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On light supernuclei

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Abstract. The possible existence of charmed analogues of the YN and light hypernuclear systems are investigated using phenomenological one-boson-exchange potential and the SU(4) symmetry. Bound light supernuclei such as C_1N ($I = 3/2$, $J = 0$) and $C_1NN(I = 2, J = 1/2)$ C_0NN ($I = 1$, $J = 1/2$, and $I = 0$, $J = 1/2$ or $3/2$) are predicted with reasonable binding energies using Faddeev formalism for the three body systems.

Keywords. Supernuclei; Faddeev formalism: SU(4) symmetry; one-boson exchange potential.

1. Introduction

The discovery of charm quantum number has led to the study of two body interactions of charmed baryons with other hadrons (Iwao 1977; Dover *et al* 1977) as well as to speculations (Tyapkin 1975; Batusov *et al* 1976) on the possible existence of the charmed analogues of hypernuclei called supernuclei. Since the charmed particles have been established to belong to SU(4) flavour multiplets, supernuclei may be treated as analogues of hypernuclei. The hyperon-nucleon (YN) and the hyperon-hyperon (YY) interactions deduced from scattering and bound state data using a SU(3) symmetric interaction Lagrangian for strong interactions (Brown *et al* 1972; Nagels *et al* 1975) lead to a consistent picture. Similar studies with charmed particles can therefore give information on the validity of the SU(4) symmetry and provide predictions on the possible existence of supernuclei. Further, recent studies on the existence of multiquark hadrons with baryon number $B > 2$ and charm $c \neq 0$ (Bhamathi *et al* 1980) through a bag model has led to the prediction of possible bound and resonant states. If such bound or resonant charmed multiquark hadronic states exist, some of them will be coupled to supernuclei and may decay through these channels if the latter exist as bound systems. Thus the study of light supernuclei becomes interesting and important in testing the validity of SU(4) symmetry and the unambiguous identification of charmed multiquark hadronic states if they exist.

Theoretical studies on supernuclei have been initiated by Iwao, Dover and Kahana. While Iwao made a general study on the plausible existence of supernuclei, Dover and Kahana made a more detailed study using two body $B_c - N$ one boson exchange potential (OBEP) under SU(4) symmetry and concluded that there is a strong possibility of the $C_1N(I = 3/2)$ and SN ($I = 1$) states being bound in the $^1S_0$ state. They had used further the B_c-nucleus potential obtained by averaging the $B_c - N$ interaction over nuclear density in the $I = 0, J = 0$ state and had found a rich spectrum of many bound levels. While this treatment may be applicable for systems with $B > 5$ it is well-known that the three and four body systems even in the nuclear case need to be handled more carefully and much more so in the case of light hypernuclei. It has been found that the application of the Faddeev formalism (Faddeev 1961) or the equivalent Schrödinger formalism (Mitra 1969) leads to a better though not completely satisfactory description for the three and four body systems. Since the SU(4) symmetry is found to lead to B_cN potentials not very different from the YN or NN potentials it is expected that the three and four body supernuclei would also be rather lightly bound. Therefore we have applied explicitly the three body formalism to investigate the possible bound states of the light supernuclei with $B = 3$.

In this paper we report the results of our analysis on light supernuclei C_0N, C_1N, C_0NN and C_1NN obtained by using the non-relativistic Schrödinger formalism to deduce the low energy two body scattering parameters and the Schrödinger equivalent of the Faddeev formalism for the three body systems.

