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Off-shell behaviour of α–α interaction potentials

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Abstract. Off-shell behaviour of representative α–α interaction potentials, both local and non-local separable, is compared through the partial wave Kowalski-Noyes half-off-shell functions. Parameters of the existing rank-one separable potentials are redetermined and an additional rank-two potential is constructed for this family. It is found that all these potentials show similar off-shell behaviour for higher partial waves. Their behaviour for low partial waves, however, particularly in the region far away from the energy-shell, is widely different. The off-shell correction for the $(\alpha, 2\alpha)$ reaction at 140 MeV is calculated, as an application, and it is found that separable potentials predict a non-negligible effect.

Keywords. Off-shell behaviour; Kowalski-Noyes function; alpha-alpha interaction potentials.

1. Introduction

The elastic scattering cross-sections of α-particles on ^{4}He have been measured extensively at a number of energies from 0·150 to 120 MeV (Russell *et al* 1956; Nilson *et al* 1958; Conzett *et al* 1960; Tombrello *et al* 1963; Darriulat *et al* 1965). These data have been analysed in terms of phase-shifts which, due to large internal binding energy and spinless bosonic nature of α-particles, are mainly real and are limited to a small number. Their imaginary parts are zero up to about 35 MeV and acquire values maximum upto 20° beyond it. In addition, the energy variation of the real parts of these phase shifts indicates that there are resonances at centre-of-mass energies of 0·094, 2·9, 11·7 and about 26 MeV for $l = 0$, 2, 4 and 6 respectively. From the nuclear structure point of view, since there is considerable evidence that α-particles constitute an important sub-structure in complex nuclei (Hodgson 1977), efforts have been made to construct interaction potentials corresponding to these scattering data. These potentials which are either purely real or have a very weak imaginary part can be classified into the following three categories:

(i) local, angular momentum dependent shallow attractive potentials with high repulsion in the first few partial waves (Darriulat *et al* 1965; Ali and Bodmer 1966);
(ii) local, angular momentum independent, deeply attractive potentials without repulsion (Neudatchin *et al* 1971; Buck *et al* 1977) and
(iii) angular momentum dependent, non-local, rank one separable potentials (Kukulin and Neudatchin 1970; Tamagaki and Fujiwara 1977).

As these potentials are obtained by fitting the phase shifts, all of them are obviously equivalent on-shell. Their off-shell behaviour, of course, may be different although it may also be restricted to certain behavioural pattern as these potentials satisfy some subsidiary conditions imposed by the more satisfactory microscopic theories, like 'resonating group method' (RGM) or 'generator co-ordinate method' (GCM), for the scattering of composite particles (Brink 1975). For α-α system, these conditions amount to suppression of the α-α relative motion wave function in $1S$, $2S$ and $1D$ states. Since the many body calculations with α-particles as sub-structure and the $(\alpha, 2\alpha)$ knock out reaction depend sensitively on the off-shell behaviour of the scattering amplitudes, in the present note we have compared the half-shell amplitudes corresponding to various potentials. As all these potentials are real, except the one of Darriulat *et al* which has a weak imaginary part, we have used only the real part of the potentials for this comparison. The Coulomb term in the potential is also excluded because the off-shell continuity of the Coulomb scattering amplitude is still not well-defined (Kok and van Haeringen 1981). Considering that the separable approximation, because of many resonances in the α-α system, may be good representation for this interaction (Brown and Jackson 1976), we have also constructed an additional separable potential which is of rank two in $l=0$ and $l=2$ partial waves. This brings about a kind of similarity between the available local and non-local potentials when both these types now have purely attractive and attractive-with-a-repulsive-core potentials. We have also re-determined the parameters of the potentials of Kukulin and Neudatchin (1970) and extended them to cover partial waves upto $l=10$. As pointed out by Tamagaki and Fujiwara (1977) the original parameters of Kukulin *et al* are in error as they are determined by fitting the calculated purely nuclear phase shifts wrongly to the measured nuclear phase shifts in the presence of the Coulomb potential.

In § 2 we list the essential expressions needed for our purpose. § 3 contains the description of various potentials and results.

