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Phase shift analysis of $\pi\pi$ scattering

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Abstract. A novel approach to energy dependent phase shift analysis of $\pi\pi$ scattering is proposed. Optimised polynomial expansions of Roskies' amplitudes are given. The parameters of these expansions are searched out by making a fit to the differential cross-sections. Then the transformation matrix between Roskies' amplitudes and the conventional partial waves is used to compute the phase shifts and inelasticities. Significant improvement is observed in so far as the number of free parameters for a fixed energy phase shift analysis is concerned. Value for the effective Chew-Mandelstam coupling constant, λ_{CM}, is also estimated.

Keywords. Roskies' amplitudes; mapped variables; phase shifts; inelasticities; Chew-Mandelstam coupling constant.

1. Introduction

The availability of high statistics data on $\pi\pi$ scattering *via* analysis of production reaction of the type

$$\pi N \rightarrow \pi\pi N,$$

$$\pi\pi\Delta,$$

has resulted in considerable progress in the field of $\pi\pi$ phase shift analysis (Protopopescu *et al* 1972; Grayer *et al* 1974; Hyams *et al* 1975; Ochs 1977; Rosselet *et al* 1977). Recent experimental and theoretical efforts have made it possible to obtain the best parameters for ρ meson, establish the negative sign of δ_0^2, and δ_2^2, find the properties of the second sheet pole S^*, remove the UP-DOWN ambiguity and confirm the coupling of $\pi\pi$ to $K\bar{K}$ channel. New ambiguities have also crept in, discrete ambiguities (Ochs and Wagner 1973; Estabrooks and Martin 1974, 1975; Manner 1974; Eoogatt and Peterson 1977) due to Barrelet zeroes (Barrelet 1972), the four bands of phase shifts (Pomponiu and Wanders 1977) allowed by analyticity, unitarity and crossing for the so far well understood $l = I = 1$ partial wave and continuous ambiguities (Atkinson *et al* 1973, 1974) that affect decisively the location and even the existence of the resonances.

As larger and larger energies are being reached, the data on $\pi\pi$ scattering at these high energies are accumulating quite fast. Higher partial waves become progressively more and more important. The addition of more partial waves brings in more uncertainties and ambiguities. It is, therefore, important to be able to obtain a large number of partial waves from a smaller number of input parameters. This is indeed possible if one uses the constraints of analyticity as has been proposed by Cutkosky and Deo (1968) and also demonstrated by them as also by several other authors. The present paper is an attempt to formulate the problem of $\pi\pi$ phase shift analysis in the best possible manner which reduces the uncertainties to a bare minimum.

The plan of the paper is as follows: in § 2 we present our method for the phase shift analysis. § 3 contains calculations and results. A brief discussion of our results is given in § 4.

2. Method of analysis

The earlier workers have parametrised the isospin amplitudes, F^I, to perform a phase shift analysis of $\pi\pi$ scattering. However each of these F^I's satisfies, the twice subtracted dispersion relation (Jin and Martin 1964), whereas the Roskies' amplitudes G_K, $K=0, 1, 2$, (Roskies 1970) satisfy either the once subtracted (Mahoux *et al* 1974) dispersion relations (for G_0 and G_1) or do not need a subtraction at all (for G_2). These G_K's have the same analytic structure as the F^I's and thus they have been used to advantage by many authors (Mahoux *et al* 1974; Atkinson and Pool 1974; Deo and Mohapatra 1977). The G_K's are related to the F^I's through the relations

$$G_0\,(s, t, u) = \tfrac{1}{3}\,[F^0\,(s, t, u) + 2F^2\,(s, t, u)], \tag{1}$$

$$G_1\,(s, t, u) = \frac{F^1\,(s, t, u)}{t-u} + \frac{F^1\,(t, u, s)}{u-s} + \frac{F^1\,(u, s, t)}{s-t}, \tag{2}$$

$$\begin{aligned} G_2\,(s, t, u) = {} & \frac{1}{s-t}\left[\frac{F^1\,(s, t, u)}{t-u} - \frac{F^1\,(t, s, u)}{s-u}\right] \\ & + \frac{1}{t-u}\left[\frac{F^1\,(t, u, s)}{u-s} - \frac{F^1\,(u, t, s)}{t-s}\right] \\ & + \frac{1}{u-s}\left[\frac{F^1\,(u, s, t)}{s-t} - \frac{F^1\,(s, u, t)}{u-t}\right]. \end{aligned} \tag{3}$$