2. Formalism

The OBEP for $B_c - N$ was calculated under full SU(4) symmetry with the symmetry-breaking introduced only through the use of the physical values for the masses of the hadrons. The H_{int} is obtained by constructing the SU(4) invariant interaction (Dover *et al* 1977) between the baryon 20-plet and the meson 15-plet and singlet. The relevant terms in the H_{int} for the C_0N and C_1N interactions with pseudo-scalar meson exchanges are given by

$$H_{\text{int}} = g\Big[2i(1-\alpha)\,\bar{\mathbf{c}}_1\times\mathbf{c}_1\cdot\pi + \frac{2}{\sqrt{3}}(1-\alpha)\,\bar{\mathbf{c}}_1\cdot\mathbf{c}_1\,\eta + \frac{2}{\sqrt{3}}\Big(1-\frac{5}{3}\alpha\Big)\bar{\mathbf{c}}_0\,\mathbf{c}_0\,\eta - \frac{1}{\sqrt{6}}(1-4\alpha)\,\bar{\mathbf{c}}_1\cdot\mathbf{c}_1\,\eta_c - \frac{1}{\sqrt{6}}\Big(1+\frac{4}{3}\alpha\Big)\bar{\mathbf{c}}_0\,\mathbf{c}_0\,\eta_c\Big], \tag{1}$$

where $g = g_{NN\pi}$ and α is the d/f ratio. The corresponding expressions for the scalar and vector meson 15 plets and singlets can be written down as usual. Since the charmed mesons belonging to the 15 plets are much heavier than the octet mesons we have neglected their contributions to the potential in calculating the low energy scattering parameters of the B_cN interactions. Further in computing the OBEP of the C_0N and C_1N systems in the $I = 1/2$ and $3/2$ states we have preferred use of the coupling constants and α obtained by the fit of Brown *et al* (1972) over that determined by Nagels *et al* (1975); to the NN and YN data. Our choice was dictated by the fact that while the conclusions of Brown *et al* differ slightly from those of Nagels *et al* with respect to the existence of the ${}^3S_1$ ΛN resonance just below the ΣN threshold the overall fit to the YN scattering data is quite good. Further, the parameters of Brown *et al* have been found adequate to fit the $K^{-}d$ reaction data (Morris 1974; Dalitz 1978) and also to yield $\Lambda\Lambda$ scattering parameters (Dalitz) consistent with those deduced from double-hypernuclear data (Bodmer and Ali 1965; Tang and Herndon 1965). A comparison of the B_cN potentials thus obtained with the corresponding NN and YN potentials was found to indicate that B_cN bound states can exist: similar to the conclusions of Dover and Kahana (1977).

Using OBEP the Schrödinger equations for scattering in the singlet and triplet states of C_1N ($I = 3/2$) and (C_0N, C_1N, $I = 1/2$) coupled channel cases were solved numerically and the scattering length a and the effective range r_0 were obtained in the standard way. The results displayed in table 1 indicate that the ${}^1S_0$ C_1N system is bound in the $I = 3/2$ state whereas there are no bound states in the $I = 1/2(C_0N, C_1N)$ coupled system. These results are in agreement with those of Dover *et al* (1977) except that the binding energy of the C_1N system is rather small being of the order of only 0·25 MeV. The next step was to formulate the coupled equations for the three body C_0NN and C_1NN systems. Since a solution of these three-body equations with OBEP for the two-body interaction would involve solving several coupled multiple integral equations numerically, we had to reduce the problem to one involving a separable form for the kernel by using equivalent separable potentials of the Yamaguchi (1954) type with appropriate strength and inverse range parameters which reproduce the previously determined a and r_0. A further complication in the C_0NN system with the C_0N interaction in the $I = 1/2$ state is the coupling to the C_1N channel in the intermediate state. In these preliminary studies we have neglected the coupling to the second channel by truncating the intermediate states to include only the diagonal interaction. However, it must be noted that in computing the two body interaction parameters of the C_0N, $I = 1/2$ state coupling to the C_1N channel has been taken into account fully. Similarly where the triplet state interaction was involved the contributions from the coupling to the D state through tensor forces had to be neglected.