2. Formalism

For a rank 2 separable potential defined as

$$V_l(p, p') = -\frac{\hbar^2}{m_\alpha}\frac{1}{2\pi^2}[g_l(p)\, g_l(p') - h_l(p)\, h_l(p')], \tag{1}$$

the fully off-shell scattering matrix for the l th partial wave is given (Tabakin 1964; Mongan 1968) by

$$T_l(p, k'; k^2) = N_l(p, k'; k^2) \,/\, D_l(k^2), \tag{2}$$

where

$$N_l(p, k'; k^2) = \frac{\hbar^2}{m_\alpha} 4\pi\, [h_l(p)\, h_l(k')\, G_{11}^+ - g_l(p)\, g_l(k')\, G_{22}^+ + (g_l(p)\, h_l(k') + g_l(k')\, h_l(p))\, G_{12}^+], \tag{3}$$

and the Fredholm determinant $D_l(k^2)$ is written as

$$D_l(k^2) = G_{11}^{+}\, G_{22}^{+} + G_{12}^{+2}, \tag{4}$$

with
$$G_{11}^{+} = 1 - \frac{2}{\pi}\int_0^a \frac{g_l^2(q)\, q^2\, dq}{q^2 - k^2 - i\epsilon}, \tag{5a}$$

$$G_{22}^{+} = 1 + \frac{2}{\pi}\int_0^a \frac{h_l^2(q)\, q^2\, dq}{q^2 - k^2 - i\epsilon}, \tag{5b}$$

and
$$G_{12}^{+} = \frac{2}{\pi}\int_0^a \frac{g_l(q)\, h_l(q)\, q^2\, dq}{q^2 - k^2 - i\epsilon}. \tag{5c}$$

The phase shifts $\delta_l(k^2)$ are given in terms of the on-shell T-matrix,

$$\tan\delta_l(k^2) = \frac{\mathrm{Im}\, T_l(k, k; k^2)}{\mathrm{Re}\, T_l(k, k; k^2)}. \tag{6}$$

In an explicit way from (2), the half-shell T-matrix is written as

$$T_l(p, k; k^2) = \frac{N_l(p, k; k^2)}{D_l(k^2)} = f_l(p, k)\, T_l(k, k; k^2), \tag{7}$$

where the Kowalski-Noyes half-off-shell function (Kowalski 1965; Noyes 1965) $f_l(p, k)$ are given by

$$f_l(p, k) = N_l(p, k; k^2)/N_l(k, k; k^2). \tag{8}$$

For the simple case of one term separable potential,

$$V_l(p, p') = -\frac{\hbar^2}{m_a}\frac{1}{2\pi^2} g_l(p)\, g_l(p'), \tag{9}$$

the expressions for $T_l(p, k; k^2)$, δ_l and $f_l(p, k)$ reduce to

$$T_l(p, k; k^2) = \frac{\dfrac{-\hbar^2}{m_a} 4\pi\, g_l(p)\, g_l(k)}{1 - \dfrac{2}{\pi}\displaystyle\int_0^a \frac{g_l^2(q)\, q^2\, dq}{q^2 - k^2 - i\epsilon}}, \tag{10}$$

$$\tan\delta_l(k^2) = k\, g_l^2(k) \Big/ \left[1 - \frac{2}{\pi}\int_0^a \frac{g_l^2(q)\, q^2\, dq}{q^2 - k^2 - i\epsilon}\right], \tag{11}$$

and
$$f_l(p, k) = g_l(p)/g_l(k). \tag{12}$$

For local potentials also, one can easily calculate the ratio of the half-off-shell T-matrix to on-shell T-matrix and thus define a sort of half-shell function for comparison with the Kowalski-Noyes functions defined for separable potentials. We thus write,

$$T_l(p, k; k^2) = 4\pi \int_0^a U_l(kr)\, V_l(r)\, j_l(pr)\, r^2\, dr, \tag{13}$$

where j_l is spherical Bessel function and

$$f_l(p, k) = \frac{\int_0^a U_l(kr)\, V_l(r)\, j_l(pr)\, r^2\, dr}{\int_0^a U_l(kr)\, V_l(r)\, j_l(kr)\, r^2\, dr}, \tag{14}$$

where $U_l(kr)$ is the radial part of the scattering solution for interaction potential $V_l(r)$. The full T-matrix, half off-shell, is calculated through

$$T(\mathbf{p}, \mathbf{k}; k^2) = \Sigma(2l + 1) f_l(p, k)\, T_l(k, k; k^2)\, p_l(\hat{p} \cdot \hat{k}), \tag{15}$$

which reduces to the on-shell value when $|p| = |k|$ and all $f_l(p, k) = 1$.

3. Interaction potentials and results

Before presenting the results, we first give here a description of the α-α interaction potentials which we have used for the purpose of comparing the half-off-shell amplitudes. As mentioned earlier, these potentials fit the scattering data and also incorporate in them the requirement of suppressing the relative motion wave function in $1S$, $2S$ and $1D$ states.

