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Generalized interacting boson model and the collective behaviour in nuclei

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Abstract. The effect of including the high spin bosons on the manifestation of collective behaviour in nuclei is examined by plotting the $B(E2; 2^+ \rightarrow 0^+)$ rates as a function of neutron number for various values of η, where η is the highest angular momentum of the bosons included in the calculation. $B(E2; 2^+ \rightarrow 0^+)$ values of a large number of nuclei in various regions of the nuclear periodic table are calculated with a single value for the effective charge in the generalized scheme. Irreducible representations of SU(3) contained in the symmetric partition [N] of U(15) are worked out for integers N upto $N = 15$, to enable the explicit inclusion of the g boson into calculations. The experimentally observed odd-K bands in ^{234}U and ^{184}W are described as a direct consequence of the g boson.

Keywords. Generalized interacting boson model; irreducible representations; symmetric partitions; g boson.

1. Introduction

The interacting boson model (IBM) of Arima and Iachello (1976, 1978) describes the collective properties of nuclei by considering pairs of protons and neutrons coupled to $S = 0$ and $J = L = 0$, and 2. These pairs are called the s and the d bosons respectively. But basing on the existing theoretical evidence (Hecht *et al* 1972 and McGrory 1978) which points to the important role of pairs of nucleons coupled to $J = 4, 6, 8$ etc. for the description of excited states, Ratna Raju (1979, 1981a) has generalized IBM by allowing the individual bosons to occupy $J = L = 4, 6, 8 \ldots J_{\max}$ states in addition to the $J = 0$ and 2 states. Then a generalized boson degeneracy g was defined as

$$g = (\eta+1)\ (\eta+2)/2, \tag{1}$$

where $\eta = J_{\max}$ The chain of groups $U(g) \supset SU(3) \supset R(3)$ will now provide a complete set of quantum numbers to describe the rotational states. This approach is hereafter called as the generalized interacting boson model (GIBM). The $U(6)$ limit of Arima and Iachello is obtained by setting $\eta = 2$. This generalization has several important consequences. It predicts the existence of odd-K bands with $K = 1$ and 3 near the first β and γ bands. Through the deduction of simple and general relations between the strength parameters of IBM and GIBM, the analytical nature of the eigen value problem is preserved intact making it easy to calculate the energies and transition rates in GIBM as in IBM. Above all, the generalized picture removes the cut-off effects in the $B(E2)$ rates involving high spin states (Ratna Raju 1979, 1981) and explains clearly why in some nuclei like Rb isotopes the $B(E2)$ values fall sharply at high spins and why not in other nuclei like ^{156}Gd.

The purpose of the present work is two-fold. We first investigate the effect of including the high spin bosons on the manifestation of collective behaviour in nuclei by studying how fast the $B(E2; 2^+ \rightarrow 0^+)$ values increase as the neutron number grows from one closed shell to another. We then establish the efficiency of the GIBM by calculating the E2 transition rates of a number of deformed nuclei in different mass regions making use of a single value for the effective charge and also by studying the structure of two specific nuclei *viz.* ^{234}U and ^{184}W.

2. Theoretical details

Before we make any calculation in GIBM, we should know which irreducible representations of SU(3) are contained in a given partition of U(g). There exists an algebraic solution to this problem in the U(6) limit (Arima and Iachello 1978). But in the general case where η can take any large even value, no solution exists in closed form in literature except for $N = 2$ (Littlewood 1940; Wybourne 1970). When $N = 2$, the partition [2] of U(g), where g is defined as in (1), will contain all representations in the expansion

$$[2] = \{2\eta\} + \{2\eta - 2, 2\} + \{2\eta - 4, 4\} + \{2\eta - 6, 6\} + \{2\eta - 8, 8\} + \dots, \tag{2}$$

which satisfy the condition that $f_1 \geqslant f_2$, f_1 and f_2 being the number of boxes in row one and row two respectively. The SU(3) representations $(\lambda\mu)$ are then obtained as $\lambda = f_1 - f_2$ and $\mu = f_2$. But to obtain such a general solution for any N is prohibitively difficult. Therefore we restrict ourselves in the present paper to the case of $\eta = 4$ which gives rise to the U(15) group. The problem of reducing any U(15) partition into its SU(3) irreducible content was solved in part already (Kota and Ratna Raju 1975). Adopting the same technique, we reduced the partitions $[N]$ for up to $N = 15$. The results are tabulated.*

The vast amount of experimental data of $B(E2)$ values of nuclei in various regions of the nuclear periodic table (Nathan and Nilsson 1965) shows two systematic trends:

(i) the $B(E2; 2^+ \rightarrow 0^+)$ rates of nuclei in a given major shell increase fast as we approach the middle of the shell and

(ii) the absolute values of these rates show a fast rising trend as we go from a lower major shell to a higher major shell.