The Schrödinger form (Mitra 1969) of the set of three coupled three body equations with a separable interaction of the Yamaguchi type which is equivalent to the Faddeev equations is given by

Table 1. a and r_0 variation with r_c.*

			$r_c = 0{\cdot}46\ f$		$r_c = 0{\cdot}50\ f$	
System	I	J	a	r_0	a	r_0
C_1N	$\frac{3}{2}$	0	16·12	1·82	−4·55	2·77
C_1N	$\frac{3}{2}$	1	−2·96	4·71	−2·28	5·54
C_0N	$\frac{1}{2}$	0	−0·69	5·77	−0·30	16·86
C_0N	$\frac{1}{2}$	1	5·29	3·71	5·29	3·70

*All quantities are quoted in units of fermi.

$$\Phi_{12}(\mathbf{p}_3)\left[1-\lambda_{12}\int\frac{\left[f_{12}\left(\mathbf{p}_1'+\frac{\mathbf{p}_3}{2}\right)\right]^2 d^3\mathbf{p}_1'}{k_1\,\mathbf{p}_1'^2+\mathbf{p}_3^2+\mathbf{p}_1'\cdot\mathbf{p}_3+a^2}\right]$$

$$=\lambda_{13}\int\frac{f_{12}\left(\mathbf{p}_2'+\frac{\mathbf{p}_3}{2}\right)f_{13}\left(\frac{\mathbf{p}_2'}{2}+\mathbf{p}_3\right)\Phi_{13}(\mathbf{p}_2')\,d^3\mathbf{p}_2'}{k_1(\mathbf{p}_3^2+\mathbf{p}_2'^2)+(2k_1-1)\,\mathbf{p}_3\cdot\mathbf{p}_2'+a^2}$$

$$+\lambda_{23}\int\frac{f_{12}\left(\mathbf{p}_1'+\frac{\mathbf{p}_3}{2}\right)g_{23}\left(\frac{\mathbf{p}_1'}{2}+\mathbf{p}_3\right)\chi_{23}(\mathbf{p}_1')\,d^3\mathbf{p}_1'}{k_1\,(\mathbf{p}_3'^2+\mathbf{p}_3^2+\mathbf{p}_3\cdot\mathbf{p}_1'+a^2},\qquad(2)$$

$$\Phi_{13}(\mathbf{p}_2)\left[1-\lambda_{13}\int\frac{f_{13}\left(\mathbf{p}_3'+\frac{\mathbf{p}_2}{2}\right)d^3\mathbf{p}_3'}{k_1\,\mathbf{p}_3'^2+\mathbf{p}_2^2+\mathbf{p}_3'\cdot\mathbf{p}_2+a^2}\right]$$

$$=\lambda_{12}\int\frac{f_{13}\left(\frac{\mathbf{p}_2}{2}+\mathbf{p}_3'\right)f_{12}\left(\mathbf{p}_2+\frac{\mathbf{p}_3'}{2}\right)\Phi_{12}(\mathbf{p}_3')\,d^3\mathbf{p}_3'}{k_1(\mathbf{p}_2+\mathbf{p}_3')^2+\mathbf{p}_2^2-(\mathbf{p}_2+\mathbf{p}_3')\cdot\mathbf{p}_2+a^2}$$

$$+\lambda_{23}\int\frac{f_{13}\left(\mathbf{p}_1'+\frac{\mathbf{p}_2}{2}\right)g_{23}\left(\frac{\mathbf{p}_1'}{2}+\mathbf{p}_2\right)\chi_{23}(\mathbf{p}_1')\,d^3\mathbf{p}_1'}{k_1\,\mathbf{p}_1'^2+\mathbf{p}_2^2+\mathbf{p}_2\cdot\mathbf{p}_1'+a^2},\qquad(3)$$

$$\chi_{23}(\mathbf{p}_1)\left[1-\lambda_{23}\int\frac{g_{23}^2\left(\mathbf{p}_3'+\frac{\mathbf{p}_1}{2}\right)d^3\mathbf{p}_3'}{k_1\,\mathbf{p}_1^2+\mathbf{p}_3'^2+\mathbf{p}_1\cdot\mathbf{p}_3'+a^2}\right]$$

