymbol{\tau}^2)_+)\, \boldsymbol{\sigma}^2 \cdot \hat{r}\, X(m_\pi r)$$
$$+ \mathbf{T}^{1\dagger} \cdot \boldsymbol{\tau}^2\, T^1_+ (\mathbf{S}^{1\dagger} \cdot \boldsymbol{\sigma}^2\, F_0(r) + S^*_{12}(\hat{r})\, F_2(r))\, \mathbf{S}^1 \cdot \mathbf{k}$$
$$\hat{R}^2_{01} = (2\lambda_1 \tau^1_+ - \hat{\lambda}_2\, i\,(\boldsymbol{\tau}^2 \times \boldsymbol{\tau}^1)_+)\, \boldsymbol{\sigma}^1 \cdot \hat{r}\, X(m_\pi r)$$
$$+ \mathbf{T}^{2\dagger} \cdot \boldsymbol{\tau}^1\, T^2_+ (\boldsymbol{\sigma}^1 \cdot \mathbf{S}^{2\dagger} F_0(r) + S^*_{21}(\hat{r})\, F_2(r))\, \mathbf{S}^2 \cdot \mathbf{k} \qquad (15)$$
and
$$\hat{R}^1_{10} = (2\lambda_1 \tau^2_- - \hat{\lambda}_2\, i\,(\boldsymbol{\tau}^1 \times \boldsymbol{\tau}^2)_-)\, \boldsymbol{\sigma}^2 \cdot \hat{r}'\, X(m_\pi r')$$
$$+ T^{1\dagger}_- \mathbf{T}^1 \cdot \boldsymbol{\tau}^2\ (\mathbf{S}^{1\dagger} \cdot \mathbf{k}'\ \mathbf{S}^1 \cdot \boldsymbol{\sigma}^2\ F_0(r') + \mathbf{S}^{1\dagger} \cdot \mathbf{k}'\ S^{*\dagger}_{12}(\hat{r}')\, F_2(r')).$$

4. Pauli blocking

While calculating the absorption potential the effects of nucleon-nucleon and nucleon-nucleus interactions in the intermediate state have been neglected and the two-nucleon propagator in the intermediate state has been taken to be

$$-\frac{m \exp[i\hat{Q}\,|\mathbf{r}-\mathbf{r}'|]}{4\pi\,|\mathbf{r}-\mathbf{r}'|} = \frac{1}{(2\pi)^3} \int \mathrm{d}^3 Q'' \frac{\exp[i\,\mathbf{Q}''\cdot(\mathbf{r}-\mathbf{r}')]}{E_{\mathrm{av}} - \dfrac{Q''^2}{m} + i\eta}, \qquad (16)$$

where $\hat{Q} = (m\, E_{\mathrm{av}})^{1/2}$. Obviously, inclusion of these interaction effects would make the calculation of imaginary potential extremely complicated. Brack *et al* (1977) have shown that final state interactions do not affect reaction cross-sections very much. Hence neglecting nucleon-nucleon interaction in the intermediate state may be justified.

The effect of nucleon-nucleus interaction can be incorporated by considering the absorption process in nuclear matter. Then the d^3Q''-integral in (16) will be restricted to the nucleon momenta larger than the Fermi momentum k_F (the Pauli blocking) and the energy denominator will be modified (binding effects). The binding effects can be approximately included by modifying E_{av} (see later). With Pauli blocking, the propagator in coordinate space becomes

$$F(\mathbf{r}-\mathbf{r}') = -\frac{1}{(2\pi)^3} \int \mathrm{d}^3 Q'' \exp[i\,\mathbf{Q}''\cdot(\mathbf{r}-\mathbf{r}')]\, \frac{Q_0(\mathbf{Q}'', k_F)}{E_{\mathrm{av}} - \dfrac{Q''^2}{m} + i\eta}, \qquad (17)$$

where $Q_0(\mathbf{Q}'', k_F)$ is the Pauli operator which excludes the occupied nucleon states. Assuming that the nucleons are at rest before absorption,

$$Q_0(\mathbf{Q}'', k_F) = 1 \quad \text{if} \quad \left|\mathbf{Q}'' \pm \frac{\mathbf{k}_0}{2}\right| > k_F,$$
$$= 0 \quad \text{otherwise,} \tag{18}$$

where $\mathbf{k}_0$ is the incident pion momentum. Thus

$$F(\mathbf{r}-\mathbf{r}') = \frac{m \exp(i\hat{Q}|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|} + \frac{m}{2\pi^2|\mathbf{r}-\mathbf{r}'|}\left\{\int_0^{\sqrt{k_F^2-k_0^2/4}} \mathrm{d}Q''\, Q''\right.$$
$$\times \frac{\sin Q''|\mathbf{r}-\mathbf{r}'|}{\hat{Q}^2-Q''^2} + \frac{1}{k_0}\int_{\sqrt{k_F^2-k_0^2/4}}^{k_F-k_0/2} \mathrm{d}Q''\left(k_F^2-\left(Q''-\frac{k_0}{2}\right)^2\right)$$
$$\left.\times \frac{\sin Q''|\mathbf{r}-\mathbf{r}'|}{\hat{Q}^2-Q''^2}\right\}. \tag{19}$$

Since $k_F + k_0/2$ is smaller than $\hat{Q}$, Pauli blocking modifies only the real part of the absorption potential. The real parts of (16) and (19) are plotted in figure 2. This

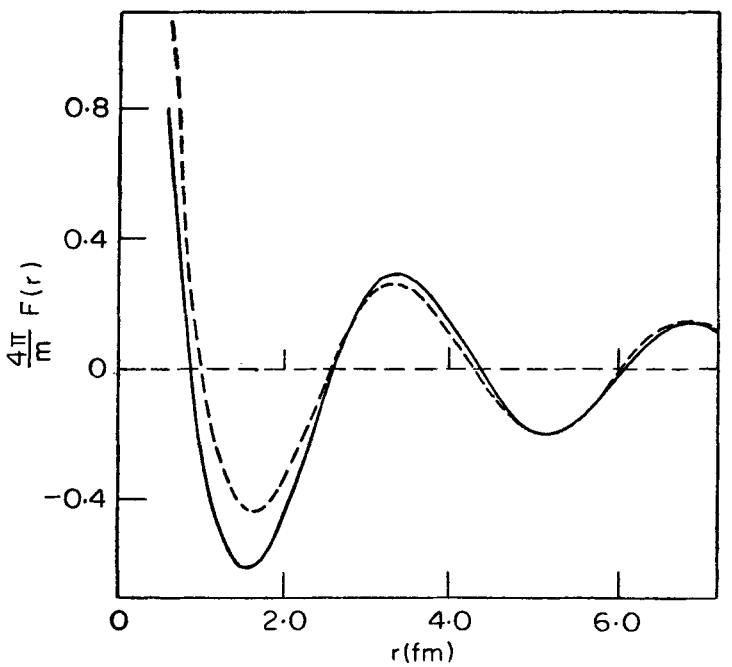

Figure 2. The real part of $4\pi/m\ F(r)$ *vs.* r. The solid and dashed curves correspond to (16) and (19) respectively.