We have used the G_K's to formulate phase shift analysis. F^I and G_K are written in the form

$$F^I = F^I_{\text{el}} + F^I_{\text{in}}, \tag{4}$$

$$G_K = G^{\text{el}}_K + G^{\text{in}}_K, \tag{5}$$

where the elastic term corresponds to the contribution from the two particle intermediate states and the inelastic term takes into account the effect of the inelastic processes starting with the opening up of the 4π channel, the inclusion of which tends to minimise (Ynd urain 1975) the continuous ambiguities which appear in such fixed energy phase-shift analysis. To evaluate $F^I_{\rm el}$ and then $G^{\rm el}_K$, we use an effective renormalisable field theory for $\pi\pi$ scattering with the simple interaction Lagrangian (Martin, *et al* 1976).

$$\mathscr{L} = -\lambda\,(\phi^2_\alpha)^2, \tag{6}$$

with

$$\lambda = \lambda_{\rm CM}\,\pi, \tag{7}$$

where $\lambda_{\rm CM}$ is the Chew-Mandelstam coupling constant.

The S-matrix elements are given by

$$S\,(p_\alpha, q_\beta;\, p'_{\alpha'}, q'_{\beta'}) = \frac{2\,i\,\delta^4\,(p+q-p'-q')}{\pi\,W^2}\,F_{\alpha\beta\,\alpha'\beta'}, \tag{8}$$

where W is the energy of either pion in the centre of mass system. The isospin decompositions of the amplitudes $F_{\alpha\beta\,\alpha'\beta'}$ are,

$$F_{\alpha\beta\,\alpha'\beta'} = \sum_{I=0}^{2} P_I\,F^I, \tag{9}$$

where P_I are the isospin projection operators (Okubo 1960) given by,

$$P_0 = 1/3\ [\delta_{\alpha\beta}\,\delta_{\alpha'\beta'}], \tag{10}$$

$$P_1 = 1/2\ [\delta_{\alpha\alpha'}\,\delta_{\beta\beta'} - \delta_{\alpha\beta'}\,\delta_{\beta\alpha'}], \tag{11}$$

$$P_2 = 1/2\ [\delta_{\alpha\alpha'}\,\delta_{\beta\beta'} + \delta_{\alpha\beta'}\,\delta_{\beta\alpha'}] - 1/3\ [\delta_{\alpha\beta}\,\delta_{\alpha'\beta'}]. \tag{12}$$

Okubo (1960) has shown that $F_{\alpha\beta\,\alpha'\beta'}$ obeys the algebraic equation,

$$\begin{aligned} F_{\alpha\beta\,\alpha'\beta'} = &-\lambda\,[\delta_{\alpha\beta}\,\delta_{\alpha'\beta'} + \delta_{\alpha\beta'}\,\delta_{\beta\alpha'} + \delta_{\alpha\alpha'}\,\delta_{\beta\beta'}] \\ &+ \frac{\lambda}{\pi} J\,[\delta_{\alpha\beta}\,\delta_{\alpha''\beta''} + \delta_{\alpha\beta''}\,\delta_{\beta\alpha''} + \delta_{\alpha\alpha''}\,\delta_{\beta\beta''}]\,F_{\alpha''\beta''\,\alpha'\beta'}, \end{aligned} \tag{13}$$

where J is the integral of a boson loop

$$J = \frac{i}{\pi^2}\int\!\!\int {\rm d}^4 p''\,{\rm d}^4 q''\,\delta^4\,(p''+q''-p-q)\,\frac{1}{p''^2-m_\pi^2}\,\frac{1}{q''^2-m_\pi^2}. \tag{13a}$$