3.1 *Local potentials*

In this category we use the potentials of Darriulat *et al* (1965) and Buck *et al* (1977). The potential of Darriulat *et al* is angular momentum dependent and its real part is written as

$$U^l(r) = U_1/[1 + \exp(r - r_1)/a_1] - U_2/[1 + \exp(r - r_2)/a_2], \tag{16}$$

and the parameters are listed in table 1. The potential of Buck *et al* is deeply attractive and angular momentum independent. It is given by,

$$U(r) = -U_0 \exp(-\beta r^2), \tag{17}$$

with $U_0 = 122{\cdot}6225$ MeV and $\beta = 0{\cdot}22$ fm^{-2}.

Table 1a. Parameters of the potential due to Darriulat *et al* (1965).

l	U_1 (MeV)	r_1 (fm)	a_1 (fm)	U_2 (MeV)	r_2 (fm)	a_2 (fm)
0	150·0	1·65	0·10	9·20	3·72	0·40
2	150·0	1·63	0·05	16·00	3·55	0·30
4	220·0	1·20	0·05	71·00	2·48	0·46
6	—	—	—	50·00	2·96	0·53
8	—	—	—	110·00	2·00	0·65

Table 1b. Parameters of the one-term separable potential (Gaussian form).

l	λ_l^2	k_0 (fm^{-1})	k_1^2 (fm^{-2})	k_2^2 (fm^{-2})	β (fm)
0	$8{\cdot}40 \times 10^{-8}$ fm^{13}	2·40	0·54	5·51	1·02
2	$1{\cdot}18 \times 10^{-9}$ fm^{13}	3·75	0·00	4·725	1·02
4	$1{\cdot}58 \times 10^{-3}$ fm^{13}	2·80	0·00	0·00	1·02
6	$4{\cdot}46 \times 10^{-5}$ fm^{17}	2·28	0·00	0·00	1·02
8	$7{\cdot}73 \times 10^{-9}$ fm^{21}	8·85	0·00	0·00	1·02
10	$3{\cdot}80 \times 10^{-11}$ fm^{25}	9·80	0·00	0·00	1·02

Table 1c. Parameters of the one-term separable potential (Yamaguchi form).

l	λ_l^2 (fm^{-3})	k_1^2 (fm^{-2})	k_2^2 (fm^{-2})	b_l (fm^{-1})
0	$7{\cdot}04 \times 10^{3}$	0·54	5·51	2·1
2	$5{\cdot}7 \times 10^{3}$	0·00	4·725	2·1
4	$3{\cdot}04 \times 10^{2}$	0·00	0·00	2·1
6	$4{\cdot}18 \times 10^{3}$	0·00	0·00	2·1
8	$4{\cdot}9 \times 10^{2}$	0·00	0·00	2·1
10	$5{\cdot}79 \times 10^{2}$	0·00	0·00	2·1

Table 1d. Parameters of the two term separable potential

l	λ_a (fm$^{l+1/2}$)	b_a (fm^{-1})	λ_r (fm$^{l+1/2}$)	b_r (fm^{-1})
0	7·25	1·03	20·00	1·49
2	1·63	1·53	10·00	2·69
4	$2{\cdot}31 \times 10^{-1}$	1·65	0·00	—
6	$5{\cdot}37 \times 10^{-2}$	1·46	0·00	—
8	$1{\cdot}86 \times 10^{-2}$	1·22	0·00	—
10	$4{\cdot}01 \times 10^{-3}$	1·10	0·00	—

3.2 *Non-local separable potentials*

The form factor constructed by Kukulin and Neudatchin (1970) for partial waves $l = 0$, 2 and 4, which we shall call as one term Gaussian form, is written as

$$g_l(k) = \lambda_l (k^2 + k_0^2)(k^2 - k_1^2)(k^2 - k_2^2) \exp(-\beta^2 k^2/2). \tag{18}$$