figure shows that except at short distances (< 2 fm) the propagator does not change appreciably and the overall effect of Pauli blocking is to make the absorption potential more attractive (or less repulsive).

We have chosen E_{av} by subtracting the average binding energy from pion energy. Thus

$$E_{\text{av}} = \omega_{k_0} - \frac{1}{4m} \langle (\mathbf{p} + \mathbf{k}_0)^2 \rangle - 2\, E_b$$

$$\simeq \omega_{k_0} - 0{\cdot}3 \frac{k_F^2}{m} - \frac{k_0^2}{4m} - 2\, E_b, \tag{20}$$

where k_F is the Fermi momentum. E_b has been chosen to be 10 MeV. In order to test the binding effect we have set $E_b = 0$ in some calculations.

5. Absorption potential

The expression (16) or (19) for two-nucleon propagator can now be substituted in (14) and spin-isospin matrix elements can be evaluated by a straightforward calculation. We have also replaced $\cos\left(\frac{\mathbf{k}\cdot\mathbf{r} \pm \mathbf{k}'\cdot\mathbf{r}'}{2}\right)$ in (14) by $\cos\left(\mathbf{k}_0 \cdot \left(\frac{\mathbf{r} \pm \mathbf{r}'}{2}\right)\right)$ where $\mathbf{k}_0$ is the on-shell momentum. This is reasonable since the dominant momentum dependence comes from $\hat{\rho}^2\,(q_\pi)$ which has a peak at $q_\pi = 0$. One of the consequences of this approximation is that the crossed scalar-tensor terms in $R_{10}^1\, R_{01}^1$, etc. do not contribute. Thus, after evaluating the spin-isospin matrix elements (see appendix 2 for details) we get,

$$U_{\text{abs}}(\mathbf{k}\cdot\mathbf{k}') = \frac{(A-1)f^2}{A\,\omega_{k_0}} \hat{\rho}^2\,(q_\pi) \int \mathrm{d}^3 r\, \mathrm{d}^3 r'\, f(r) f(r')$$

$$\times F(\mathbf{r} - \mathbf{r}') \{B(\mathbf{r}, \mathbf{r}') + C(\mathbf{r}, \mathbf{r}')\, \mathbf{k}\cdot\mathbf{k}'\}$$

$$= \frac{(A-1)}{A} \hat{\rho}^2\,(q_\pi)\,(\hat{B} + \hat{C}\, \mathbf{k}\cdot\mathbf{k}'). \tag{21}$$

$B(\mathbf{r}, \mathbf{r}')$ and $C(\mathbf{r}, \mathbf{r}')$ are given in appendix 2.

Fourier transform of (21) gives the more familiar coordinate-space absorption potential. Hence

$$U_{\text{abs}}(\mathbf{r}) = \frac{(A-1)}{A} [\hat{B}\, \rho^2(r) - \hat{C}\, \nabla \cdot (\rho^2(r)\, \nabla)], \tag{22}$$

where the nuclear density $\rho(r)$ is normalized to number of nucleons in the nucleus.

The conventional absorption coefficients, B_0 and C_0 (Ericson and Ericson 1966) are related to the absorption potential through

$$2\mu\, U_{\text{abs}}(\mathbf{r}) = -\frac{(A-1)}{A} 4\pi\, [B_0\, \rho^2(r) - C_0\, \nabla \cdot (\rho^2(r)\, \nabla)], \tag{23}$$

where μ is the pion-nucleus reduced mass, $\mu = \dfrac{\omega_{k_0} Am}{\omega_{k_0} + Am}$ ($\sim \omega_{k_0}$ for practical purposes since $\omega_{k_0} \ll Am$). Hence,

$$B_0 = -\frac{\omega_{k_0}}{2\pi}\hat{B}, \quad C_0 = -\frac{\omega_{k_0}}{2\pi}\hat{C}. \tag{24}$$

So far we have not included the so-called angle-transformation effects (Thomas and Landau 1980) which arise due to the coordinate transformation between pion-two nucleon c.m. frame and pion-nucleus c.m. (or lab) frame. These coordinate transformation effects can be incorporated by noticing that the pion-momenta appearing in (2) are pion-two nucleon c.m. momenta. Thus, by using semi-relativistic kinematics,

$$\mathbf{k} \to \frac{\mathbf{k} - \epsilon\,\mathbf{p}}{1+\epsilon}, \quad \mathbf{k}' \to \frac{\mathbf{k}' - \epsilon\,\mathbf{p}'}{1+\epsilon}, \tag{25}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are two-nucleon momenta before pion-absorption and after pion emission, respectively and $\epsilon = \omega_{k_0}/2m$. In addition, the product of absorption and emission operators in (2) should be multiplied by a factor

$$\gamma = (\bar{\omega}_k\, \bar{\omega}_{k'}/\omega_k\, \omega_k')^{1/2}, \tag{26}$$

where $\bar{\omega}_k$ and $\bar{\omega}_{k'}$ are pion energies in pion-two nucleon c.m. system. Using (25) and (26) and averaging over two-nucleon momenta, the modified absorption potential becomes,

$$U_{\text{abs}}(\mathbf{r}) = -\frac{(A-1)}{A}[B\,\rho^2(r) - C\nabla\cdot(\rho^2(r)\,\nabla) + D\,\nabla^2\,\rho^2(r)], \tag{27a}$$

where

$$\left.\begin{aligned} B &= -\hat{B} - \frac{3}{5}\frac{k_F^2\epsilon^2}{(1+\epsilon)^2}\hat{C}, \\ C &= -\frac{1}{(1+\epsilon)^2}\hat{C}, \\ D &= -\frac{\epsilon(1-\epsilon/2)}{2(1+\epsilon)^2}\hat{C}. \end{aligned}\right\} \tag{27b}$$

While obtaining (27) we have neglected terms containing factors of $1/A$.