To be more specific about the second point, in the framework of IBM, ^{154}Sm and ^{230}U both correspond to $N = 11$ active bosons. For ^{154}Sm, $B(E2; 2^+ \rightarrow 0^+) = 4{\cdot}2\ e^2b^2$ where as for ^{230}U, it is $8{\cdot}9\ e^2b^2$. Therefore, in IBM one has to double the value of α_2^2 used for ^{154}Sm to get the right value for ^{230}U. It would be ideal if a single value of

*Can be had from the author on request.

a_2 can be used at least for all such nuclei which correspond to the same active boson number, if not for all boson numbers.

It can be shown trivially that for transitions within the $K = 0$ ground state band

$$B(E2; L+2 \rightarrow L) = a_2^2 \tfrac{3}{4} \frac{(L+2)(L+1)(\eta N - L)(\eta N + L + 3)}{(2L+3)(2L+5)}, \quad (3)$$

where a_2 is the effective charge of the bosons. We do not distinguish the proton bosons from neutron bosons as long as the valence protons and neutrons are both hole-like or both particle-like (Arima and Iachello 1976). The implications of the above formula were recently examined (Ratna Raju 1981b) to explain the behaviour of $B(E2)$ values at high spins.

Chakraborty *et al* (1981) computed the intrinsic quadrupole moments in the SU(3) limit of IBA and compared them to the variational calculations. Their results show the necessity to include high-spin bosons ($\eta > 2$) in order to describe the heavy deformed nuclei. We plot in figure 1 the $B(E2; 2^+ \rightarrow 0^+)$ rates generated by the above equation in the units of a_2^2 as a function of η and N, in the ground state bands of nuclei with three active proton bosons for $\eta = 2$, 4, and 6 in exactly the same way as Arima and Iachello (1978) did for the case of $\eta = 2$.

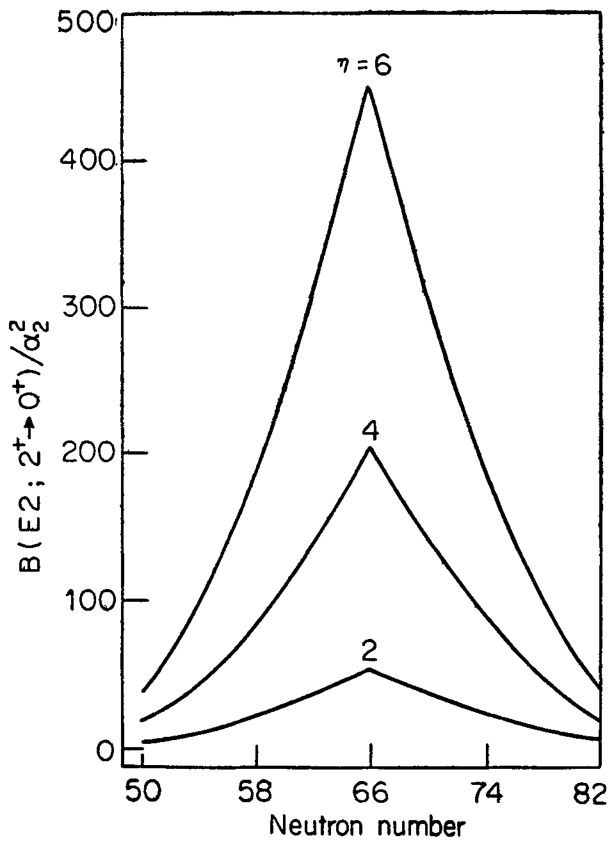


Figure 1. Schematic behaviour of equation (3).

The highest angular momentum that can be associated with a spin zero pair of identical particles will depend upon the major shell which the valence particles are populating. The higher the major shell number, higher will be the possible values of η. Bearing this in mind we infer two things from this diagram:

(i) the steep rise in the curves for higher values of η is an indication that as a result of including high spin bosons collective behaviour sets in faster in such nuclei for which η can be large, as the neutron number increases towards the middle of a major shell and

(ii) for the same number of valence particles, GIBM predicts naturally larger $B(E2)$ values for nuclei with valence particles in the higher shells and smaller $B(E2)$ values for nuclei with valence particles in the lower shells.

