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Strong isospin violation and η-π mixing in $|\Delta S| = 1$ weak transition

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Abstract. We consider the effects of $\eta - \pi$ mixing on the violation of the $|\Delta I| = 1/2$ rule in $|\Delta S| = 1$ weak transitions. The processes considered are the $K \to 2\pi$, $K \to 3\pi$, Λ, Ξ and Λ hyperon decays.

Keywords. Isospin violation: $\eta - \pi$ mixing; $|\Delta I| = 1/2$ rule; kaon decay; hyperon decay.

1. Introduction

In the theory of quantum chromodynamics, the strong interaction Lagrangian is written in terms of a small number of parameters namely, a dimensionless colour gauge coupling and a quark mass for all the quark flavours (Gross *et al* 1979). By dimensional transmutation the coupling constants may be represented in terms of a mass scale (M), say, the mass of a standard hadron consisting of light quarks. One may as well identify M as the renormalization scale parameter to be determined from experimental observations. The pattern of the quark masses, on the other hand, reflects the character of flavour symmetry of the strong interactions. If isospin violation is purely accidental then as Weinberg (1978) has stated, there might be special cases where this violation has noticeable effects and which in most cases will be set by the ratio $(m_d - m_u)/M$. However, the values of the quark mass ratios evaluated from the meson mass spectrum runs counter to such an expectation—the ratio $m_d - m_u/m_d + m_u$ turns out to be $\sim 1/3$ thereby implying the presence of a fairly substantial isospin breaking in strong interactions whose effects are clearly not observed. This problem was essentially solved by Gross *et al* following the observation by 't Hooft that the divergence of the U(1) current contains an anomaly term which ensures that there is no Goldstone boson associated with it even in the chiral limit. Gross *et al* showed that the existence of the anomaly term reduces the degree of isospin violation from $O\,(m_d - m_u/m_d + m_u)$ to $O\,(m_d - m_u/m_s)$ which is phenomenologically the correct order of magnitude. They also showed that the three cases where isospin violation might produce noticeable effects are the pion mass difference problem, the decay process $\eta' \to 3\pi$ and the $\Sigma^{\pm} \to \Lambda e^{\pm} \nu$ decay. Recent measurement of the decay rate $\Gamma(\Psi' \to \Psi\pi^0)$ by the Crystal Ball (Oregila *et al* 1980) and the Mark II (Himel *et al* 1980) groups has given rise to another possibility where also the effects of isospin violation (through isoscalar-isovector pseudoscalar mixing) can be equally important. In fact the measured experimental values for the branching ratio R given by*

$$R = \frac{\Gamma(\Psi' \to \Psi\pi^\circ)}{\Gamma(\Psi' \to \Psi\eta)} = \left(\frac{k_\pi}{k_\eta}\right)^3 \theta^2, \tag{1}$$

show that the η—π mixing parameter θ needs to be as large as

$$\theta = (4{\cdot}5 \pm 0{\cdot}6) \times 10^{-2}$$

$$\text{or } (5{\cdot}5 \pm 1) \times 10^{-2}, \tag{2}$$

corresponding to $R_{\text{exp}} = (4 \pm 1) \times 10^{-2}$ or $(6 \pm 2) \times 10^{-2}$ respectively.

Lately, a number of papers have appeared which have attempted to explain this large branching ratio (Langacker 1980; Gerard *et al* 1980; Ioffe and Shifman 1980; Lahiri *et al* 1980; Lahiri and Bagchi 1981; for earlier works see Segre and Weyers 1976; Genz 1978). While some of them (Langacker 1980; Gerard *et al* 1980) extend the possibility of the mixing phenomenology to include the effects of η' too, the chief reason being that a combined study of baryon and meson mass splittings, the $\eta \to 3\pi$ decay, and $\rho - \omega$ mixing yield a value for $\theta = 0{\cdot}013 \pm 0{\cdot}002$ (Langacker 1980) which is about a factor of 3 too small to explain the decay ratio R; some (Ioffe and Shifman 1980; Lahiri *et al* 1980; Lahiri and Bagchi 1981) hold the view that $\eta - \pi$ mixing is sufficient to explain R. In one of these papers the time ordered product of axial currents has been suitably evaluated to estimate the $\eta - \pi$ mixing angle θ (Lahiri and Bagchi 1981); in another, Weinberg's spectral function sum rule in the asymptotic limit has been made use of to evaluate θ (Lahiri *et al* 1980). In both the approaches, θ has turned out to be very large in comparison with the value just cited above.

