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Schrödinger vs. Dirac bound state spectra of $Q\bar{Q}$-systems and a plausible Lorentz structure of the effective power-law potential

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Abstract. It is shown that a non-relativistic power-law potential model for the heavy quarks in the form $V(r) = Ar^{\nu} + V_0$, $(A, \nu > 0)$ acquires relativistic consistency in generating Dirac bound states of $Q\bar{Q}$-system in agreement with the Schrödinger spectroscopy if the interaction is modelled by equally mixed scalar and vector parts as suggested by the phenomenology of fine-hyperfine splittings of heavy quarkonium systems in a non-relativistic perturbative approach.

Keywords. Relativistic consistency; Lorentz vector and scalar; quark; confinement; Schrödinger spectroscopy; fine-hyperfine splittings.

1. Introduction

It has been shown recently (Martin 1980; Barik and Jena 1980) that a purely phenomenological approach to the non-relativistic potential model study of Ψ and Υ spectra can very well lead to a static non-coulombic power-law potential of the form,

$$V(r) = Ar^{\nu} + V_0. \tag{1}$$

Here ν is close to 0·1 and $A > 0$. The non-coulombic short distance behaviour of this potential; which is in apparent contradiction with the predictions of quantum chromodynamics (QCD) does not pose any problem to explain the fine-hyperfine splittings, if we formally prescribe the spin-dependence to be generated through this static confining potential in the form of an approximately equal admixture of scalar and vector parts with no contributions from the anomalous quark magnetic moments (Barik and Jena 1980). This non-relativistic formalism, when extended to a unified study of the entire meson spectra including the ordinary light and the heavy mesons, gives a very good account of the meson masses, fine-hyperfine splittings, electromagnetic transition rates and leptonic decay widths, without reflecting any inadequacy in the short and long-range behaviour of this simple effective power-law potential (Barik and Jena 1981 a, b). Therefore one can argue that even if this phenomenological potential does not possess certain behaviours expected from the theoretical approaches, it definitely simulates the actual potential in a wide range of the quark-antiquark separation distance probed in the study of the entire meson spectra.

However, in spite of the quantitative success in understanding the experimental data on meson spectra through non-relativistic Schrödinger type approach, a potential model must also be realized to yield relativistic quark confinement; if flavour independent quark confinement really exists. Therefore, for a relativistic consistency we should expect the power-law potential to generate relativistic bound states of quark-antiquark systems in conformity with the experimental data. Since more numerous and convincing experimental data are available for the bound states of $c\bar{c}$ and $b\bar{b}$—systems; we would concentrate mainly on these heavy $Q\bar{Q}$-systems in studying their relativistic bound states. Such systems being fairly non-relativistic one would be justified in expecting the relativistic bound state spectra to come out in close agreement with the corresponding Schrödinger spectra as well. The study of relativistic bound states of $Q\bar{Q}$-systems would appropriately require an equation designed for two equal mass particles like the Bethe-Salpeter one. Although Bethe-Salpeter equation has been employed for this purpose by some authors (Mitra 1979, 1980) its solution is by no means simple and straightforward. Nevertheless one might hope to gain some insight by using the Dirac equation in an independent particle model approach. Although this is not a very realistic approach (unless perhaps, one quark is much heavier than the other), several authors (Gunion and Li 1975; Critchfield 1975; Rein 1977; Ferreira 1977; Ferreira and Zagury 1977; Ferreira *et al* 1980) have used it to study the relativistic bound state problems of $q\bar{q}$-mesons and S-wave baryons. Therefore following these authors we would choose here to use the Dirac-equation for simplicity.