Both these observations are consistent with the experimental situation (see figure 14 of Nathan and Nilsson 1965). It now looks possible that the $B(E2)$ values of all even-even rotational nuclei can be described using a single value for the effective charge but including successively higher spin bosons as we go from medium mass nuclei to heavy mass regions. In table 1 we give results of such a calculation with $\alpha_2 = 2/15\ eb$ for nuclei with various active boson numbers. For all these nuclei the valence protons and neutrons are both hole-like or both particle-like. In the medium mass region we include only the s and d bosons. For heavy nuclei with $150 \leqslant A \leqslant 190$, the g boson and for nuclei with $A \sim 230$ the i boson are also included. We would like to mention that by redefining the effective charge α_2 one can choose another sequence of increasing values of η to obtain the same $B(E2)$ values in various regions of the nuclear periodic table with a single α_2. We have only suggested here one such sequence which seemed reasonable to us since we based our arguments on the degree of collectivity in that region. In fixing α_2, we did not look for an exact fit to

Table 1. $B(E2; 2^+ \rightarrow 0^+)$ values of some collective nuclei with $\alpha_2 = 2/15\ eb$

η	Nucleus	Number of Bosons N	$B(E2; 2^+ \rightarrow 0^+)$ Values Theoretical	Experimental*
2	^{120}Xe	10	0·817	0·92
4	^{188}Pt	10	3·057	3·05
	^{154}Gd	11	3·674	3·68
	^{186}W	11	3·674	3·50
	^{156}Gd	12	4·352	4·64
	^{184}Pt	12	4·352	4·1
	^{158}Gd	13	5·084	4·95
	^{158}Dy	13	5·084	4·8
	^{160}Gd	14	5·877	5·67
6	^{226}Ra	10	6·72	6·78
	^{228}Th	10	6·72	7·05
	^{230}Th	11	8·096	8·00
	^{230}U	11	8·096	8·9
	^{232}Th	12	9·6	9·49
	^{232}U	12	9·6	9·9
	^{234}U	13	11·23	10·4
	^{236}U	14	12·992	12·0

*Experimental values are taken from Ross and Bhaduri (1972) and Andrejtscheff *et al* (1975) any particular nucleus. The nuclei shown in table 1 are those for which the experimental and theoretical values agree well. For example for ^{154}Sm the theoretical value with $a_2 = 2/15\ eb$, is 3·674 e^2b^2 whereas the experimental value is 4·2 e^2b^2. This can be obtained with $a_2 = 0{\cdot}1425$ which is different from the value we have used by 0·0092. This case is not shown in the table. By a very slight and smooth variation of the above value used by us one can obtain accurate results for all the left out cases.

To generate the energy spectrum we have used the model Hamiltonian introduced in the U (6) ⊃ SU (3) ⊃ R (3) limit of the IBM (Arima and Iachello 1978) *viz.*

$$H = -K \sum_{i,j} \mathbf{Q}_i \cdot \mathbf{Q}_j - K' \sum_{i,j} \mathbf{L}_i \cdot \mathbf{L}_j, \tag{4}$$

the eigen values of which can be written as

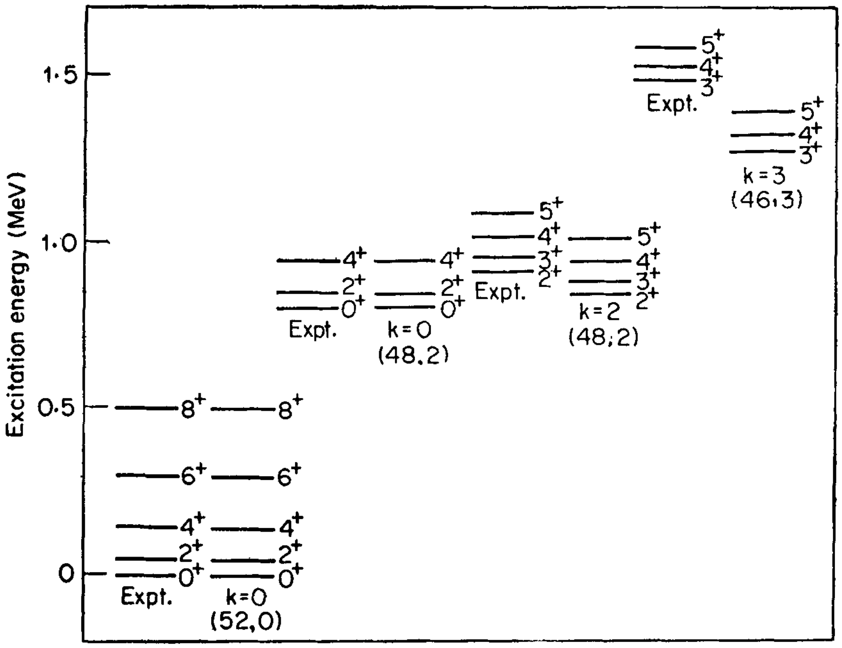
$$E\left((\lambda\,\mu)\,L\right) = a\,L\,(L+1) - \beta\,C\,(\lambda\,\mu). \tag{5}$$