Here k_0 is introduced to simulate some kind of resonance-width at the resonance while k_1 and k_2 are put in to reproduce the nodal behaviour of the wave function. The parameters k_0, k_1 and k_2 are l-dependent while the range parameter β is kept fixed for all partial waves, at a value derived from the strength parameter of the shell model wave function in the harmonic oscillator basis. The values of k_1 and k_2, in each partial wave, are also fixed from similar calculations. They then search λ_l and k_0 (l) and fit the real parts of α-α phase shifts up to 60 MeV. Tamagaki and Fujiwara (1977) have pointed out that the phase shifts to be fitted by this nuclear part of the interaction should be δ_l^N and not δ_l^{NC} which are reported in the literature as the nuclear phase shifts in the presence of the Coulomb potential. It is known that δ_0^N is different from δ_0^{NC} in the entire energy range while other δ_l^N are different from δ_l^{NC} up to around 30 MeV. They thus correct the original parameters of Kukulin and Neudatchin and also construct potential for $l = 6$. We checked these potentials for phase shifts at higher energies and found that they have a very disappointing trend beyond 50 MeV. D-wave phase shift changes sign and S-wave phase shift goes very close to $-180°$ around 60 MeV while both these effects occur around or above 100 MeV. We have corrected this potential, keeping β at the value given by Tamagaki and Fujiwara but changing k_1 and k_2 such that the change of sign in the phase shifts occur at the right place, and found new values of λ_l and k_0 (l) so that the data up to 120 MeV is well-fitted. Similar potentials for $l = 8$ and 10 are also constructed. It may be mentioned that for $l > 4$, the form factor (18) is multiplied by k^{l-4} to ensure proper threshold behaviour. The parameters of the potential are given in table 1 and fits to the phase-shifts are displayed in figure 1.

Another form factor suggested by Kukulin and Neudatchin (1970) for partial waves $l = 0$, 2 and 4, which we may call one term Yamaguchi form, is given as:

$$g_l(k) = \lambda_l (k^2 - k_1^2)(k^2 - k_2^2)/(k^2 + b_l^2)^3. \tag{19a}$$

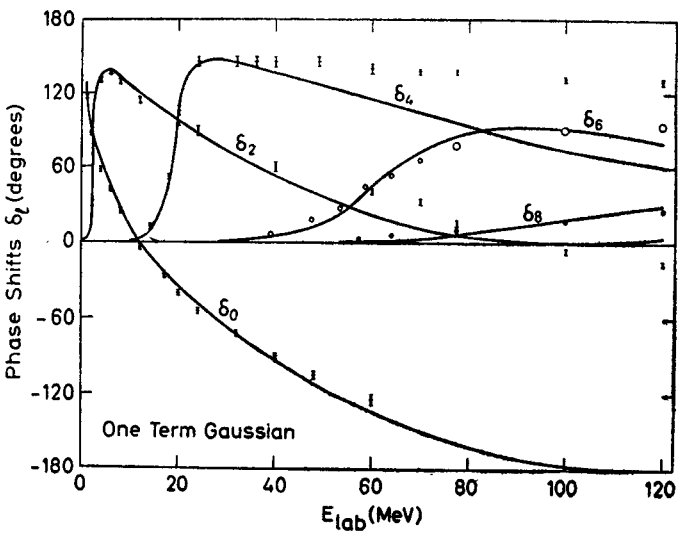

Figure 1. Fits to the α-α phase-shifts (nuclear part) with the one-term separable potential of Gaussian form.

This form is also unsatisfactory at high energies. We have also corrected and extended this potential so as to reproduce all $\delta_l^N(k^2)$ up to 120 MeV. For partial waves with $l > 4$ the form factor is modified as

$$g_l(k) = \lambda_l\, k^l/(k^2 + b_l^2)^{\frac{l+2}{2}}. \tag{19b}$$

We have fixed the range parameter b_l, for all partial waves, at 2·1 fm^{-1}, a value very close to the range parameter of the Gaussian potential if expressed as $\exp(-\beta^2 k^2/2) = \exp(-k^2/b^2)$ so that the energy-independence of the position of nodes is maintained. So, this in essense is a one parameter potential, the values of which are given in table 1 and fits to the phase shifts are displayed in figure 2. A noticeable improvement is the better quality fit to $\delta_4^N(k^2)$.

Though these one term potentials very correctly reproduce the phase-shifts and resonances in α-α scattering, they are incapable of predicting the position of Breit-Wigner poles on the second sheet of the complex energy plane. The Fredholm determinant $D_l(k^2)$ must vanish at some value of k to show this effect. In case of one-term separable potentials, it is only the real part of $D_l(k^2)$ identified as the denominator in (10), that can vanish while the imaginary part, given by $kg_l^2(k)$ is always non-zero (except at $k = k_1$ and $k = k_2$ which anyway are not the positions of resonances) and positive definite. This condition can be achieved with a two term potential through (4). Further it has been shown (Beam 1969) using Tabakin's one term potential (Tabakin 1968) for nucleon-nucleon interaction that by putting factors like $k^2 - k_1^2$ or $k^2 - k_2^2$ in the form factor $g_l(k)$ one does not produce a genuine change of sign in the phase-shifts. We have, therefore, constructed another potential with simple Gaussian form factors which is of rank 2 for the S and D partial waves. The form factors are written as