In (13) the second term effectively sums up all the chain diagrams. However, to use Okubo's formalism to evaluate λ through this phase shift analysis, we confine ourselves to order λ^2 and assume that it represents the effective contribution from the two particle intermediate states. Then the discontinuities $\bar{F}^I_{\rm el}$ of $F^I_{\rm el}$, to the order λ^2, as calculated from (13) are,

$$\bar{F}^0_{\rm el} = -\frac{5\lambda^2}{\pi}(5\bar{J}(s) + 3\bar{J}(t) + 3\bar{J}(u)), \tag{14}$$

$$\bar{F}^1_{\rm el} = -\frac{5\lambda^2}{\pi}(\bar{J}(t) - \bar{J}(u)), \tag{15}$$

$$\bar{F}^2_{\rm el} = -\frac{\lambda^2}{\pi}(2\bar{J}(s) + 9\bar{J}(t) + 9\bar{J}(u)), \tag{16}$$

where,

$$\bar{J}(s) = -\pi\left(1 - \frac{4m_\pi^2}{s}\right)^{1/2}\theta(s - 4m_\pi^2). \tag{17}$$

The discontinuities of $G_K^{\rm el}$ are then obtained using (1), (2) and (3) and we calculate (Deo and Mohapatra 1977) $G_K^{\rm el}$ by evaluating the dispersion relation with one subtraction at the symmetry point.

To construct $G_K^{\rm in}$ we note that in $x = \cos\theta$ plane Re $(G_K^{\rm in})$ is analytic every where except for the cuts from $\pm\left(1 + \dfrac{32m_\pi^2}{s - 4m_\pi^2}\right)$ to $\pm\,\infty$. To determine the analytic structure of Im $G_K^{\rm in}$ we use the equations to the boundary of the spectral functions (Chew and Mandelstam 1960)

$$(s - 4m_\pi^2)(t - 16m_\pi^2) = 64m_\pi^2; \quad t > s, \tag{18}$$

$$(t - 4m_\pi^2)(s - 16m_\pi^2) = 64m_\pi^2; \quad s > t. \tag{19}$$

We then obtain that Im $(G_K^{\rm in})$ has a cut from

$$\cos\theta = 1 + \frac{2t_0}{s - 4m_\pi^2},$$

to $\cos\theta = \infty$ and a symmetrical left hand cut, where

$$t_0 = -(s + 8m_\pi^2) + (s^2 + 32s\,m_\pi^2)^{1/2}. \tag{20}$$

To optimally exploit this analytic structure, the cut $\cos\theta$ plane of analyticity of the real and the imaginary parts of $G_K^{\rm in}$ is mapped into two separate unifocal ellipses (Cutkosky and Deo 1968). Mapping of Re $(G_K^{\rm in})$ and Im $(G_K^{\rm in})$ into separate ellipses is necessary as their domains of analyticity are different. The new variables, thus obtained, are denoted by Z_r and Z_i respectively, where,

$$Z = \sin \phi\,(x, k_0), \tag{21}$$

$$\phi\,(x, k_0) = \pi\, F\,(\sin^{-1} x, k_0)/2K\,(k_0), \tag{22}$$

and $F(\phi, k_0)$ and $K = F(\pi/2, k_0)$ are the incomplete and complete elliptic integrals of the first kind respectively and k_0 is their modulus. Then we perform the following expansions.