$$=\lambda_{12}\int\frac{g_{23}\left(\frac{\mathbf{p}_1}{2}+\mathbf{p}_3'\right)f_{12}\left(p_1+\frac{\mathbf{p}_3'}{2}\right)\Phi_{12}(\mathbf{p}_3')\,d^3\mathbf{p}_3'}{k_1\,p_1^2+(\mathbf{p}_1+\mathbf{p}_3')^2-\mathbf{p}_1\cdot(\mathbf{p}_1+\mathbf{p}_3')+a^2}$$

$$+\lambda_{13}\int\frac{g_{23}\left(\frac{\mathbf{p}_1}{2}+\mathbf{p}_3'\right)f_{13}\left(\mathbf{p}_1+\frac{\mathbf{p}_3'}{2}\right)\Phi_{13}(\mathbf{p}_3')\,d^3\mathbf{p}_3'}{k_1\,\mathbf{p}_1^2+\mathbf{p}_3'^2+\mathbf{p}_1\cdot\mathbf{p}_3'+a^2},\qquad(4)$$

when the three particles are distinct.

When two of them are identical the equations reduce to a simpler form involving only two coupled integral equations.

$$\Phi_{12}(\mathbf{p}_3)=\frac{\lambda_{12}}{[1-\lambda_{12}f_0]}\int\frac{f_{12}\left(\mathbf{p}_2'+\frac{\mathbf{p}_3}{2}\right)f_{12}\left(\mathbf{p}_3+\frac{\mathbf{p}_2'}{2}\right)\Phi_{12}(p_2')\,d^3\,p_2'}{k_1\,(p_2'+p_3)^2-p_2'\cdot p_3+a^2}$$

$$+\frac{\lambda_{23}}{[1-\lambda_{12}f_0]}\int\frac{f_{12}\left(\mathbf{p}_2'+\frac{\mathbf{p}_3}{2}\right)g_{23}\left(\mathbf{p}_3+\frac{\mathbf{p}_2'}{2}\right)}{k_1\,\mathbf{p}_2'+\mathbf{p}_3^2+\mathbf{p}_3\cdot(\mathbf{p}_3+\mathbf{p}_2')+a^2)}\chi_{23}(\mathbf{p}_2')\,d^3\,\mathbf{p}_2',\qquad(5)$$

$$\chi_{23}(p_1)=\frac{2\lambda_{12}}{[1-\lambda_{23}\,g_0]}\int\frac{g_{23}\left(\mathbf{p}_1+\frac{\mathbf{p}_2'}{2}\right)f_{12}\left(\mathbf{p}_1+\frac{\mathbf{p}_2'}{2}\right)}{(k_1\,\mathbf{p}_1^2+\mathbf{p}_2'+\mathbf{p}_1\cdot\mathbf{p}_2'+a^2)}\Phi_{12}(\mathbf{p}_2')\,d^3\,\mathbf{p}_2'.\qquad(6)$$

The kernels were chosen to be of the form

$$f_{12}(p) = \frac{1}{p^2 + \beta_{12}^2}$$

$$f_{13}(p) = \frac{1}{p^2 + \beta_{12}^2}$$

and $$g_{23}(p) = \frac{1}{p^2 + \beta_{23}^3}$$

λ_{ij} and β_{ij} are the strength and inverse range parameters of the interaction between the ith and jth particles and $E = -\alpha^2/\mu$ is the binding energy of the system.

The bound state energies of the supernuclei C_0NN and C_1NN for their various possible states of spin and isospin were obtained by converting the integral equations to a finite number of coupled linear algebraic equations by the Gaussian quadrature method and searching for the zeros of the determinant of the coefficient matrix. Since the binding energies were expected to be small and to depend sensitively on the two body interactions, even though the n and p were taken to be identical for determining the symmetry of the state, the potential parameters of np, nn and pp were deduced independently from the a and r_0 appropriate to each system as displayed in table 2 along with the B_cN parameters. Apart from computing the total binding energies of the three body systems under consideration we also computed their approximate separation energies by solving for the energy of the charmed baryon interacting independently with the other two nucleons. The results displayed in table 3 lead to the following conclusions.