In order to solve for the Dirac bound states for $Q\bar{Q}$-system one ought to know the exact Lorentz character of the potential to be used. It has been observed by Critchfield (1975) that in spite of their quantitative success in the Schrödinger spectroscopy, the logarithmic potential as well as other conventional forms of the confining potentials of heavy quarks, if considered in the rest frame of the quarks as the time component of a 4-vector, cannot generate relativistic bound states of quarks in the Dirac equation. There has been various attempts in avoiding this difficulty. One of the possibilities is to consider the confining potential as a Lorentz scalar (Critchfield 1975; Gunion and Li 1975). But it has been shown that although a linear scalar potential realizes a relativistically consistent confinement (Critchfield 1975; Gunion and Li 1975), the oscillator like scalar potential does not generate real eigenvalues of the Dirac equation (Ram and Halesa 1979). Another prescription to this effect is to consider the confining potential as a mixture of a (static) 4-vector part V_v and a scalar part V_s (Smith and Tassie 1971; Critchfield 1976; Rein 1977). In this context it is worthwhile to look for some possible clues in the non-relativistic perturbative study of the meson spectra. Of course the gross features alone of the Schrödinger spectra of heavy mesons of Ψ and Υ family which are believed to be fairly non-relativistic, do not reflect the Lorentz-structure of the static potential used in a model. But the phenomenological study of fine-hyperfine splittings of $c\bar{c}$-levels in particular reveals the Lorentz character of the static potential in the process of generating adequate spin-dependent correction to the potential necessary for the purpose. Such a study in power-law potential model (Barik and Jena 1980) leads to a Lorentz-structure of this potential in the form of an approximately equal admixture of scalar and vector parts. This observation about the Lorentz structure of the static confining potential is in line with what was reported by Beavis *et al* (1979) in the context of a different potential model. Such a conclusion was also reached in the phenomenological parametrizations of the potential model by Appelquist *et al* (1978), and also in a gauge-invariant formalism of Eitchen and Feinberg (1979).

It has further been pointed out by Magyari (1980a) that an equally mixed 4-vector and scalar interaction represented by the class of static potentials satisfying the decomposition rule.

$$V(\lambda r) = A(\lambda) f(r) + B(\lambda), \tag{2}$$

has a comfortable feature in reducing the Klein-Gordon and the Dirac equations to the corresponding Schrödinger equation. The power-law potential in (1) as well as the logarithmic potential obviously belong to the above class. It has been shown (Magyari 1980b) that the logarithmic potential realizes a relativistically consistent confinement of the independent quarks with the above prescription. Following Magyari we would show here that a power-law potential of the form given in (1), considered formally as an equal admixture of Lorentz vector and scalar parts, when used appropriately in the Dirac equation in the independent particle model of quarks leads to the $c\bar{c}$ and $b\bar{b}$ bound state masses in exact agreement with the corresponding experimental data as well as the Schrödinger-bound state masses. Such a relativistic consistency can be interpreted as another point in favour of the aforesaid Lorentz-structure observed phenomenologically in a non-relativistic perturbative study of the fine-hyperfine spectrum. In § 2 we derive an expression for the Schrödinger bound state masses of $Q\bar{Q}$-systems in the power-law potential model. Then in the process of obtaining the gross features like the bound state masses and leptonic widths in conformity with the experimental data corresponding to the $c\bar{c}$ and $b\bar{b}$-systems; we can get a good estimate of the potential parameters A, V_0, ν as well as the quark masses m_c and m_b. Such a static potential extracted in this manner from the study of the gross features of $c\bar{c}$ and $b\bar{b}$-systems in a Schrödinger approach; when used in an appropriate manner in the independent particle Dirac equation, with its Lorentz-character as discussed before is found (in § 3) to yield relativistic bound state masses in close agreement with the Schrödinger masses and also with the corresponding experimental levels.

2. Schrödinger bound states and the potential parameters

Here we would like to extract the effective power-law potential representing the static $Q\bar{Q}$-interaction by adequately describing the observed gross-features like the mass-spectra and leptonic decay widths of $c\bar{c}$ and $b\bar{b}$–systems in a non-relativistic Schrödinger formalism. In this process we can get a good estimate of the quark masses m_c, m_b and the potential parameters ν, A and V_0 in (1).