At this stage we would like to stress that coordinate transformation does not produce significant changes since ϵ is rather small. For example, at threshold $\epsilon \sim 0{\cdot}07$.

6. Results and discussion

The integrations in (21) are done numerically and absorption coefficients, B_0 and C_0 are obtained through (27). One of the aims of the present work is to determine the effect of short range nucleon-nucleon correlations and Pauli-blocking on absorption potential. Hence rest of the parameters, such as absorption vertex form factors, coupling constants, etc. have been chosen from the work of Brack *et al.* This ensures an agreement with pion absorption data on deuteron.

6.1 *The S-wave potential*

The results for real and imaginary parts of B_0 (Re B_0 and Im B_0) *vs.* pion kinetic energy are plotted in figures 3 and 4 respectively. Figure 3 shows that at lower pion energies Re B_0 is weakly attractive ($\sim 0{\cdot}012 = m_\pi^{-4}$ at threshold) and becomes repulsive at higher pion energies. This contradicts earlier estimates (Hüfner 1975) and is in agreement with Miller and Noble (1980). Our result also disagrees with the data on

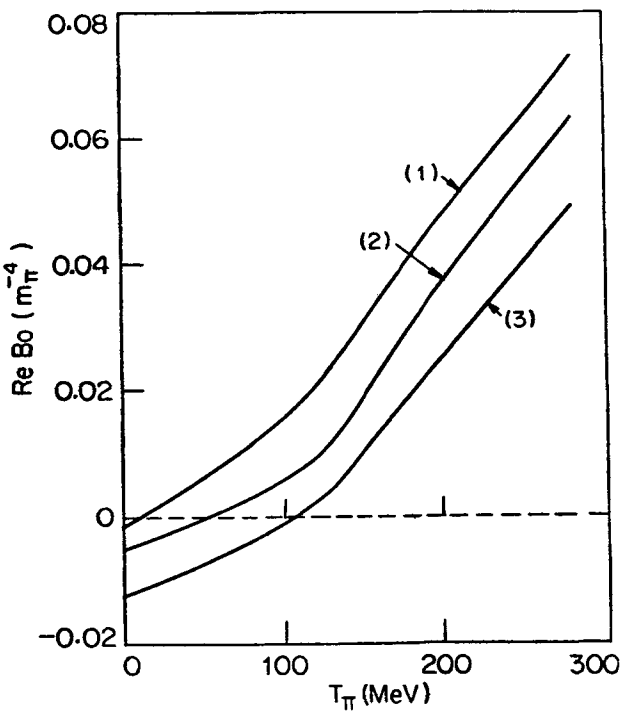

Figure 3. The real part of B_0 *vs.* pion kinetic energy. Curves labelled 1, 2 and 3 correspond to hard core correlations (no Pauli-blocking), no correlations and hard core correlations (with Pauli-blocking) respectively.

pionic atoms which require a repulsive S-wave potential having the strength of about 0·04 m_π^{-4}. One should, however, note that the determination of Re B_0 from pionic atoms data is somewhat ambiguous since the effect of Re B_0 ρ^2 (r) can be approximated by ρ_{eff} Re B_0 ρ (r) where ρ_{eff} is some effective nuclear density (Stricker *et al* 1979). In other words, the pionic atoms data do not determine both Re B_0 and b_0 (the S-wave term coming from first order optical potential) uniquely.

Figure 3 displays effects of Pauli-blocking, short-range correlations and nuclear binding. As discussed in § 4 Pauli-blocking makes the potential more attractive (or less repulsive). The calculation shows that Pauli-blocking modifies Re B_0 significantly. Short-range correlations, on the other hand, produce somewhat smaller changes and, to some extent, cancel Pauli-blocking. Nuclear binding on the other hand had negligible effect on B_0.

The results for Im B_0 are displayed in figure 4. These results show that short-range correlations decrease Im B_0 and their effect is larger at higher energies. It appears that the energy-dependence of Im B_0 (and also Re B_0) comes primarily from the energy dependence of absorption and emission operators. Nuclear binding increases the strength of Im B_0. The threshold value of Im B_0 is 0·028 m_π^{-4}, which is roughly 30 per cent smaller than that determined from pionic atoms data and is close to earlier calculations. Regarding this discrepancy we would like to make the following comments. Firstly, the local-Laplacian term D ∇^2 ρ^2 (r), arising from coordinate transformation is not included in the analysis of pionic-atoms data. Although the coefficient of local-Laplacian term is small (Im D = 0·004 m_π^{-6} at threshold), inclusion of this term would probably reduce the disagreement between the calculated and experimental values of B_0. With this point in mind, it may be useful to reanalyse pionic-atoms data with local-Laplacian term included.

Part of the discrepancy may also be due to the interaction of virtual pion that is exchanged between two nucleons, with the rest of the nucleons. This is a many-body

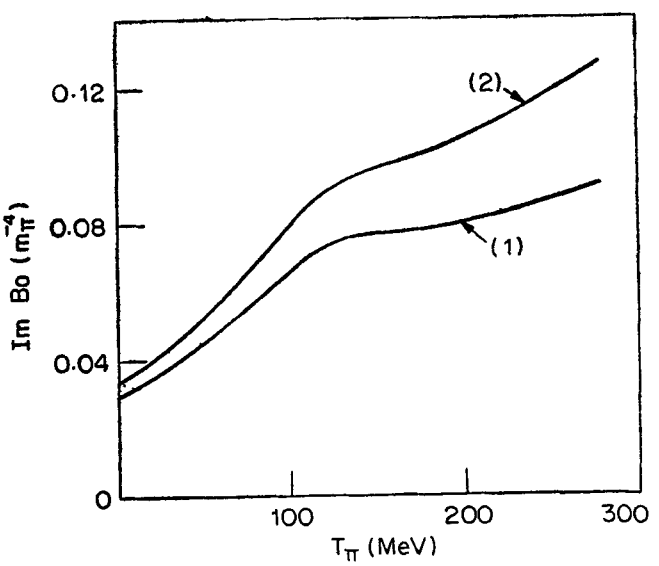

Figure 4. The imaginary part of B_0 *vs.* pion kinetic energy. Curves labelled 1 and 2 correspond to hard core and no correlations respectively.

effect and it is expected to change the propagator of virtual pion. We are, at present, attempting to estimate this effect.