That the effects of $\eta - \pi$ mixing may be important and can have an appreciable effect on the violation of the $\Delta I = 1/2$ rule in $\Delta S = 1$ Kl_3 decays was pointed out long ago by Oneda *et al* (1970). It may be mentioned that in an asymptotic symmetry framework they had determined the values of the mixing angles between the π°, η and η' states. In particular, for the $\eta - \pi$ mixing they had obtained two distinct values for θ, one about 2×10^{-2} and the other approximately -6×10^{-3}. They had also pointed out that if the former solution was correct, a sizeable violation of the $\Delta I = 1/2$ rule was expected while if the $\Delta I = 1/2$ rule was well satisfied, the second solution was preferred.

More recently, the effects of the η—π mixing in the violations of the $\Delta I = 1/2$ rule in the $\Delta S = 1$ weak decays in general were studied by Holstein (1979). The result of such a calculation was that the $\eta - \pi$ mixing did produce a significant effect on the measured size of the intrinsic $\Delta I = 3/2$ weak amplitudes. However, the value of θ used to evaluate the $\eta - \pi$ mixing effects was the canonical value obtained by Gross *et al* (1979) in the tadpole approximation *viz*.

$$\theta = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s} = (1 \pm 0{\cdot}2) \times 10^{-2}. \tag{3}$$

*If the $\eta - \pi$ mixing alone is considered

As mentioned earlier, this value of θ is much too small to explain the branching ratio R given by (1).

In this paper we inquire into the effects of the $\eta-\pi$ mixing once again on the $\Delta S=1$ weak transitions in the light of the values of θ obtained by us (Lahiri *et al* 1980; Lahiri and Bagchi 1981) and the value estimated by Oneda *et al* (1970) that differs in sign from the standard values. Our conclusions shall be presented as we go along.

2. Violation of the $\Delta I=\frac{1}{2}$ rule in strangeness changing decays

2.1 $K\to 2\pi$ decays

Following Holstein (1979)*, we consider first of all the non-leptonic weak decays $K\to 2\pi$. The various amplitudes can be expressed in terms of the $\Delta I=1/2$ and $\Delta I=3/2$ components as

$$A(K^\circ\to\pi^3\pi^3)=-\sqrt{\frac{1}{3}}f_1+\frac{2}{\sqrt{15}}f_3,$$

$$A(K^\circ\to\pi^+\pi^-)=\sqrt{\frac{1}{3}}f_1+\frac{1}{\sqrt{15}}f_3,$$

$$A(K^+\to\pi^+\pi^3)=\sqrt{\frac{3}{10}}f_3, \tag{4}$$

where π^3 corresponds to the triplet pseudoscalar state that mixes with the octet state to give the physical π° and η

$$\pi^\circ=P^3+\theta P^8,$$

$$\eta=-\theta P^3+P^8, \tag{5}$$

θ being the $\eta-\pi$ mixing parameter.

By defining y as

$$y=\frac{\sqrt{2}\,A(K^+\to\pi^+\pi^\circ)}{2\,A(K^\circ\to\pi^+\pi^-)-A(K^\circ\to\pi^\circ\pi^\circ)}, \tag{6}$$

$$=0{\cdot}032\pm 0{\cdot}001\text{ (exp.)}$$

one can interpret it as a 3% $\Delta I=1/2$ rule violating parameter

$$\frac{1}{\sqrt{5}}\frac{f_3}{f_1}=0{\cdot}032. \tag{7}$$

*We follow the notations and equations of Holstein (1979).

If $\eta - \pi$ mixing is included, y becomes

$$y = \frac{1}{\sqrt{5}} \frac{f_3}{f_1} + \frac{2}{\sqrt{3}} \theta. \tag{8}$$

by using (4) and (5) and assuming $\Delta I = 1/2$ dominance for $A(K \to \pi\eta)$. We now discuss the effects of various values of θ on (8). For definiteness we consider for θ the value $\theta = 4 \times 10^{-2}$ obtained by us and which lies within the limits dictated by (1), the value $\theta = -6 \times 10^{-3}$ obtained by Oneda *et al* and the canonical value given by (3).