First of all let us obtain an expression for the spin-averaged mass of a non-relativistic $Q\bar{Q}$-bound state system from the Schrödinger equation ($\hbar = c = 1$)

$$\frac{d^2 U(r)}{dr^2} + \left[m_Q (E - V(r)) - \frac{l(l+1)}{r^2}\right] U(r) = 0. \tag{3}$$

Taking $V(r)$ as given in (1) and using a substitution $\rho = (r/r_0)$ with the scale factor r_0 chosen for convenience as

$$r_0 = (m_Q A)^{-1/(\nu+2)}, \tag{4}$$

equation (3) would reduce to the form,

$$\frac{d^2\,U(\rho)}{d\,\rho^2}+\left[\epsilon-\rho^{\nu}-\frac{l\,(l+1)}{\rho^2}\right]U(\rho)=0, \tag{5}$$

where

$$\epsilon=m_Q\,(E-V_0)\,[m_Q\,A]^{-2/(\nu+2)}. \tag{6}$$

Now for $\nu>0$ a solution to (5) is always possible either by an exact numerical method or by the semi classical WKB-method (Quigg and Roener 1979; Feldman *et al* 1979) which would give $\epsilon=\epsilon_{nl}$ (say) corresponding to any particular radial and orbital state (n, l) with the quark-antiquark binding energy $E=E_{nl}$ obtained from (6) as

$$E_{nl}=a\,(a/m_Q)^{\nu/(\nu+2)}\;\epsilon_{nl}+V_0, \tag{7}$$

where $a=(A)^{1/(\nu+1)}$. (8)

Then the non-relativistic Schrödinger bound-state mass of the $Q\bar{Q}$-system follows immediately as,

$$M_{nl}^{\text{Sch}}\,(Q\bar{Q})=2m_Q+V_0+a\,(a/m_Q)^{\nu/(\nu+2)}\,\epsilon_{nl}. \tag{9}$$

If we take the semiclassical WKB results (Quigg and Rosner 1979) of (5), then we have ϵ_{nl} and the absolute square of the S-state wave functions at the origin $|\,\psi_{ns}\,(0)\,|^2$ in the following form;

$$\epsilon_{nl}=[G(\nu)\,(n+l/2-\tfrac{1}{4})]^{2\nu/(\nu+2)}, \tag{10}$$

$$|\,\psi_{ns}\,(0)\,|^2=\frac{1}{2\pi^2}\left(\frac{\nu}{\nu+2}\right)[am_Q\,\{aG(\nu)\}^{\nu}]^{3/(\nu+2)}\,(n-\tfrac{1}{4})^{2(\nu-1)/(\nu+2)}, \tag{11}$$

where, $G(\nu)=\dfrac{2(\pi)^{1/2}\,\Gamma\,(3/2+1/\nu)}{\Gamma\,(1+1/\nu)}$. (12)

Now using these results we can obtain the gross features like the spin-averaged bound state masses and the leptonic decay widths of $Q\bar{Q}$-systems. Equation (9) aided with (10) and (12) gives obviously the Schrödinger bound state masses of $Q\bar{Q}$-systems corresponding to various possible radial ($n=1, 2, 3$) and orbital ($l=0, 1, 2, \ldots$) states. The leptonic decay widths can be obtained using the Van Royen-Weisskopf formula (van Royen and Weisskopf 1967) in the form

$$\Gamma\,(ns\to e^+\,e^-)=\frac{16\,\pi\,\alpha^2\,e_Q}{M_{ns}^2\,(Q\bar{Q})}\,|\,\psi_{ns}\,(0)\,|^2. \tag{13}$$

However, this formula should not be too much trusted in its absolute value; as it is affected by correction factors not quite certain. In that case the leptonic decay width ratio is given by

$$R_{ns} = \frac{\Gamma\,(ns \to e^+\,e^-)}{\Gamma\,(1s \to e^+\,e^-)} = \left[\frac{M_{1s}\,(Q\bar{Q})}{M_{ns}\,(Q\bar{Q})}\right]^2 \times \frac{|\,\psi_{ns}\,(0)\,|^2}{|\,\psi_{1s}\,(0)\,|^2}, \tag{14}$$

would serve as a meaningful and reliable quantity.