Table 2. Potential parameters λ and β.*

			$r_c = 0{\cdot}46\,f$		$r_c = 0{\cdot}5\,f$	
System	I	J	$\lambda(a_0^3)$	$\beta(a_0)$	$\lambda(a_0^3)$	$\beta(a_0)$
C_1N	3/2	0	2773	29·7	1316	24·7
C_1N	3/2	1	198·5	14·06	1805	13·93
C_0N	1/2	0	1468	28·57	942·2	26·28
C_0N	1/2	1	52·26	5·60	52·26	5·60
np	0	1	414·0	14·49		
np	1	0	149	11·64		
nn	1	0	138·3	11·47		
pp	1	0	146·1	12·05		

*$a_0 = (10f)^{-1}$

Table 3. Binding and separation energies in MeV.

				$r_c = 0{\cdot}46\,f$		$r_c = 0{\cdot}5\,f$	
System	I_1	J_T	J_{NN}	B.E.	S.E.	B.E.	S.E.
C_1NN	2	1/2	0	0·13	0·13	not bound	
C_0NN	1	1/2	0	5·63	4·98	5·13	4·88
C_0np	0	1/2	1	6·77	4·65	6·33	4·23
C_0np	0	3/2	1	11·53	9·23	11·53	9·23

3. Results and discussion

3.1 C_1NN ($I_{\text{tot}} = 2, J_{\text{tot}} = 1/2$)

This system is found to be very lightly bound for $r_c = 0{\cdot}46\,f$ and unbound when r_c increases to $0{\cdot}5\,f$. This is due to the fact that the NN system in this case will be in the $^1S_0$ state according to Pauli's principle. Thus the C_1N interaction will be a combination of singlet and triplet states with $I = 3/2$. It can be seen from the C_1N scattering parameters that even though the singlet state binds the $I = 3/2$ triplet state interaction which contributes more to the three body system is not strong enough to bind and neither does the NN interaction in the $^1S_0$ state. Hence the overall binding energy is very small. Further if the Coulomb repulsion of the pp is also taken into account the only likely bound systems with $I = 2$ are C_1nn and C_1np which cannot decay into the C_0NN system due to isospin conservation. However these will be very weakly bound if at all. The other three body C_1NN systems with $I = 1$ or 0 will decay rapidly into C_0NN and therefore have not been considered.

3.2 C_0np, C_0nn and C_0pp ($I_{\text{tot}} = 1, J_{\text{tot}} = \frac{1}{2}, J_{NN} = 0$)

For these systems we find that the total binding energy is much larger than the corresponding hypernuclei. This is quite consistent with the fact that C_0N interaction in this case is dominated by the $I = 1/2$ triplet state interaction which even though not bound is considerably stronger than the $I = 3/2$ C_1N state and the YN case and also due to the C_0 particle being much heavier than the hyperon. The separation energy is smaller than anticipated but as we mentioned earlier this is only an approximate estimate since the NN correlations have not been taken into account in computing them.

3.3 C_0np ($I_{\text{tot}} = 0, J_{\text{np}} = 1, J_{\text{tot}} = 3/2$ and $1/2$)

We find that the C_0np system has the largest binding energy, comparable to that of the corresponding ordinary nuclei. This is not unreasonable, for $J_{\text{tot}} = 3/2$ state since the NN and C_0N interaction are in the triplet state besides which the C_0 is heavier than the nucleon which reduces the kinetic energy of the system. As is to be expected the binding energy in the $J_{\text{tot}} = 1/2$ state is smaller since the C_0N interaction is now a linear combination of the singlet and triplet state interactions and the singlet state interaction is much weaker. The separation energies in both cases are close to the expected values.

Thus we find that a three body analysis of light supernuclei leads to the prediction of fairly strongly bound C_0NN systems and possibly only a very lightly bound C_1NN system with total isospin and spin values 2 and 1/2 respectively. These results warrant a more accurate and detailed analysis of light supernuclei as well as double supernuclei and charmed particle binding in nuclear matter. Results of some of these studies which are already in progress will be reported elsewhere.

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