6.2 *The P-wave potential*

The results for Re C_0 and Im C_0 are displayed in figures 5 and 6 respectively. Both Re C_0 and Im C_0 peak around $T_\pi = 100$ MeV. This is due to the Δ_{33} resonance in pion absorption and emission operators. The peak position is however shifted from the Δ_{33} resonance energy ($T_\pi = 195$ MeV in πN c.m. frame). The shift in the peak position is due to the factors of $\omega_{k_0}^{-1/2}$ in pion absorption and emission operators and lab. to pion-two nucleon c.m. transformation.

The Re C_0 is attractive in the energy-range considered here. The magnitude of Re C_0 at threshold is close to the magnitude of Im C_0 (0·115 m_π^{-6} *vs.* 0·10 m_π^{-6}). This result disagrees, in sign, with the estimate of Hüfner (1975) and with the phenomenological estimate of Re C_0. As in the case of Re B_0, the pionic atoms data do not determine Re C_0 unambiguously, since its effect can be absorbed into the (effective) P-wave multiple scattering potential.

The effect of Pauli-blocking and short-range correlations of Re C_0 is shown in figure 5. As in case of Re B_0, Pauli-blocking makes Re C_0 more attractive. Short range correlations, on the other hand, decrease the attraction, cancelling the effect of Pauli-blocking to some extent.

The threshold value of Im C_0 is $\simeq$ 0·1 m_π^{-6} and this is somewhat larger than Im C_0 obtained from pionic atoms data (0·04 — 0·08 m_π^{-6}). Short range correlations and nuclear binding have rather small effect on Im C_0 ($\sim$ 8 per cent and less than 1 per cent respectively). The Re C_0, on the other hand, is more sensitive to short-range correlations.

In order to see the effect of ρ-meson exchange we have performed one calculation where ρ-meson exchange contribution was switched off (curve 3 in figures 5 and 6). The cancellation between π and ρ exchange contributions, which takes place in the tensor term (see (12d)) is apparent in these figures. In fact, our calculation shows that the tensor term gives a major fraction of C_0 and the contribution of scalar term is only a few percent.

Finally, we would like to comment on the relative sizes of pion-absorption potential and pion-nucleus potential obtained from multiple scattering. Although one cannot compare the two potentials as such since the former is proportional to the square of

Table 1. The pion-absorption potential is compared with the first other pion optical potential in the nuclear interior. The parameters b_0 and c_0 are taken from a calculation which includes impulse approximation correction (Phatak 1980). The nuclear density ρ_0 is chosen to be 0·17 $fm^{-3} = 0·482\ m_\pi^3$.

T_π (MeV)	$b_0\,\rho_0$ (m_π^{-2})	$B_0\,\rho_0^2$ (m_π^{-2})	$c_0\,\rho_0$	$C_0\,\rho_0^2$
20	−0·0161+*i*0·0048	0·0024+*i*0·0082	0·0825+*i*0·0025	0·0290+*i*0·0302
60	−0·0219+*i*0·0092	0·0015+*i*0·0113	0·0942+*i*0·0137	0·0363+*i*0·0519
100	−0·0279+*i*0·0131	0·0002+*i*0·0152	0·1082+*i*0·0418	0·0428+*i*0·0822
160	−0·0495+*i*0·0171	−0·0032+*i*0·0180	0·0753+*i*0·1069	0·0163+*i*0·0482
240	−0·0465+*i*0·0269	−0·0087+*i*0·0198	−0·0122+*i*0·1017	0·0014+*i*0·0076

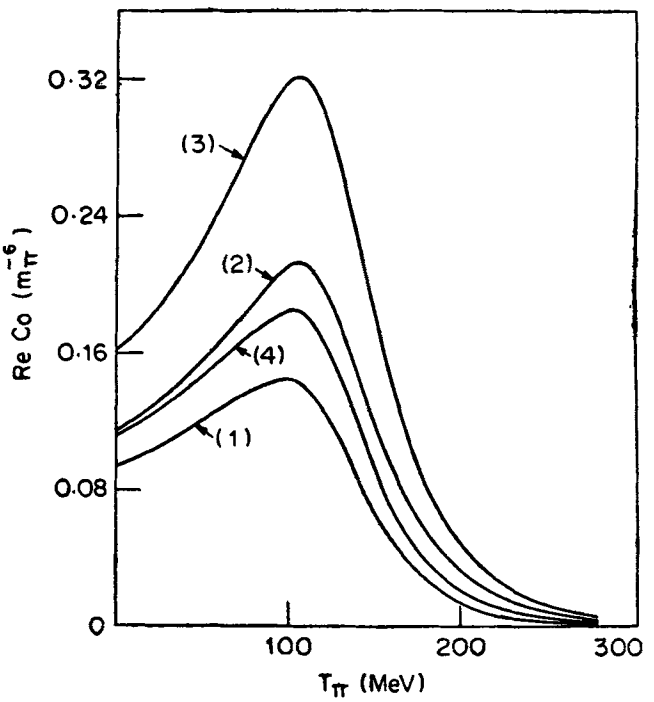

Figure 5. The real part of C_0 *vs.* pion kinetic energy. Different curves are **1.** hard core correlations (no Pauli-blocking) **2.** no correlations (no Pauli-blocking) **3.** no ρ-meson exchange (hard-core, no Pauli blocking) and **4.** hard core correlations (with Pauli-blocking)