Case 1

Taking the canonical value of θ first *viz.*,

$$\theta = 1 \times 10^{-2},$$

we note that this value of θ gives (Holstein 1979) $(2\theta/\sqrt{3}) \approx 0{\cdot}012$ which takes up about 40% of the experimental $\Delta I = 1/2$ violating amplitude so that only 60% need be a result of intrinsic $\Delta I = 3/2$ terms:

$$\frac{1}{\sqrt{5}} f_3 / f_1 = 0{\cdot}02$$

Case 2

$$\theta = 4 \times 10^{-2}.$$

Such a large value of θ gives $(2\theta/\sqrt{3}) \approx 0{\cdot}048$ which is bigger than the experimental value of y and indicates that the intrinsic $\Delta I = 3/2$ terms may be present with the wrong sign

$$\frac{1}{\sqrt{5}} \frac{f_3}{f_1} \approx -0{\cdot}016.$$

Case 3

$$\theta = -6 \times 10^{-3}.$$

This gives $2\theta/\sqrt{3} \approx -0{\cdot}007$. Thus the $\eta - \pi$ mixing accounts for the experimental $\Delta I = 1/2$ violating amplitude by about 25%. The $\Delta I = 3/2$ effects are then given by

$$\frac{1}{\sqrt{5}} \frac{f_3}{f_1} \approx 0{\cdot}039.$$

2.2 $K \to 3\pi$ decays

Following the standard parametrisation, the $K \to 3\pi$ decays can be written as

$$A(K \to \pi^a \pi^a \pi^b) = A_0 \left[1 - \lambda \frac{2m_K}{m_\pi^2}\left(E_b - \frac{m_K}{3}\right)\right], \tag{9}$$

where the mean decay amplitude is given by A_0 and λ is the slope parameter. The $\Delta I = 1/2$ rule violating parameter can be introduced as (Holstein 1979)

$$v_1 = \frac{1}{4}\left(\frac{A_0^{++-}}{A_0^{+-0}}\right)^2 - 1 = 0{\cdot}216 \pm 0{\cdot}020,$$

$$v_2 = -\frac{1}{2}\frac{\lambda^{+-0}}{\lambda^{++-}} - 1 = 0{\cdot}308 \pm 0{\cdot}051. \tag{10}$$

In the limit when the $\eta - \pi$ mixing is neglected, standard current algebra calculations yield (Bricman *et al* 1978)

$$v_1 = 6y \approx 0{\cdot}19,$$

$$v_2 = \frac{27}{2}y \approx 0{\cdot}43; \tag{11}$$

however if the $\eta - \pi$ mixing is invoked, it can be shown that

$$v_1 = 6y + 2\sqrt{3}\,\theta,$$

$$v_2 = \frac{27}{2}y - 6\sqrt{3}\,\theta. \tag{12}$$

Case 1

$$\theta = 1 \times 10^{-2}.$$

Holstein (1980) has shown that for this value of θ, v_1 and v_2 become

$$v_1 \approx 0{\cdot}228,$$

$$v_2 \approx 0{\cdot}32,$$

in reasonable agreement with (10).

Case 2

$$\theta = 4 \times 10^{-2}.$$

In this case, v_1 and v_2 turn out to be

$$v_1 = 0{\cdot}192 + 0{\cdot}138 = 0{\cdot}33,$$

$$v_2 = 0{\cdot}432 - 0{\cdot}416 = 0{\cdot}016.$$

Clearly, the present experimental values on v_1 and v_2 [see (10)] rule out such a large value of θ. It may be noted that in this case the effects of $\eta - \pi$ mixing on v_1 and v_2 are of the same order as the ones obtained in (11) by neglecting $\eta - \pi$ mixing.

Case 3

$$\theta = -6 \times 10^{-3}.$$

This value of θ leads to

$$v_1 = 0{\cdot}192 - 0{\cdot}021 = 0{\cdot}171,$$

$$v_2 = 0{\cdot}432 + 0{\cdot}062 = 0{\cdot}496,$$

which also disagree with their respective experimental values.