Now as a phenomenological exercise we first of all fix the parameter $\nu = 0{\cdot}1$ in close agreement with the findings of Martin (1980) and Barik and Jena (1980). Then taking the experimental masses of Ψ, Ψ' and Υ, Υ' as inputs in (9), we determine the potential parameters a, V_0 and the quark masses m_c and m_b as follows:

$$(a,\ V_0) \equiv (5{\cdot}4072,\ -\ 7{\cdot}452;\ \text{GeV}), \tag{15}$$

$$(m_c,\ m_b) \equiv (1{\cdot}77,\ 5{\cdot}11;\ \text{GeV}). \tag{16}$$

Then using these parameters we can calculate the bound state mass spectrum and the leptonic decay width ratios (14) for $c\bar{c}$ and $b\bar{b}$-systems. These results in comparison with the corresponding experimental values are presented in tables 1 and 2. In view of the close agreement between the calculated and the experimental values, we believe to

Table 1. Schrödinger bound state spectrum of $b\bar{b}$ and $c\bar{c}$ systems.

$n,\ l$	$\epsilon_{nl}^{\text{WKB}}$	$M_{nl}^{\text{Sch}}\,(c\bar{c})$ GeV		$M_{nl}^{\text{Sch}}\,(b\bar{b})$ GeV	
		Theory	Expt.	Theory	Expt.
1 S	1·229	3·0969	3·097 ± 0·002	9·43393	9·4336 ± 0·0002
2 S	1·3323	3·6859	3·686 ± 0·003	9·9939	9·9944 ± 0·00004
3 S	1·3909	4·02	4·03 ± 0·01	10·3116	10·3231 ± 0·00004
4 S	1·4326	4·25789	—	10·5377	10·5476 ± 0·00011
5 S	1·4653	4·44389	4·417 ± 0·01	10·7145	—
1 P	1·2903	3·44632	—	9·76612	—
2 P	1·3646	3·86996	—	10·1689	—
3 P	1·4132	4·1473	—	10·4325	—

Table 2. Ratio of decay width $R_{ns} = \dfrac{\Gamma\,(ns \to e^+\,e^-)}{\Gamma\,(1s \to e^+\,e^-)}$ for $c\bar{c}$ and $b\bar{b}$ systems.

$n\ \ l$	$R_{ns} = \Gamma\,(ns \to e^+\,e^-)/\Gamma\,(1s \to e^+\,e^-)$			
	$c\bar{c}$ systems		$b\bar{b}$ systems	
	Theory	Experiment	Theory	Experiment
1 S	1	1	1	1
2 S	0·341	0·45 ± 0·09	0·431	0·44 ± 0·06
3 S	0·194	0·156	0·275	0·32 ± 0·04
4 S	0·133	0·102	0·201	0·2 ± 0·06

have a fairly good estimate of the quark masses and the potential parameters of the effective static potential used on this process [equation (1)].

3. Dirac bound states and the Lorentz structure

Now we would like to use the static potential extracted in § 2 in an appropriate manner to obtain spin-averaged mass of the $Q\bar{Q}$-bound system from the Dirac equation written in the independent particle model of quarks. As observed in the phenomenology of the fine-hyperfine splittings of the mesons (Barik and Jane 1980; Beavis *et al* 1979; Appelquist *et al* 1978) and also following the prescriptions of Magyari if we consider the potential $V(r)$ in (1) as an admixture of scalar and vector components like

$$V(r) = g_v V(r) + (1 - g_v) V(r), \tag{17}$$

with the vector fraction $g_v = 1/2$, then in the independent particle model of quarks, the potential appropriately would be $V'(r) = 1/2\, V(r) = V_s(r) + V_v(r)$. Hence the scalar part $V_s(r)$ and the vector part $V_v(r)$ would be each equal to $1/4\, V(r)$. Thus with the static interactions $V_s(r)$ and $V_v(r)$ simultaneously present, the Dirac equation can be written as (with $\hbar = c = 1$)

$$(\boldsymbol{\alpha} \cdot \mathbf{P} + m_Q \beta)\, \psi(\mathbf{r}) = [E' - V_v(r) - V_s(r)\, \beta]\, \psi(\mathbf{r}). \tag{18}$$