$$g_l(k) = \lambda_a\, k^l \exp(-k^2/b_a^2), \tag{20a}$$

$$h_l(k) = \lambda_r\, k^l \exp(-k^2/b_r^2), \tag{20b}$$

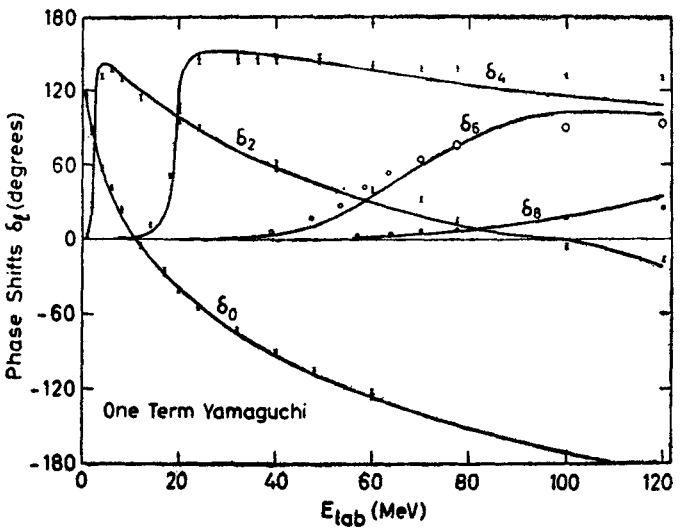

Figure 2. Fits to the α-α phase-shifts (nuclear part) with the one-term separable potential of Yamaguchi form.

and thus four parameters are searched for $l = 0$ and $l = 2$ while for higher partial waves only λ_a and b_a are searched as $h_l(k)$ is put equal to zero. The parameters are listed in table 1 and the phase shifts thus fitted are displayed in figure 3. The quality of fits is slightly better than those in figure 1. In all these cases fits to $\delta_{10}^N(k^2)$ are not shown as this phase shift is very small and attains a value of only 7° at 120 MeV.

It is worthwhile to investigate the unitary and analytic properties of the on-shell amplitudes generated by these potentials. As these amplitudes are obtained by solving the Lippmann-Schwinger equation (Mitra 1969), they automatically satisfy the condition of unitarity. To have some kind of confidence as regards their off-shell continuation, these amplitudes should have analytic properties similar to those shown by the amplitudes generated by an acceptable local potential. Mitra (1961) has discussed this criteria, for the nucleon-nucleon case, with respect to a Yukawa type interaction and translates the conditions of analyticity of the amplitudes onto the form-factors $g_l(k)$ of the separable potential if the latter are to be reliably used to generate the former. These conditions are (i) $g_l(k) \to 0$ as k^l for $k \to 0$ (ii) $g_l(k)$ has branch cut behaviour for k^2 lying between $-\alpha$ and $-\beta_{l/4}^2$ with $\beta_{l/4}^2 > a_l^2$ where a_l^2 is the energy of the bound state, if any, in that partial wave and (iii) $g_l(k) \to 0$ at least as fast as $(1/k^2) \ln k^2$ for $k^2 \to \alpha$. We, however, believe that the dominant long range part of the local α-α interaction is of the Gaussian form (Ali and Bodmer 1966; Buck *et al* 1977). Then the form factor is not required to have any left hand singularity (except at $k^2 = -\alpha$ itself) and the appearance of logarithmic term in condition (iii) is also not necessary. Hence the one term Gaussian potential which Kukulin and Neudatchin (1970) constructed on the basis of detailed consideration of the dynamic behaviour (presence of resonances, oscillatory nature of the relative motion wave function at the interior and its Gaussian fall-off in the asymptotic region) of the α-α system, is undoubtedly more reliable for its off-shell behaviour as compared to the Yamaguchi form which has undesirable isolated left hand singularities. For the same reasons and as discussed earlier we expect the two term Gaussian potential to show most acceptable off-shell behaviour.