2.3 Λ-*hyperon decays*

Here the $\Delta I = 1/2$ rule violations can be read off from the following ratio

$$R_\Lambda = \frac{A(\Lambda^0 \to p\,\pi^-) + \sqrt{2}\,A(\Lambda^0 \to n\,\pi^0)}{\sqrt{2}\,A(\Lambda^0 \to p\,\pi^-) - A(\Lambda^0 \to n\,\pi^0)} = \sqrt{\frac{2}{3}}\left(\theta - 2\,\rho\,\frac{D-F}{D+3\,F}\right) + z, \tag{13}$$

which has been obtained by taking into account the effects of $\Lambda^0 - \Sigma^0$ mixing as (Gross *et al* 1979)

$$\Lambda^0 = B^8 + \rho\, B^3,$$

$$\Sigma^0 = -\rho\, B^8 + B^3,$$

with $\rho \approx 0{\cdot}0096$. In (13), D and F are the octet $\langle B' \,|\, H_\omega \,|\, B\rangle$ coupling constants and $z = \sqrt{2}\,\sigma_3/(D + 3\,F)$, σ_3 being the bonafide $\Delta I = 3/2$ amplitude contribution. It may be noted that

$$R_\Lambda^{\exp} = -0{\cdot}027 \pm 0{\cdot}008. \tag{14}$$

Taking the approximation $F \approx -2D$, we now consider the following cases

Case 1

$$\theta = 1 \times 10^{-2}.$$

R_Λ turns out to be (Holstein 1979) $R_\Lambda \approx 0{\cdot}018 + z$ which shows that the $\eta - \pi$ mixing effects are working opposite in sign to the experimental $\Delta I = 1/2$ violating effects. Thus the intrinsic $\Delta I = 3/2$ amplitude contribution is now larger *i.e.* $z \approx -0{\cdot}043$ than what was expected by neglecting the $\eta - \pi$ mixing effects *viz.* $z = -0{\cdot}027$.

Case 2

$$\theta = 4 \times 10^{-2}.$$

This gives $R_\Lambda \approx 0{\cdot}042 + z$ which implies that the intrinsic $\Delta I = 3/2$ amplitude contribution ought to be still bigger *viz.* $z \approx -0{\cdot}07$ than what was obtained in the previous case.

Case 3

$$\theta = -6 \times 10^{-3}.$$

This case is quite different from the previous two since here one gets

$$R_\Lambda = 0{\cdot}0045 + z,$$

indicating that the intrinsic $\Delta I = 3/2$ amplitude contribution is *not* much different from naive expectations.

2.4 Ξ *hyperon decay*

The $\Delta I = 1/2$ rule violating ratio is given by

$$R_\Xi = \frac{A(\Xi^- \to \Lambda\pi^-) + \sqrt{2}\, A(\Xi^\circ \to \Lambda\pi^\circ)}{\sqrt{2}\, A(\Xi^- \to \Lambda\pi^-) - A(\Xi^\circ \to \Lambda\pi^\circ)}$$

$$= \sqrt{\frac{2}{3}}\left(\theta - 2\rho\, \frac{D+F}{D-3F}\right) + z', \tag{15}$$

where $z' = \dfrac{\sqrt{2}\,\sigma_3}{D-3F}$. Experimentally, R_Ξ is

$$R_\Xi^{\text{exp}} = -0{\cdot}030 \pm 0{\cdot}011. \tag{16}$$

Case 1

$$\theta = 1 \times 10^{-2}.$$

By substituting this value of θ in (15) one has

$$R_\Xi = 0{\cdot}11 + z'.$$

As with the Λ hyperon decay, here too, the $\eta - \pi$ mixing effects are present with an opposite sign from the experimental value. The value of z' now is $z' = -0{\cdot}41$ which is larger than what was obtained by neglecting the $\eta - \pi$ mixing: $z' = -0{\cdot}030$.

Case 2

$$\theta = 4 \times 10^{-2}.$$

This value leads to

$$R_{\Xi} = 0{\cdot}035 + z',$$

indicating that the bonafide $\Delta I = 3/2$ contribution needs to be $z' = -0{\cdot}065$.

Case 3

$$\theta = -6 \times 10^{-3}.$$

In this case the contribution of the $\eta - \pi$ mixing effects has a sign similar to the value of R_{Ξ}^{exp}:

$$R_{\Xi} = -0{\cdot}003 + z'.$$

Moreover, the magnitude of the $\eta - \pi$ mixing effects is about an order of magnitude smaller than R_{Ξ}^{exp} which is in keeping with the usual expectations. z' in this case is given by $z' = -0{\cdot}033$.

