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# Heavy quark systems and hadron mass spectra

G Q SOFI and TEJ K ZADOO
Department of Physics, S P College, Srinagar 190 001, India

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**Abstract.** The influence of chromomagnetic hyperfine interaction on $B$ and $T$ hadrons is studied. The colour magnetic moments are related to the hyperfine splittings of hadrons.

**Keywords.** Heavy quarks; colour magnetic moment; spectroscopy; heavy hadrons.

## 1. Introduction

For the last twenty years the number of hadron states has been increasing and most of them have been successfully classified using various dimensions of special unitary group *viz.* SU(2), SU(3) and SU(4), etc. Four quarks $u$, $d$, $s$ and $c$ are postulated to inhabit the basic representation of SU(4) (Ahmad and Zadoo 1977; Gaillard *et al* 1975; Moffat 1975; Einhorn 1975). The discovery of a new class of resonance state $\Upsilon$(9.44) (Herb *et al* 1977; Innes *et al* 1977) necessitated the introduction of a new quark $b$ carrying a new flavour bottomness (Ellis *et al* 1977; Hagiwara *et al* 1978).

There have been many theories (Fritzsch *et al* 1975; Gourdin 1976) where a new quark $t$ has also been introduced carrying a flavour topness, thus increasing the number of quarks to six. With these heavier quarks, heavier mesons and baryons are expected. Several authors (Boal 1978; Aubrecht and Scott 1979; Singh *et al* 1980; Martin 1979; Zadoo 1981; Sofi 1981a, b) have studied some of the properties of these heavy hadrons. The purpose of this paper is to consider the hyperfine mass splitting of these heavy hadrons under the influence of chromomagnetic hyperfine interaction and quantum chromodynamics (QCD).

## 2. Mass formula and spectroscopy

The interaction energy of two magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ in a nonrelativistic approximation is given by (Fritzsch 1976)

$$H^{\text{mag}} = -8\pi/3\, \vec{\mu}_1 \cdot \vec{\mu}_2\, \delta(\mathbf{r}) + 1/r^3\, [(\vec{\mu}_1 \cdot \vec{\mu}_2 - 3\,(\mathbf{r}.\,\vec{\mu}_1)\,(\mathbf{r}.\,\vec{\mu}_2)], \tag{1}$$

where $\mathbf{r}$: relative coordinate; $|\mu| = e/2m$ for Dirac fermions. The first term of this is relation is the Fermi hyperfine interaction while the second term is the dipole-dipole interaction (which is zero for $s$ wave states).

This interaction can be applied to study the chromomagnetic forces between the quarks if one makes the following substitution:

$$a = -\frac{e^2}{4\pi} = -\frac{4}{3}k \text{ (in case of mesons)}$$

$$a = -\frac{2}{3}k \text{ (in case of baryons).} \tag{2}$$

Adopting the QCD inspired picture that the spin-spin forces of the constituents are proportional to their colour magnetic moments (Fritzsch 1976; Ahmad and Sofi 1979) and the electromagnetic mass difference of baryons is also caused by the magnetic hyperfine interaction (Minamikawa *et al* 1966) the mass splitting of hadrons can be studied using the above relations. One observes from the above that the mass splitting between pseudoscalar states and vector states of bottomed ($B$) and topped ($T$) hadrons decreases rapidly with the increasing mass. Following (1) and assuming that only the chromomagnetic Fermi interaction contributes we write for the states composed of the same set of quarks

$$M = M_0 + \xi \left\langle \sum_{i>j} \lambda_i \lambda_j \mathbf{S}_i \cdot \mathbf{S}_j \right\rangle, \tag{3}$$

where $\xi$ is a positive parameter, depending on the space parts of the wave functions, and $\lambda_i$ is the chromomagnetic moment of the quark which is assumed to be proportional to the electromagnetic moment divided by the quark charge (Fritzsch and Minkowski 1980). Further the isospin symmetry requires $\lambda_u = \lambda_d$.

Applying (1) and (3) for baryons $\Sigma_b^{*0}$, $\Sigma_b$ and $\Lambda_b$ we have

$$M = M_0 + \xi \langle \lambda_u^2 \mathbf{s}_u \cdot \mathbf{s}_d + \lambda_u \lambda_b (\mathbf{s}_u \cdot \mathbf{s}_b + \mathbf{s}_d \cdot \mathbf{s}_b) \rangle. \tag{4}$$

The wave function for the $\Sigma_b^0$ and $\Sigma_b^{*0}$ state contains the $(u, d)$ quark system in an $I = 1$ state. Consequently the spins are aligned, and we have $\mathbf{s}_u \cdot \mathbf{s}_d = +\frac{1}{4}$.

Then we find that $\mathbf{s}_b . (\mathbf{s}_u + \mathbf{s}_d) = -1$ ($\Sigma_b^0$) and

$$\mathbf{s}_b . (\mathbf{s}_u + \mathbf{s}_d) = +\tfrac{1}{2} (\Sigma_b^{*0}).$$

We get then

$$M_{\Sigma_b^0} = M_0 + \xi [\lambda_u^2 \tfrac{1}{4} + \lambda_u \lambda_b \cdot (-1)]$$

$$= M_0 + \xi \lambda_u^2 (\tfrac{1}{4} - \rho) \tag{5}$$

$$M_{\Sigma_b^{*0}} = M_0 + \xi [ \lambda_u^2 (\tfrac{1}{4} + \lambda_u \lambda_b \cdot \tfrac{1}{2})$$

$$= M_0 + \xi \lambda_u^2 [\tfrac{1}{4} + \tfrac{1}{2} \rho], \tag{6}$$

where we have introduced the ratio $\rho = \lambda_b/\lambda_u$, the ratio of the colour magnetic moments of $b$ and $u$ quarks. In the $\Lambda_b$ state the $u$, $d$ system has $I = 0$; consequently the spins are antiparallel, then $\mathbf{s}_u \cdot \mathbf{s}_d = -\frac{3}{4}$ and $\mathbf{s}_b \cdot (\mathbf{s}_u + \mathbf{s}_d) = 0$.