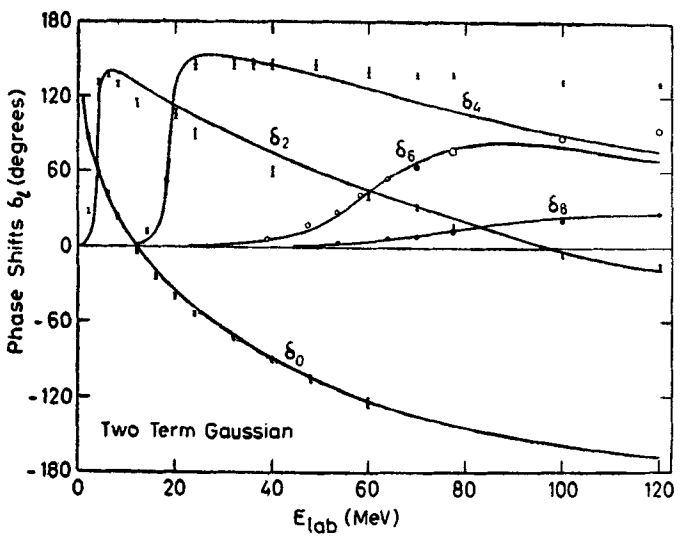

Figure 3. Fits to the α-α phase-shifts (nuclear part) with the two-term separable potential of Gaussian form.

3.3 *Half off-shell factors*

Corresponding to these potentials, described in §§ 3.1 and 3.2, half off-shell factors $f_l(p, k)$ are calculated at a few fixed values of k such that the on-shell centre of mass energy, $E_{on} = \hbar^2 k^2/m_a$ varies from 10 MeV to 60 MeV. The off-shell parameter p goes from 0·25 fm^{-1} to 5 fm^{-1} spanning a range of off-shell energy, $E_{off} = \hbar^2 p^2/m_a$ from 0·65 MeV to 260 MeV. The criteria for inferring that a potential has large off-shell effect in some partial wave and at some on-shell energy E_{on} is that either the corresponding $f_l(p, k)$ is large in magnitude or it is negative in sign. A negative value of $f_l(p, k)$ completely reverses the contribution of that partial wave to the total scattering amplitude. These amplitudes are always real for separable potentials while for local potentials they are expected to be complex in general. It has been found that up to $E_{on} = 60$ MeV, the imaginary part of $f_l(p, k)$ as generated by the potential of Darriulat *et al* is identically zero while that produced by the potential of Buck *et al* have very small imaginary part (less than 10% of the real part) for values of E_{on} above 40 MeV. We thus compare only the real parts of the half-shell factors produced by these potentials.

For partial waves $l \geqslant 4$, $f_l(p, k)$ are positive for all values of E_{on} over almost the entire range of p. They are large in magnitude at low energies except the one for $l = 4$ but that is because the on-shell T-matrix for these partial waves is identically zero in that range. These factors vary smoothly around the on-shell point and tend

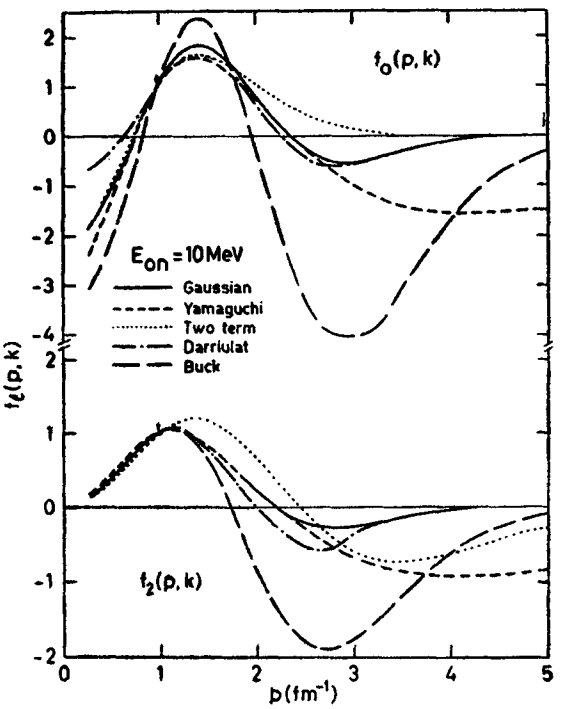

Figure 4. Comparison of the Kowalski-Noyes half-off-shell factors $f_l(p, k)$, in the $l = 0$ and $l = 2$ partial waves, as generated by various potentials. The values of the on-shell energy E_{on} is 10 MeV. The position of the on-shell point is shown by arrow-head.