2.5 *Σ — hyperon decays*

Here R_{Σ} is given by

$$R_{\Sigma} = \frac{A(\Sigma^{+} \to n\pi^{+}) - A(\Sigma^{-} \to n\pi^{-}) + \sqrt{2}\, A(\Sigma^{+} \to p\pi^{\circ})}{A(\Sigma^{-} \to n\pi^{-})}$$

$$\approx -\sqrt{3} \sin\theta + z'', \tag{17}$$

where

$$z'' = -\frac{3\sqrt{3}}{4\sqrt{2}} \frac{\sigma_{3}''}{D - F} \text{ and } R_{\Sigma}^{\text{exp}} = 0{\cdot}12 \pm 0{\cdot}05. \tag{18}$$

Case 1

$$\theta = 1 \times 10^{-2}.$$

One obtains here (Holstein 1979)

$$R_{\Sigma} = -0{\cdot}018 + z'',$$

which, like R_{Λ} and R_{Ξ}, differs in sign with R_{Σ}^{exp}. Here z'' needs to be $z'' \approx 0{\cdot}14$ in comparison with the naive value $z'' \approx 0{\cdot}12$.

Case 2

$$\theta = 4 \times 10^{-2}.$$

R_{Σ} turns out to be

$$R_{\Sigma} = -0{\cdot}072 + z'',$$

which implies that $z'' \approx 0{\cdot}19$, slightly larger than what has been obtained in case 1.

Case 3

$$\theta = -6 \times 10^{-3}.$$

Here one gets

$$R_{\Sigma} = +0{\cdot}012 + z'',$$

which shows that the magnitude (but differing in sign) of the $\eta - \pi$ mixing effects is exactly equal to the value of R_{Σ}^{exp} thus ruling out the possibility of any bonafide $\Delta I = 3/2$ amplitude contribution.

3. Summary

To summarize, we have studied in this paper the effects of the $\eta - \pi$ mixing on the violation of the $\Delta I = 1/2$ rule in $\Delta S = 1$ weak transitions. For the $\eta - \pi$ mixing angle θ we have considered (i) the value $\theta = 4 \times 10^{-2}$ obtained by us in earlier papers and which lies within the limits dictated by the experimental value of $\Gamma(\Psi' \to \Psi\pi^{0}) / \Gamma(\Psi' \to \Psi\eta)$ ratio, (ii) the value $\theta = -6 \times 10^{-3}$ obtained by Oneda *et al* that differs in sign from the standard values and (iii) the canonical value $\theta = 1 \times 10^{-2}$ obtained by Gross *et al.* For the weak processes we have considered the $K \to 2\pi$, the $K \to 3\pi$, Λ, Ξ and Σ hyperon decays. We have found that for the values $\theta = 4 \times 10^{-2}$ and $\theta = 1 \times 10^{-2}$, the $\eta - \pi$ mixing effect on the violation of the $\Delta I = 1/2$ rule is non-negligibly large although the $K \to 3\pi$ decays do seem to rule out the value $\theta = 4 \times 10^{-2}$. It may be noted that these decays favour the smaller value of θ *viz.* $\theta = 1 \times 10^{-2}$. For the negative value of $\theta\,(= -6 \times 10^{-3})$ we have found that for the Λ and Ξ hyperon decays such a value is favoured since in these processes the intrinsic $\Delta I = 3/2$ amplitude contributions are not much different from what one expects naively. However, the $K \to 3\pi$ decays disfavour the value $\theta = -6 \times 10^{-3}$. For Σ hyperon decay, the presence of the $\eta - \pi$ mixing term rules out the possibility of any bonafide $\Delta I = 3/2$ amplitude contribution if the negative value of θ is taken as an input.

3.1 *Note*

After this work was submitted for publication, two papers have appeared (Gusbin 1981; Holstein 1981) which have also dealt with the effects of isoscalar-isovector pseudoscalar mixing in non-leptonic weak decays. In one of these papers, Gusbin (1981) has shown that the η' mixing effects are negligible in $K^+ \to \pi^+\pi^0$ decay and do not affect the $\Delta I = 3/2$ transition strength. In the other, Holstein (1981) has argued that the presence of $\Delta I = 3/2$ term in the $K \to \pi\eta$ amplitude seems unlikely.

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