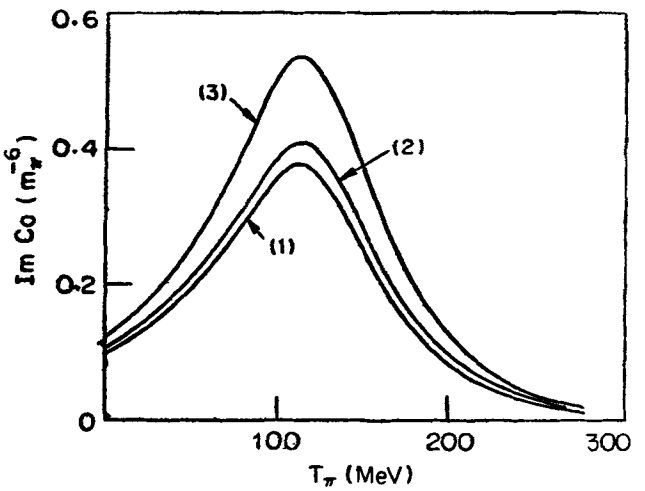

Figure 6. The imaginary part of C_0 *vs.* pion kinetic energy. Different curves are **1.** hard core correlations, **2.** no correlations and **3.** no ρ-meson exchange (hard core).

nuclear density whereas the latter is linear in density (in the first order approximation), some idea about their relative strengths can be obtained by comparing them in the nuclear interior ($\rho(r) = \rho_0 \sim 0{\cdot}17$ fm^{-3}). Such a comparison is done in table 1 where

we have listed $B_0 \rho_0^2$, $C_0 \rho_0^2$ and the S- and P-wave contributions coming from first-order optical potential ($b_0 \rho_0$ and $c_0 \rho_0$ respectively) for various pion energies. It is apparent from this table that the pion-absorption potential is important for low energy pions ($T_\pi \lesssim 100$ MeV).

Acknowledgement

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Appendix A

The two-nucleon density-matrix for a nuclear system having A nucleons is

$$\psi^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2}, \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2}, \mathbf{R}-\frac{\mathbf{r}}{2}\right)$$

$$=\int \psi_0^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2}, \mathbf{R}-\frac{\mathbf{r}'}{2}\ldots \mathbf{r}_A\right)\psi_0\left(\mathbf{R}+\frac{\mathbf{r}}{2}, \mathbf{R}-\frac{\mathbf{r}}{2}, \ldots, \mathbf{r}_A\right)\prod_{i=3}^{A} \mathrm{d}^3 r_i, \tag{A1}$$

where ψ_0 is the ground state wavefunction of the nuclear system. Using independent pair approximation for the ground-state wave function we get,

$$\psi^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2}, \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2}, \mathbf{R}-\frac{\mathbf{r}}{2}\right)$$

$$=\frac{1}{2}\sum_{\alpha,\beta}\left(\phi_\alpha^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2}\right)\phi_\beta^*\left(R-\frac{\mathbf{r}'}{2}\right)-\phi_\beta^*\left(R+\frac{\mathbf{r}'}{2}\right)\phi_\alpha^*\left(\mathbf{R}-\frac{\mathbf{r}'}{2}\right)\right)$$

$$\times\left(\phi_\alpha\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right)\phi_\beta\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right)-\phi_\beta\left(\mathbf{R}+\frac{\mathbf{r}'}{2}\right)\phi_\alpha\left(\mathbf{R}-\frac{\mathbf{r}'}{2}\right)\right) f_{\alpha\beta}(r') f_{\alpha\beta}(r), \tag{A2}$$

where the state labels α, β, which include spin-isospin labels, are summed over occupied states. Correlation functions $f_{\alpha\beta}(r)$ generally depend on states α, β. In the following, we shall adopt Jastrow ansatz and neglect the state-dependence of correlation functions.

For spin-isospin saturated nucleus the sum over occupied states can be done and (A2) can be written in terms of one-nucleon density matrices as,

$$\psi^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2},\ \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2},\ \mathbf{R}-\frac{\mathbf{r}}{2}\right)$$

$$= f(r)f(r')\left[\rho\left(\mathbf{R}+\frac{\mathbf{r}'}{2},\ \mathbf{R}+\frac{\mathbf{r}}{2}\right)\rho\left(\mathbf{R}-\frac{\mathbf{r}'}{2},\ \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\delta_{\substack{m_{\sigma_1}\ m'_{\sigma_1}\\ m_{\sigma_2}\ m'_{\sigma_2}}}\delta_{\substack{m_{T_1}\ m'_{T_1}\\ m_{T_2}\ m'_{T_2}}}\right.$$

$$\left.-\rho\left(\mathbf{R}+\frac{\mathbf{r}'}{2},\mathbf{R}-\frac{\mathbf{r}}{2}\right)\rho\left(\mathbf{R}-\frac{\mathbf{r}'}{2},\ \mathbf{R}+\frac{\mathbf{r}}{2}\right)\delta_{\substack{m_{\sigma_1}\ m'_{\sigma_2}\\ m_{\sigma_2}\ m'_{\sigma_1}}}\delta_{\substack{m_{T_1}\ m'_{T_2}\\ m_{T_2}\ m'_{T_1}}}\right], \quad \text{(A3)}$$

where $$\rho\left(\mathbf{R}+\frac{\mathbf{r}'}{2},\ \mathbf{R}+\frac{\mathbf{r}}{2}\right)=\sum_{\alpha}\phi^*_{\alpha}\left(\mathbf{R}+\frac{\mathbf{r}'}{2}\right)\phi_{\alpha}\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right), \quad \text{(A4)}$$

and the Kroneker δ-functions in spin-isospin states follow from spin-isospin saturation. With local-density approximation (Negele and Vouthurin 1972) for one-nucleon density matrix,

$$\rho\left(\mathbf{R}+\frac{\mathbf{r}'}{2},\ \mathbf{R}+\frac{\mathbf{r}}{2}\right)\simeq\rho\left(\mathbf{R}+\frac{\mathbf{r}+\mathbf{r}'}{4}\right)\hat{j}_1\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right), \quad \text{(A5)}$$