$$\text{Re}\,(G_K^{\text{in}}) = \sum_n a_n^k T_n\,(Z_r), \tag{22}$$

$$\text{Im}\,(G_K^{\text{in}}) = \sum_n b_n^k T_n\,(Z_i), \tag{23}$$

where $T_n\,(x)$ is Techebyschef polynomial of order n in the variable x. The Techebyschef polynomials converge inside the ellipses enclosing the entire cut planes. Substituting (22) and (23) in (5) and using the transformation matrix (Mahoux *et al* 1974) between F^I and G_K one observes that the coefficients a_n^k and b_n^k are directly related to the F^I's which are expanded into partial waves as

$$F^I = \sum_l \frac{1 + (-1)^l}{2}\,(2l + 1)\, f_l^I P_l\,(\cos\theta), \tag{24}$$

where

$$f_l^I = \frac{\eta_l^I \exp\,(2i\delta_l^I - 1)}{2ik}, \tag{25}$$

$$\eta^I = 1 \text{ for elastic processes}, \tag{26}$$

$$\eta_l^I < 1 \text{ for inelastic processes}. \tag{27}$$

We note that to get a tolerable fit to the data by using a least χ^2 search procedure one can terminate the convergent series (22) and (23) at a suitable finite value of n. Once these truncated series are obtained then one can obtain the F^I's from them and hence the phase shifts and inelasticities.

As an example, if we are trying to get a tolerable fit to the data by taking only the constant term of the six equations of (22) and (23) then we are using only six free parameters. This amounts to dealing with the first three phase shifts and inelastics occurring in (24). In our analysis we observed that two terms of each of the series approximating Re (G_0^{in}) and Im (G_0^{in}) and just one term in each of the rest of the series *i.e.* a total of only eight free parameters give a good fit to the experimental data at energies below 1 GeV.

3. Calculations and results

In the absence of any published result on the differential cross-sections of pion-pion scattering, we construct the same using the equation

$$\frac{d\sigma}{d\Omega} = \frac{\sigma}{\sqrt{4\pi}} \sum_{l=0} (2l+1)^{1/2} A_l P_l (\cos\theta), \tag{28}$$

where

$$A_l = \langle Y_l^0 \rangle = \frac{\int \frac{d\sigma}{d\Omega} Y_l^0 \, d\Omega}{\Omega}, \tag{29}$$

normalised in such a way that

$$\langle Y_0^0 \rangle = 1/\sqrt{4\pi}, \tag{30}$$

The series, (28), is terminated at $l = 6$ because it is found experimentally that in the $\pi\pi$ mass range, $s < 1{\cdot}8$ GeV, moments with $l > 6$ are vanishingly small within errors. We shall take it as a statement of experimental fact (Ochs and Wagner 1975; Manner 1974; Estabrooks and Martin 1974, 1975) that the moments with $l > 6$ are identically equal to zero. For $\langle Y_l^0 \rangle$ we use the values of Protopopescu *et al* (1972) and Hyams *et al* (1975).

To demonstrate the proposed method of phase shift analysis we consider the data at 750 MeV, 880 MeV, 940 MeV, 1·0 GeV and 1·07 GeV. These energies have been chosen with the additional aim in view to study the effect of the $K\bar{K}$ threshold and the onset of inelasticity, particularly in the isosinglet *s*-wave channel, $l = 0$, $I = 0$.

3.1 *Determination of phase shifts and inelasticities*

After the $d\sigma/d\Omega$ data were constructed, we tried to fit them starting with only one parameter in each of the six series of (22) and (23). However, after several trials with arbitrary starting values for the parameters it was observed that (a) to give a good fit to $d\sigma/d\Omega$ and (b) to reproduce the moments and the total cross-sections (Protopopescu *et al* 1972; Hyams *et al* (1975), at least eight parameters, two each in Re (G_0^{in}) and Im (G_0^{in}) and one in each of the rest of the four series are needed. After suitably approximating Re (G_K^{in}) and Im (G_K^{in}) by optimal polynomial series, in so far as a tolerable fit to the data is concerned, the F^I's were constructed out of them by using the transformation matrix (Mahoux, *et al* 1974)