Hence

$$M_{\Lambda_b} = M_0 + \lambda_u^2 \, \xi \, (-\tfrac{3}{4}). \tag{7}$$

From (5), (6) and (7), we have

$$\frac{\lambda_u}{\lambda_b} = \frac{1}{\rho} = 1 + \frac{3}{2} \, \frac{M_{\Sigma_b^0} - M_{\Lambda_b}}{M_{\Sigma_b^{*0}} - M_{\Sigma_b}}. \tag{8}$$

Similarly for the charm sector we can write

$$\frac{\lambda_u}{\lambda_c} = \frac{1}{\rho} = 1 + \frac{3}{2} \, \frac{M_{\Sigma_c} - M_{\Lambda_c}}{M_{\Sigma_c^*} - M_{\Sigma_c}}. \tag{9}$$

Using (8) and (9) we write

$$\frac{M_{\Sigma_b^0} - M_{\Lambda_b}}{M_{\Sigma_c} - M_{\Lambda_c}} = \frac{\lambda_u^2 \left[\frac{1}{4} - \frac{\lambda_b}{\lambda_u}\right] - \lambda_u^2 (-\frac{3}{4})}{\lambda_u^2 \left[\frac{1}{4} - \frac{\lambda_c}{\lambda_u}\right] - \lambda_u^2 (-\frac{3}{4})} = \frac{1 - \frac{\lambda_b}{\lambda_u}}{1 - \frac{\lambda_c}{\lambda_u}}. \tag{10}$$

If the chromomagnetic moment ratios be inversely proportional to the quark masses, we will have

$$\lambda_t/\lambda_u = m_u/m_t \; ; \quad \lambda_b/\lambda_u = m_u/m_b \; ; \quad \lambda_c/\lambda_u = m_u/m_c.$$

Taking the effective mass of $u$ quark as 330 MeV and that of $c$ quark as 1500 MeV (Litchenberg 1975; Zadoo 1979; McGregor 1980), and the assumption that $m_b \simeq 3\, m_c$; $m_t \simeq 4\, m_b$, we have $\lambda_c/\lambda_u = 0{\cdot}22$; $\lambda_b/\lambda_u = 0{\cdot}07$, $\lambda_t/\lambda_u = 0{\cdot}02$ (also see Sofi 1981b). Using these values and the experimental values $M_{\Sigma_c} = 2430$ MeV; $M_{\Lambda_c} = 2273$ MeV (Review of particle properties 1980) we have from (10)

$$M_{\Sigma_b^0} - M_{\Lambda_b} = 187{\cdot}19 \text{ MeV}.$$

Using a similar calculation for top-sector we have

$$M_{\Sigma_t} - M_{\Lambda_t} = 197{\cdot}25 \text{ MeV}$$

In the case of mesons the analogue of (3) is

$$M(Q\bar{q}) = M_0(Q\bar{q}) + \eta \, \lambda_Q \, \lambda_{\bar{q}} \, \langle \mathbf{S}_{\bar{q}} \cdot \mathbf{S}_Q \rangle, \tag{11}$$

where $\eta$ is a positive parameter. Here

$$\mathbf{S}_{\bar{q}} \cdot \mathbf{S}_Q = \tfrac{1}{2}\,[\,(\mathbf{S}_{\bar{q}} + \mathbf{S}_Q)^2 - (\mathbf{S}_{\bar{q}})^2 - (\mathbf{S}_Q)^2]$$

$$= \tfrac{1}{4} \text{ for } s\text{-wave vector states and}$$

$$= -\tfrac{3}{4} \text{ for } s\text{-wave pseudoscalar states.}$$

From (11) we get the relation

$$\frac{M_\rho - M_\pi}{M_{B^*} - M_B} = \frac{\lambda_u}{\lambda_b} = \frac{M_{D^*} - M_D}{M_{B_c^*} - M_{B_c}}$$

since $\dfrac{\lambda_b}{\lambda_u} = 0{\cdot}07$,

$$B^* - B = 44{\cdot}8 \text{ MeV; and}$$

$$B_c^* - B_c = 9{\cdot}8 \text{ MeV}$$

Similarly we get

$$B_s^* - B_s = 35 \text{ MeV.}$$

These results agree with the predicted results of Sofi(1981). Applying the same technique to $T$-mesons, one gets the hyperfine mass splitting as

$$T_u^* - T_u = 10{\cdot}85 \text{ MeV}$$

$$T_s^* - T_s = 6{\cdot}6 \text{ MeV}$$

$$T_c^* - T_c = 2{\cdot}31 \text{ MeV}$$

$$T_b^* - T_b = 0{\cdot}78 \text{ MeV}$$

We can also get nearly same results if we use the relation

$$\frac{M(T_q^*) - M(T_q)}{M(D^*) - M(D)} \simeq \frac{m_c}{m_t} \otimes \frac{m_u}{m_q}$$

## 3. Conclusion

We have studied the hyperfine splitting of bottom and top hadrons using the idea of chromomagnetic forces among quarks. According to Ono (1978) who has also calculated the mass splitting, the mass differences of baryons are caused by: (i) The mass difference between $u$ and $d$ quarks. (ii) The Coulomb forces among quarks. (iii) The magnetic hyperfine interaction.

On the other hand we assume that the splitting is because of the interaction energy of the two magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ in a nonrelativistic approximation. Our results agree well with his predicted results for bottom hadrons.

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