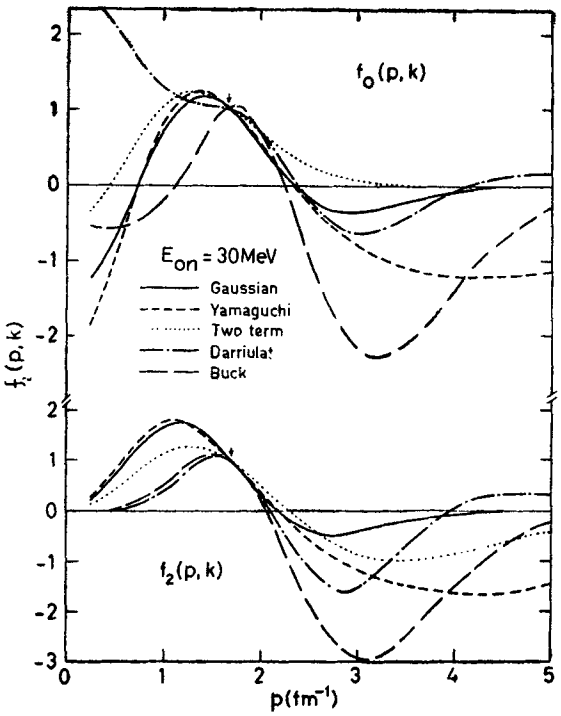

Figure 5. Same as in figure 4. The value of the on-shell energy E_{on} is 30 MeV.

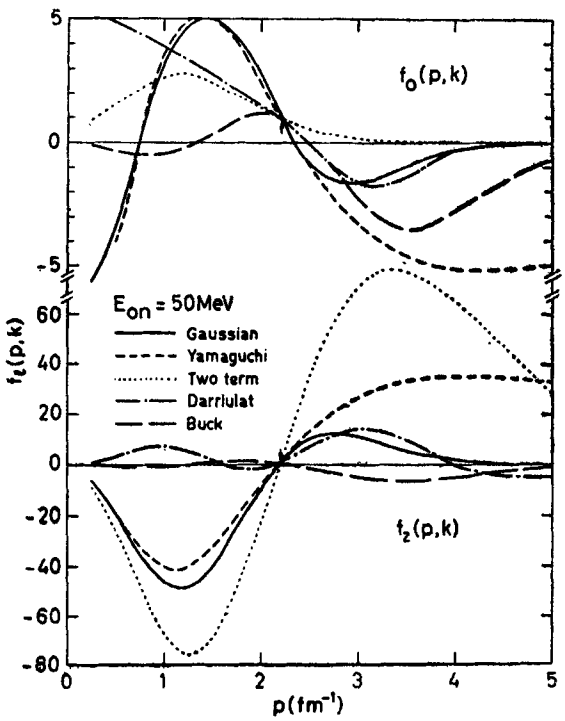

Figure 6. Same as in figure 4. The value of the on-shell energy E_{on} is 50 MeV.

to zero very fast as the off-shell distance increases. While at 30 MeV they take values between 0 and 4, this range comes down to lie between 0 and 1·5 by 50 MeV. So in these partial waves all the potentials have very smooth off-shell behaviour which thus cannot produce very large off-shell effects. The results are drastically different for $f_0(p, k)$ and $f_2(p, k)$. They show large variations and often become negative. Their behaviour is different for different potentials throughout the energy range. We have, therefore, shown only the variation in $f_0(p, k)$ and $f_2(p, k)$ at three values of on-shell energy. At 10 MeV $f_0(p, k)$ varies between $\pm$ 4 and $f_2(p, k)$ is within $\pm$ 2, (figure 4), while at 30 MeV these ranges are $\pm$ 2·5 and $\pm$ 3·0 respectively (figure 5). Though the magnitude variations are small, the sign reversal is not similar for all the potentials and in that sense they are different. By 50 MeV, though $f_0(p, k)$ is within $\pm$ 6, $f_2(p, k)$ changes through + 80 to − 80 (figure 6). Calculation at 60 MeV shows that it is $f_0(p, k)$ which shows large variations while $f_2(p, k)$ remains within $\pm$ 6 (not shown). In this range, the two term separable potential does not produce sign reversal in $f_0(p, k)$ and the variations in the magnitude of $f_2(p, k)$ as produced by Buck's potential are clearly different than those produced by other potentials. A calculation at 90 MeV shows that while other potentials start generating smaller values for all $f_l(p, k)$ these two potentials continue to generate large values for $f_0(p, k)$. Also the imaginary part of $f_0(p, k)$ as produced by Buck's potential at 90 MeV is no longer insignificant as compared to the real part.