Since $V_v(r)$ and $V_s(r)$ are spherically symmetric, (18) decomposes as (Schiff 1968)

$$(E' - V_v - V_s - m_Q)\, \phi(r) + \left(\frac{k+1}{r} + \frac{d}{dr}\right) \chi(r) = 0, \tag{19}$$

$$(E' - V_v + V_s + m_Q)\, \chi(r) + \left(\frac{k-1}{r} - \frac{d}{dr}\right) \phi(r) = 0, \tag{20}$$

where $k = l + 1$, when the total angular momentum $j = l + 1/2$ and $k = -l$ when $j = l - 1/2$. Here ϕ and χ are the radial parts of the 'large' and 'small' components of the spinor $\psi(r)$ respectively. Now with $\phi(r) = U'(r)/r$ and $V_s(r) = V_v(r) = \frac{1}{4}(Ar^{\nu} + V_0)$ we can obtain from (19) and (20), the equation for the 'large' component $\phi(r)$ in the form (Magyari 1980a)

$$\frac{d^2 U'(r)}{dr^2} + \left[(E' + m_Q)(E' - m_Q - 2\, V_s(r)) - \frac{l(l+1)}{r^2}\right] U'(r) = 0. \tag{21}$$

Introducing a dimensionless variable $\rho = (r/r_0')$ with the scale factor chosen conveniently as,

$$r_0' = [(m_Q + E')\, A/2]^{-1/(\nu+2)}, \tag{22}$$

equation (21) would reduce to the Schrödinger form like,

$$\frac{d^2U'(\rho)}{d\rho^2}+\left[\epsilon' - \rho_\nu - \frac{l\,(l+1)}{\rho^2}\right] U'\,(\rho) = 0, \tag{23}$$

where, $$\epsilon' = (E' - m_Q - \tfrac{1}{2}\,V_0)\,[(m_Q + E')\,(2/A)^{\,2/\nu}\,]^{\nu/\nu+2}. \tag{24}$$

Equation (23) being identical to its non-relativistic counterpart, would yield the value of $\epsilon' = \epsilon_{nl} > 0$ corresponding to the confined bound states of quarks which must be the same as that obtained from (5). Thus we realize that there exists bounded solutions describing confined Dirac bound states of quarks with energy eigen values $E' = E'_{nl} > |\,(m_Q + \frac{1}{2}\,V_0)\,|$, which can be obtained from (24) with $\epsilon' = \epsilon_{nl}$. If we write with $A = a^{\nu+1}$,

$$(E'_{nl} - m_Q - \tfrac{1}{2}\,V_0) = a\,x_{nl}, \tag{25}$$

and $$(2m_Q + \tfrac{1}{2}\,V_0) = a\,b,$$

equation (24) can be converted to the form,

$$x_{nl}^{(\nu+2)/\nu}\,(x_{nl} + b) = (2)^{-2/\nu}\,[\epsilon_{nl}]^{(\nu+2)/\nu}. \tag{26}$$

If we eliminate V_0 from the expression for b in (25) using (9) we can write

$$b = \left[\frac{M_{nl}^{\mathrm{Sch}}\,(Q\bar{Q})}{2a} + \frac{m_Q}{a}\right] - \tfrac{1}{2}\,(a/m_Q)^{\nu/(\nu+2)}\,\epsilon_{nl}. \tag{27}$$

Now to a first approximation if we consider $M_{nl}^{\mathrm{Sch}}\,(Q\bar{Q}) \simeq 2m$ (which is quite true at least for the ground states for the heavy mesons) we can set the constant b in (26) as,

$$b \simeq \left(\frac{2m_Q}{a}\right) - \frac{1}{2}\,(a/m_Q)^{\nu/(\nu+2)}\,\epsilon_{nl}. \tag{28}$$

Then from inspection we can find that (26) has a positive root solution given by

$$x_{nl} = \tfrac{1}{2}\,(a/m_Q)^{\nu/(\nu+2)}\,\epsilon_{nl}, \tag{29}$$

which when used in (25) leads to the Dirac bound state mass $M_{u\,l}^{\mathrm{Dirac}}\,(Q\bar{Q}) = 2\,E'_{nl}$ as,

$$M_{nl}^{\mathrm{Dir}}\,(Q\bar{Q}) = 2m_Q + V_0 + a\,(a/m_Q)^{\nu/(\nu+2)}\,\epsilon_{nl} \tag{30}$$

From (30) and (9) it obviously follows that the Dirac bound states of $Q\bar{Q}$-system in the power-law potential model exists with the bound state mass $M_{nl}^{\mathrm{Dirac}}(Q\bar{Q})$ in good agreement with the Schrödinger one $M_{nl}^{\mathrm{Sch}}\,(Q\bar{Q})$.