(where $\hat{j}_1(x)=\frac{3}{x}\left(\frac{\sin x}{x^2}-\frac{\cos x}{x}\right)$

and ρ is the single-nucleon density), (13) becomes;

$$\psi^*\left(\mathbf{R}+\frac{\mathbf{r}'}{2},\ \mathbf{R}-\frac{\mathbf{r}'}{2}\right)\psi\left(\mathbf{R}+\frac{\mathbf{r}}{2},\ \mathbf{R}-\frac{\mathbf{r}}{2}\right)$$

$$= f(r)f(r')\left[\rho\left(\mathbf{R}+\frac{\mathbf{r}+\mathbf{r}'}{4}\right)\rho\left(\mathbf{R}-\frac{\mathbf{r}+\mathbf{r}'}{4}\right)\hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right)\delta_{\substack{m_{\sigma_1}\ m'_{\sigma_1}\\ m_{\sigma_2}\ m'_{\sigma_2}}}\delta_{\substack{m_{T_1}\ m'_{T_1}\\ m_{T_2}\ m'_{T_2}}}\right.$$

$$\left.-\rho\left(\mathbf{R}+\frac{\mathbf{r}-\mathbf{r}'}{4}\right)\rho\left(\mathbf{R}-\frac{\mathbf{r}-\mathbf{r}'}{4}\right)\hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right)\delta_{\substack{m_{\sigma_1}\ m'_{\sigma_2}\\ m_{\sigma_2}\ m'_{\sigma_1}}}\delta_{\substack{m_{T_1}\ m'_{T_2}\\ m_{T_2}\ m'_{T_1}}}\right],$$

$$\simeq f(r)f(r')\rho^2(R)\left[\hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right)\delta_{\substack{m_{\sigma_1}\ m'_{\sigma_1}\\ m_{\sigma_2}\ m'_{\sigma_2}}}\delta_{\substack{m_{T_1}\ m'_{T_1}\\ m_{T_2}\ m'_{T_2}}}\right.$$

$$\left.-\hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right)\delta_{\substack{m\sigma_1\ m'_{\sigma_2}\\ m_{\sigma_2}\ m'_{\sigma_1}}}\delta_{\substack{m_{T_1}\ m'_{T_2}\\ m_{T_2}\ m'_{T_1}}}\right]. \quad \text{(A6)}$$

In the last step above we have assumed that $\rho\left(\mathbf{R}+\frac{\mathbf{r}-\mathbf{r}'}{4}\right) \simeq \rho(\mathbf{R})$. This is reasonable since the pion-absorption and emission operators are short-range operators.

Appendix B

Here we shall evaluate spin-isospin matrix elements and obtain $B(\mathbf{r}, \mathbf{r}')$ and $C(\mathbf{r}, \mathbf{r}')$. The absorption potential expressed in (21) is

$$
\begin{aligned}
U_{\mathrm{abs}}(\mathbf{k}, \mathbf{k}') = \frac{(A-1)f^2}{A\omega_{k_0}} \hat{\rho}^2(q_\pi) \int \mathrm{d}^3r\, \mathrm{d}^3r'\, f(r) f(r')\, F(\mathbf{r}-\mathbf{r}') \\
\left\{ \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}-\mathbf{r}'}{2}\right|\right)\left[\cos\left(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\right)\sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_1} & m_{\sigma_2} \\ m_{\tau_1} & m_{\tau_2} \end{matrix} \middle| \hat{R}^1_{10}\hat{R}^1_{01} \middle| \begin{matrix} m_{\sigma_1} & m_{\sigma_2} \\ m_{\tau_1} & m_{\tau_2} \end{matrix} \right\rangle \right. \\
+ \cos\left(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\right) \sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_1} & m_{\sigma_2} \\ m_{\tau_1} & m_{\tau_2} \end{matrix} \middle| \hat{R}^1_{10}\hat{R}^2_{01} \middle| \begin{matrix} m_{\sigma_1} & m_{\sigma_2} \\ m_{\tau_1} & m_{\tau_2} \end{matrix} \right\rangle \Big] \\
- \hat{j}_1^2\left(k_F\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right) \times \left[\cos\left(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\right)\right. \\
\sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_2} & m_{\sigma_1} \\ m_{\tau_2} & m_{\tau_1} \end{matrix} \middle| \hat{R}^1_{10}\hat{R}^1_{01} \middle| \begin{matrix} m_{\sigma_1} & m_{\sigma_2} \\ m_{\tau_2} & m_{\tau_2} \end{matrix} \right\rangle \\
\left. + \cos\left(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\right) \sum_{\substack{m_{\sigma_1} m_{\sigma_2} \\ m_{\tau_1} m_{\tau_2}}} \left\langle \begin{matrix} m_{\sigma_2} & m_{\sigma_1} \\ m_{\tau_2} & m_{\tau_1} \end{matrix} \middle| \hat{R}^1_{10}\hat{R}^2_{01} \middle| \begin{matrix} m_{\sigma_1} & m_{\sigma_2} \\ m_{\tau_1} & m_{\tau_2} \end{matrix} \right\rangle \right] \Big\}. \qquad \text{(B1)}
\end{aligned}
$$

The spin-isospin operators $\hat{R}^1_{01}$, etc. are defined in (15). Inspection of (A1) shows that after the evaluation of spin-isospin matrix-elements, and angle integration, only the terms containing $\hat{r}\cdot\hat{r}'$ (or its powers) survive. In other words, crossed terms (S-wave $\times$ scalar S-wave $\times$ tensor and scalar $\times$ tensor) in $\hat{R}^1_{10}\hat{R}^1_{01}$, etc. need not be evaluated. We have used the identity (Brown and Weise (1975))

$$
\mathbf{S}'^{+}\,\mathbf{A}\,\mathbf{S}'\cdot\mathbf{B} = \mathbf{A}\cdot\mathbf{B} - \tfrac{1}{3}\boldsymbol{\sigma}'\,\mathbf{A}\,\boldsymbol{\sigma}'\,\mathbf{B}, \qquad \text{(B2)}
$$

for transition spin and isospin operators. With this the calculation of spin-isospin matrix elements boils down to evaluation of traces of products of σ-matrices. The results are listed below—

$$\left.\begin{aligned}
&\langle m_{\sigma_1} m_{\sigma_2} | \boldsymbol{\sigma}^2 \cdot \hat{r}' \, \boldsymbol{\sigma}^2 \cdot \hat{r} | m_{\sigma_1} m_{\sigma_2} \rangle = 4 \, \hat{r} \cdot \hat{r}', \\
&\langle m_{\sigma_2} m_{\sigma_1} | \boldsymbol{\sigma}^2 \cdot \hat{r}' \, \boldsymbol{\sigma}^2 \cdot \hat{r} | m_{\sigma_1} m_{\sigma_2} \rangle = 2 \, \hat{r} \cdot \hat{r}', \\
&\langle m_{\sigma_1} m_{\sigma_2} | \boldsymbol{\sigma}^2 \cdot \hat{r}' \, \boldsymbol{\sigma}^1 \cdot \hat{r} | m_{\sigma_1} m_{\sigma_2} \rangle = 0, \\
&\langle m_{\sigma_2} m_{\sigma_1} | \boldsymbol{\sigma}^2 \cdot \hat{r}' \, \boldsymbol{\sigma}^1 \cdot \hat{r} | m_{\sigma_1} m_{\sigma_2} \rangle = 2 \, \hat{r} \cdot \hat{r}',
\end{aligned}\right\} \tag{B3}$$