We thus conclude that as far as higher partial waves are concerned all these potentials have very similar off-shell behaviour. The non-local potentials are further similar in producing large variations in $f_0(p, k)$ and $f_2(p, k)$ at 60 MeV and 50 MeV respectively. However, for all values of on-shell energy and for a considerable range of the off-shell parameter p, slightly away from the on-shell point, the values of $f_0(p, k)$ and $f_2(p, k)$ generated by different potentials are widely different in magnitude and sometimes in sign. Thus it is expected that for any value of on-shell energy, these potentials will produce quite different half-off-shell cross-section for a very large range of off-shell energies. Many body calculations, which require a knowledge of this behaviour for small off-shell distances at low on-shell energies, are not likely to distinguish between one potential or the other.

The $(\alpha, 2\alpha)$ knock out reaction is more suited for this purpose because of the many choices of kinematical conditions that it offers. However, the dynamics of this reaction is not well established and there are differences of opinion (Jain 1979; Jain and Sarma 1979) about its localization. Furthermore the analysis, in the factorized distorted wave impulse approximation (DWIA), of the most recently studied $(\alpha, 2\alpha)$ reaction on light nuclei at 140 MeV (Wang *et al* 1980) underestimates the energy sharing distribution by a factor of 100 when reasonable parameters are used for the distorting potentials and bound state wave function. Unless these large discrepancies are sorted out, it will be very difficult to extract off-shell effects of the α-α interaction from these experiments. We can, however, study these potentials for a comparatively smaller correction which will improve the shape agreement of the distribution instead of its absolute magnitude. One of the factors in the DWIA expression for the $(\alpha, 2\alpha)$ cross-section is $|\langle \mathbf{p} | T_{\alpha\alpha}(k^2, \theta_{\alpha\alpha}^{\mathrm{CM}}) | \mathbf{k} \rangle|^2$ where $\mathbf{p}$ and $\mathbf{k}$ are the asymptotic c.m. momenta of the α-α system before and after knock out and $\cos\theta_{\alpha\alpha}^{\mathrm{CM}} = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}$. In every analysis, this term is invariably approximated by its on shell reduction $|\langle k\hat{p} | T_{\alpha\alpha}(k^2, \theta_{\alpha\alpha}^{\mathrm{CM}})$

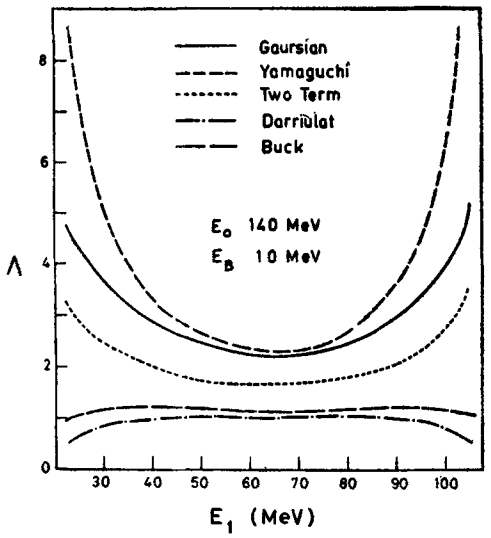

Figure 7. The off-shell correction Λ, defined in the text, for the energy sharing distribution of the $(\alpha, 2\alpha)$ reaction at 140 MeV, as produced by different potentials.

$|\mathbf{k}\rangle|^2$. We define the correcting factor $\Lambda = |\langle \mathbf{p}|T|\mathbf{k}\rangle|^2 / |\langle k\hat{p}|T|\mathbf{k}\rangle|^2$ which can be easily calculated using (15).

The values of Λ as generated by different potentials, for a model quasi free reaction $^{16}O(\alpha, 2\alpha)^{12}C$ at 140 MeV assuming α-binding energy E_B to be 10 MeV, for various values of energy E_1 of one of the outgoing α-particles are shown in figure 7. It may be mentioned that while E_{on} is very nearly 60 MeV at all points, the value of E_{off} is 70 MeV at the centre and goes up to 90 MeV on either side and $\theta_{\alpha\alpha}^{CM}$ is 90° at the centre and varies up to 80°. As is evident the correction is unity for local potentials while separable potentials produce appreciable and quite different values of Λ. When the reaction is better understood and required off-shell correction is reliably estimated we will be able to choose our potential in preference to others. Again we predict, from a look at figure 7, that the two term Gaussian potential may provide an acceptable correction.

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