However to demonstrate the existence of Dirac bound states in close conformity with the experimental bound state masses of a $Q\bar{Q}$-system considered in a power-law potential model (1) with equally mixed vector-scalar Lorentz structure, we go for some numerical details with reference to the $b\bar{b}$ and $c\bar{c}$ system. Now using the parameters given in (15) and (16) we can obtain b from (25). We take WKB-value for ϵ_{nl} from a semi-classical solution of (23) which is always possible for $\nu > 0$. (Quigg and Rosner 1979; Feldman *et al* 1979). Then according to (10) and (12)

$$\epsilon_{nl} = \left[(n + l/2 - \tfrac{1}{4})\frac{2(\pi)^{1/2}\,\Gamma(3/2 + 1/\nu)}{\Gamma(1 + 1/\nu)}\right]^{2\nu/(\nu+2)}. \tag{31}$$

With b and ϵ_{nl} known, we can always solve (26) for an unique positive root x_{nl}, which would yield from (25) the Dirac bound state mass for the $Q\bar{Q}$-bound state with radial quantum number, $n = 1, 2, 3, \ldots$ etc. and orbital quantum number $l = 0, 1, 2, \ldots$ etc., in the following form

$$M_{nl}^{\rm Dirac}(Q\bar{Q}) = 2\,E_{nl} = (2a\,x_{nl} + 2m_Q + V_0). \tag{32}$$

In this way we can obtain the Dirac bound state masses of $c\bar{c}$ and $b\bar{b}$-systems. We have presented these results in table 3 along with the corresponding experimental data. We have taken Schrödinger bound state masses calculated from (9) for comparison in the absence of the corresponding experimental data. A comparison shows that the Dirac bound state masses are in good agreement with the Schrödinger and also the experimental values.

4. Conclusion

Thus we conclude that it is possible to generate bound state mass spectrum of $c\bar{c}$ and $b\bar{b}$-systems in an effective power-law potential used in the Dirac equation with an equally mixed vector-scalar Lorentz structure. We must point out that since our

Table 3. Dirac bound state spectrum of the $c\bar{c}$ and $b\bar{b}$ system. Within bracket are Schrödinger mass values which are taken in the absence of experimental data for comparison.

n, l	$\epsilon_{nl}^{\rm WKB}$	$M_{nl}^{\rm Dirac}(c\bar{c})$ GeV		$M_{nl}^{\rm Dirac}(b\bar{b})$ GeV	
		Theory	Expt.	Theory	Expt.
1 S	1·229	3·1175	3·097 ± 0·002	9·4462	9·4336 ± 0·0002
2 S	1·3323	3·6789	3·686 ± 0·003	9·9977	9·9944 ± 0·00004
3 S	1·3909	3·9966	4·03 ± 0·01	10·3101	10·3231 ± 0·00004
4 S	1·4326	4·2222	(4·25789)	10·5321	10·5476 ± 0·00011
5 S	1·4653	4·3985	4·417 ± 0·01	10·7057	(10·7145)
1 P	1·2903	3·4508	(3·4463)	9·7735	(9·7661)
2 P	1·3646	3·8539	(3·87699)	10·1698	(10·1689)
3 P	1·4132	4·1173	(4·1473)	10·4289	(10·4325)

observation in (30) was independent of any particular $\nu > 0$, any confining potential, sogarithmic, linear or harmonic with the above Lorentz structure will also be conlistent in giving relativistic quark confinement. This relativistic consistency can be interpreted as a further support for the phenomenological observation about the equally mixed vector-scalar Lorentz-structure of the power-law potential in the context of the fine-hyperfine splittings of $c\bar{c}$-spectrum (Barik and Jena 1980).

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