$$\begin{pmatrix} F^0(s,t,u) \\ F^1(s,t,u) \\ F^2(s,t,u) \end{pmatrix} = \begin{pmatrix} 5/3 & 2(3s-4m_\pi^2)/9 & -2(3s^2+6tu-16m_\pi^4)/27 \\ 0 & (t-u)/3 & (t-u)(3s-4m_\pi^2)/9 \\ 2/3 & -(3s-4m_\pi^2)/9 & (3s^2+6tu-16m_\pi^4)/27 \end{pmatrix} \begin{pmatrix} G_0(s,t,u) \\ G_1(s,t,u) \\ G_2(s,t,u) \end{pmatrix}. \tag{31}$$

These F^I's are then convergent, but being expressed as functions of the mapped variable, contain all the partial waves.

By projecting out various partial waves we obtained the first six phase shifts and six inelasticities which are given in table 1, and they compare well with those obtained by other workers. However, they used these six phase shifts and six inelasticities, a total of twelve, as free parameters in their analysis whereas we obtained them by using only eight parameters in our analysis. Thus our analysis saved four parameters in a single energy phase shift analysis. This saving in the number of free parameters is expected to become more and more pronounced as higher and higher energies are reached.

Table 1 also shows that the isosinglet S-wave phase shift, δ_0^0, does reach 90° around 900 MeV, rises there after crossing 180° near the S^* region. This confirms previous analysis of the region. It is also interesting to note that η_0^0, though not exactly equal to unity, remains close to it even at 940 MeV. Immediately after 940 MeV, η_0^0 curve shows a sharp dip with a minimum around 1 GeV. This demonstrates convincingly the predominance of inelastic effects. This is the $K\bar{K}$ threshold, which has opened up being kinematically permissible. This agrees well with the results obtained by other workers for $l = I = 0$ partial wave. It is also encouraging to note that δ_0^2, δ_2^2 as obtained from our analysis are negative at all the energies as demonstrated by earlier workers.

Since the last column of table 1 shows that our procedure is quite reliable one important consequences immediately follows. By projecting out higher and higher partial waves, one can now find the highest partial wave, $f_{l_{\max}}^I$ that is effectively excited to contribute at any particular energy instead of ascertaining the same by making a phase shift analysis with the parameters of all the partial waves consistent with $l_{\max}$.

3.2 *Estimation of* λ_{CM}

To examine the sensitivity of this analysis to the self-coupling constant and hence to obtain λ_{CM}, we note that (Mahoux *et al* 1974)

$$G_0^{el}(s, t) = \tfrac{1}{3}\left(F_{el}^0(s, t) + 2F_{el}^2(s, t)\right), \tag{32}$$

$$G_1^{el}(s, t) = \frac{3s - 4m_\pi^2}{6(s-t)(s-u)}\left(2F_{el}^0(s, t) - 5F_{el}^2(s, t)\right) + \left(\frac{1}{t-u} - \frac{t-u}{2(s-t)(s-u)}\right)F_{el}^1(s, t), \tag{33}$$

Table 1. The first six phase shifts and inelasticities as computed from the analysis and the χ^2/NDF of the fit

Energy in MeV	δ_0^0	δ_0^2	δ_1^1	δ_2^0	δ_2^2	δ_3^1	η_0^0	η_0^2	η_1^1	η_2^0	η_2^2	η_3^1	χ^2/NDF
730	74·4	−13·9	60·3	0·18	−0·68	0·06	0·98	1·0	1·0	1·0	0·999	0·99	0·44
880	83·4	−35·2	140·7	7·3	−1·26	0·094	0·85	1·0	0·89	1·0	0·998	0·99	1·13
940	150·1	−20·0	146·2	18·0	−4·2	0·11	0·85	1·0	0·85	1·0	0·86	0·99	1·08
1000	169·5	−6·1	155·5	18·4	−2·3	0·12	0·61	1·0	0·81	0·85	0·78	0·99	0·39
1070	222·6	−15·6	161·0	21·4	−8·49	0·14	0·59	1·0	0·98	0·66	0·84	0·99	1·41