$$\left.\begin{aligned}
&\langle m_{\tau_1} m_{\tau_2} | (2\lambda_1 \tau_-^2 - i \hat{\lambda}_2 (\boldsymbol{\tau}^2 \times \boldsymbol{\tau}^1)_-) \\
&\quad + (2\lambda_1 \tau_+^2 - i \hat{\lambda}_2 (\boldsymbol{\tau}^2 \times \boldsymbol{\tau}^1)_+) | m_{\tau_1} m_{\tau_2} \rangle = 8 \, (\lambda_1^2 + \hat{\lambda}_2^2/2), \\
&\langle m_{\tau_2} m_{\tau_1} | \, (2\lambda_1 \tau_-^2 - i \hat{\lambda}_2 (\boldsymbol{\tau}^2 \times \boldsymbol{\tau}^1)_-) \\
&\quad \times (2\lambda_1 \tau_+' - i \hat{\lambda}_2 (\boldsymbol{\tau}^2 \times \boldsymbol{\tau}^1)_+) | m_{\tau_1} m_{\tau_2} \rangle = 4 \, \lambda_1 \, (\lambda_1 - 2\hat{\lambda}_2) \\
&\langle m_{\tau_2} m_{\tau_1} | \, (2 \, \lambda_1 \tau_-^2 - i \hat{\lambda}_2 (\boldsymbol{\tau}^2 \times \boldsymbol{\tau}^1)_-) \\
&\quad \times (2\lambda_1 \tau_+^1 - i \hat{\lambda}_2 (\boldsymbol{\tau}^1 \times \boldsymbol{\tau}^2)_+) | m_{\tau_1} m_{\tau_2} \rangle = 4\lambda_1 \, (\lambda_1 + 2\hat{\lambda}_2),
\end{aligned}\right\} \tag{B4}$$

$$\left.\begin{aligned}
&\langle m_{\sigma_1} m_{\sigma_2} | \, \mathbf{S}^{1\dagger} \cdot \mathbf{k}' \, \mathbf{S}^1 \cdot \boldsymbol{\sigma}^2 \, \bar{\mathbf{S}}^{1\dagger} \cdot \boldsymbol{\sigma}^2 \, \mathbf{S}^1 \cdot \mathbf{k} \, | m_{\sigma_1} m_{\sigma_2} \rangle = \frac{8}{3} \mathbf{k} \cdot \mathbf{k}' \\
&\quad = \langle m_{\sigma_2} m_{\sigma_1} | \, \bar{\mathbf{S}}^{1\dagger} \cdot \mathbf{k}' \, \mathbf{S}^1 \cdot \boldsymbol{\sigma}^2 \, \mathbf{S}^{2\dagger} \cdot \boldsymbol{\sigma}^1 \, \mathbf{S}^2 \cdot \mathbf{k} \, | m_{\sigma_1} m_{\sigma_2} \rangle \\
&\langle m_{\sigma_2} m_{\sigma_1} | \, \mathbf{S}^{1\dagger} \cdot \mathbf{k}' \, \mathbf{S}^1 \cdot \boldsymbol{\sigma}^2 \, \mathbf{S}^{1\dagger} \cdot \boldsymbol{\sigma}^2 \, \mathbf{S}^1 \cdot \mathbf{k} \, | m_{\sigma_1} m_{\sigma_2} \rangle = -\frac{8}{9} \mathbf{k} \cdot \mathbf{k}' \\
&\quad = \langle m_{\sigma_1} m_{\sigma_2} | \, \mathbf{S}^{1\dagger} \cdot \mathbf{k}' \, \mathbf{S}^1 \cdot \boldsymbol{\sigma}^2 \, \mathbf{S}^{2\dagger} \cdot \boldsymbol{\sigma}^1 \, \mathbf{S}^2 \cdot \mathbf{k} \, | m_{\sigma_1} m_{\sigma_2} \rangle,
\end{aligned}\right\} \tag{B5}$$

$$\left.\begin{aligned}
&\langle m_{\tau_1} m_{\tau_2} | \, T_-^{1\dagger} \, \mathbf{T}^1 \cdot \boldsymbol{\tau}^2 \, \mathbf{T}^{1\dagger} \cdot \boldsymbol{\tau}^2 \, T_+^1 \, | m_{\tau_1} m_{\tau_2} \rangle = \frac{8}{3}, \\
&\quad = \langle m_{\tau_2} m_{\tau_1} | \, T_-^{1\dagger} \, \mathbf{T}^1 \cdot \boldsymbol{\tau}^2 \, \mathbf{T}_-^{2\dagger} \, \boldsymbol{\tau}^1 \, T_+^2 \, | m_{\tau_1} m_{\tau_2} \rangle, \\
&\langle m_{\tau_2} m_{\tau_1} | \, T_-^{1\dagger} \, \mathbf{T}_-^1 \cdot \boldsymbol{\tau}^2 \, \mathbf{T}^{1\dagger} \cdot \boldsymbol{\tau}^2 \, T_+^1 \, | m_{\tau_1} m_{\tau_2} \rangle = -\frac{8}{9}, \\
&\quad = \langle m_{\tau_1} m_{\tau_2} | \, T_-^{1\dagger} \, \mathbf{T}^1 \cdot \boldsymbol{\tau}^2 \, \mathbf{T}^{2\dagger} \cdot \boldsymbol{\tau}^1 \, T_+^2 \, | m_{\tau_1} m_{\tau_2} \rangle
\end{aligned}\right\} \tag{B6}$$

$$\left.\begin{aligned}
&\langle m_{\sigma_1} m_{\sigma_2}|\, \mathbf{S}^{1\dagger}\cdot\mathbf{k}'\, S_{12}^{*\dagger}(\hat{r}')\, S_{12}^{*}(\hat{r})\, \mathbf{S}^{1}\cdot\mathbf{k}\,|m_{\sigma_1} m_{\sigma_2}\rangle = \frac{16}{3}\mathbf{k}\cdot\mathbf{k}'\, P_2(\hat{r}\cdot\hat{r}'),\\
&\langle m_{\sigma_2} m_{\sigma_1}|\, \mathbf{S}^{1\dagger}\cdot\mathbf{k}'\, S_{12}^{*\dagger}(\hat{r}')\, S_{12}^{*}(\hat{r})\, \mathbf{S}^{1}\cdot\mathbf{k}\,|m_{\sigma_1} m_{\sigma_2}\rangle = \frac{44}{9}\mathbf{k}\cdot\mathbf{k}'\, P_2(\hat{r}\cdot\hat{r}'),\\
&\langle m_{\sigma_1} m_{\sigma_2}|\, \mathbf{S}^{1\dagger}\cdot\mathbf{k}'\, S_{12}^{*\dagger}(\hat{r}')\, S_{21}^{*}(\hat{r})\, \mathbf{S}^{2}\cdot\mathbf{k}\,|m_{\sigma_1} m_{\sigma_2}\rangle = -\frac{8}{9}\mathbf{k}\cdot\mathbf{k}'\, P_2(\hat{r}\cdot\hat{r}'),\\
&\langle m_{\sigma_2} m_{\sigma_1}|\, \mathbf{S}^{1\dagger}\cdot\mathbf{k}'\, S_{12}^{*\dagger}(\hat{r}')\, S_{21}^{*}(\hat{r})\, \mathbf{S}^{2}\cdot\mathbf{k}\,|m_{\sigma_1} m_{\sigma_2}\rangle = -\frac{4}{3}\mathbf{k}\cdot\mathbf{k}'\, P_2(\hat{r}\cdot\hat{r}').
\end{aligned}\right\} \quad \text{(B7)}$$

With this the absorption potential becomes,

$$\begin{aligned}
U_{\text{abs}}(\mathbf{k},\mathbf{k}') = {} & \frac{(A-1)f^2}{A\,\omega_{k_0}}\hat{\rho}^2(q_\pi)\int \mathrm{d}^3r\,\mathrm{d}^3r'\, f(r)\, f(r')\, F(\mathbf{r}-\mathbf{r}')\\
& \times\Big\{16\,\hat{r}\cdot\hat{r}'\times(m_\pi r')\times(m_\pi r')\Big[(\hat{\lambda}_2^2+2\lambda_1^2)\,\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}-\mathbf{r}'}{2}\Big|\Big)\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\Big)\\
& + \lambda_1(\hat{\lambda}_2+\hat{\lambda}_1/2)\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}-\mathbf{r}'}{2}\Big|\Big)\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\Big) + \lambda_1(\hat{\lambda}_2-\lambda_1/2)\\
& \times\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}+\mathbf{r}'}{2}\Big|\Big)\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\Big)\Big] + \frac{64}{9}\mathbf{k}\cdot\mathbf{k}'\, F_0(r)\, F_0(r')\\
& \Big[\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}-\mathbf{r}'}{2}\Big|\Big)\Big(\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\Big)+\frac{1}{9}\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\Big)\Big) - \hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}+\mathbf{r}'}{2}\Big|\Big)\\
& \times\Big(\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\Big)+\frac{1}{9}\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\Big)\Big)\Big] + \frac{32}{9}\mathbf{k}\cdot\mathbf{k}'\, F_2(r)\, F_2(r')\\
& \times P_2(\hat{r}\cdot\hat{r}')\Big[\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}-\mathbf{r}'}{2}\Big|\Big)\Big(4\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\Big)+\frac{2}{9}\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\Big)\Big)\\
& + \hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}+\mathbf{r}'}{2}\Big|\Big)\Big(\frac{11}{9}\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\Big)+\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\Big)\Big)\Big]\Big\}. \qquad \text{(B8)}
\end{aligned}$$

Thus

$$\begin{aligned}
B(\mathbf{r},\mathbf{r}') = {} & 16\,\hat{r}\cdot\hat{r}'\, X(m_\pi r)\, X(m_\pi r')\Big[(\hat{\lambda}_2^2+2\lambda_1^2)\\
& \times\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}-\mathbf{r}'}{2}\Big|\Big)\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}-\mathbf{r}'}{2}\Big) + \lambda_1\Big((\hat{\lambda}_2+\lambda_1/2)\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}-\mathbf{r}'}{2}\Big|\Big)\\
& + (\lambda_2-\lambda_1/2)\hat{j}_1^2\Big(k_F\Big|\frac{\mathbf{r}+\mathbf{r}'}{2}\Big|\Big)\Big)\cos\Big(\mathbf{k}_0\cdot\frac{\mathbf{r}+\mathbf{r}'}{2}\Big)\Big], \qquad \text{(B9)}
\end{aligned}$$

and

$$C(\mathbf{r}, \mathbf{r}') = \frac{64}{9} F_0(r) F_0(r') \left[\hat{j}_1^2 \left(k_F \left| \frac{\mathbf{r}-\mathbf{r}'}{2} \right| \right) \left(\cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}-\mathbf{r}'}{2} \right) \right.\right.$$

$$\left. + \frac{1}{9} \cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}+\mathbf{r}'}{2} \right) \right) - \hat{j}_1^2 \left(k_F \left| \frac{\mathbf{r}+\mathbf{r}'}{2} \right| \right) \left(\cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}+\mathbf{r}'}{2} \right) \right.$$

$$\left.\left. + \frac{1}{9} \cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}-\mathbf{r}'}{2} \right) \right) \right] + \frac{32}{9} F_2(r) F_2(r') \left[\hat{j}_1^2 \left(k_F \left| \frac{\mathbf{r}-\mathbf{r}'}{2} \right| \right) \right.$$

$$\times \left(4 \cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}-\mathbf{r}'}{2} \right) + \frac{2}{9} \cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}+\mathbf{r}'}{2} \right) \right) + \hat{j}_1^2 \left(k_F \left| \frac{\mathbf{r}+\mathbf{r}'}{2} \right| \right)$$

$$\left. \times \left(\frac{11}{9} \cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}-\mathbf{r}'}{2} \right) + \cos\left(\mathbf{k}_0 \cdot \frac{\mathbf{r}+\mathbf{r}'}{2} \right) \right) \right]. \tag{B10}$$

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