$$G_2^{el}(s,t) = -\frac{1}{2(s-t)(s-u)}(2F_{el}^0(s,t) - 5F_{el}^2(s,t))$$
$$+\frac{3(3s-4m_\pi^2)}{2(t-u)(s-t)(s-u)} F_{el}^1(s-t), \tag{34}$$

where,

$$F_{el}^0(s,t) = -\frac{5\lambda^2}{\pi}(5J(s) + 3J(t) + 3J(u)), \tag{35}$$

$$F_{el}^1(s,t) = -\frac{5\lambda^2}{\pi}(J(t) - J(u)), \tag{36}$$

$$F_{el}^2(s,t) = -\frac{\lambda^2}{\pi}(2J(s) + 9J(t) + 9J(u)). \tag{37}$$

We used the data at 730 MeV and repeated the computation in § 3.1 varying the strength of the G_K^{el} term. We obtained that $|\lambda_{CM}| = (\lambda^2/\pi^2)^{1/2} \simeq 0{\cdot}091$ gives the best fit to the data as exhibited by the minimum in the $(x^2 - \lambda^2)$ curve (figure 1).

The most rigorous bound (Chung and Vin Mau 1975) obtainable from constraints of unitarity and axiomatic analyticity is

$$|\lambda_{CM}| \leqslant 0{\cdot}2. \tag{38}$$

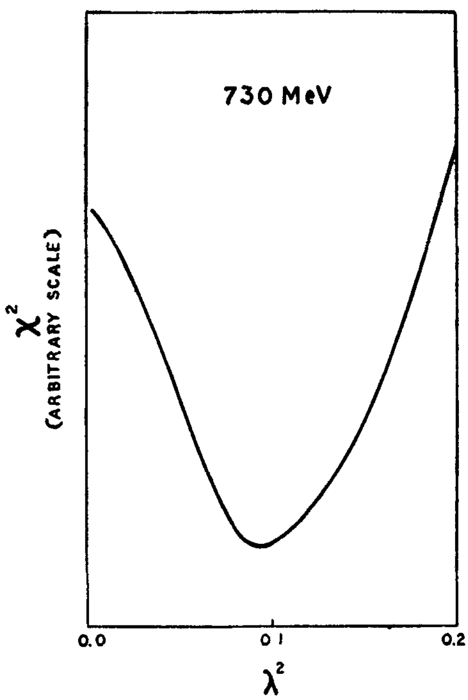

Figure 1. χ^2 curve as a function of λ^2 at 730 MeV.

On the other hand semiphenomenological and model-dependent bounds (Auberson *et al* 1975; Morgan and Shaw 1970; Shaw 1968) are known to be

$$|\lambda_{CM}| \leqslant 0{\cdot}2 \text{ to } 0{\cdot}4. \tag{39}$$

Recently Auberson *et al* (1976) have deduced that if $I = 2$ scattering length, $a_2 = 7{\cdot}0 \times 10^{-4}$, then the bound on the coupling is,

$$-0{\cdot}164 < \lambda_{CM} < 0{\cdot}162. \tag{40}$$

In view of the above bounds our value of $|\lambda_{CM}| \simeq 0{\cdot}091$ should be considered highly acceptable.

4. Conclusion

In conclusion, we note that the above method has made very significant improvements in the analysis of π–π scattering. It saved four parameters in a single-energy phase-shift analysis and in principle is able to give the phase shifts and inelasticities of all the partial waves that may be excited at a particular energy. Thus with new data coming up at higher and higher energies for $\pi\pi$ scattering, our method will be perhaps more convenient for phase shift analysis in the sense that one will have to handle a lesser number of free parameters. We have also concluded that it seems possible to get a conservative estimate of λ_{CM} by a phase shift analysis of the scattering data. However, this estimate will have the limitation of a model dependent calculation of G_K^{el}.

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