With the above Hamiltonian, like in IBM the β and γ bands coming from ($\eta N-4$, 2) representation are degenerate now also. The $K=1$ and 3 bands coming out of ($\eta N-6$, 3) lie just above the first β and γ bands. They are labelled as the V and the W bands (Ratna Raju 1981). The $K=3$ band has been identified experimentally in many nuclei like ^{172}Yb, ^{184}W and ^{234}U.

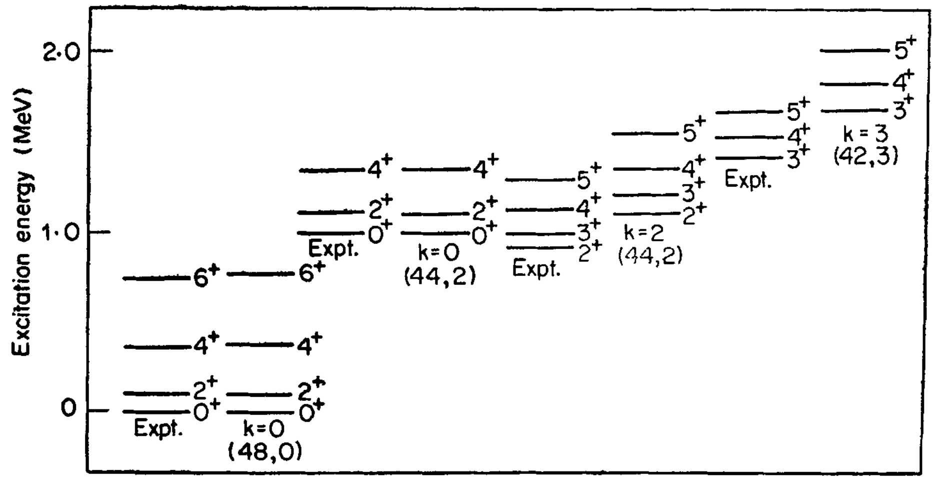
Since the first β and γ bands are almost degenerate in ^{234}U, it is an ideal case to be studied in both IBM and GIBM. In both the proton and neutron spaces the valence particles have particle-like states and they correspond to 13 active bosons. ^{184}W is not such an ideal choice since the 2^+ level of the $K=2$ (γ) band lies about 99 keV below the 2^+ level of the $K=0$ (β) band. The spectra of these two nuclei are generated including the g boson explicitly into the calculation with the following values for the strength parameters:

for ^{234}U $a = 6{\cdot}90$ keV and $\beta = 2{\cdot}65$ keV
for ^{184}W $a = 18{\cdot}50$ keV and $\beta = 3{\cdot}55$ keV.

The resulting spectra are shown in figures 2 and 3. In both the cases, we did not show in the figures the second β and γ bands coming from ($\eta N-8$, 4) although they were observed experimentally. With such a simple Hamiltonian as is used here, it is impossible to get correctly all the positions of various bands. For the same reason Arima and Iachello (1978) also did not show these bands in ^{234}U which they studied in the U (6) ⊃ SU (3) ⊃ R (3) limit. The second β band coming from (18, 4) would be 445 keV above the experimental band if one generates it with the strengths used by Arima and Iachello in the U (6) limit. The same thing happens in the GIBM also. But we should mention here that in both IBM and GIBM, the level spacing within the second β and γ bands are in fairly good agreement with the experimental values. Similarly in the case of ^{184}W, the theoretical second β band starts about 572 keV above the experimental band in the U (15) limit. The experimental values are taken from Lederer *et al* (1978). With the strength factors used here, the theoretical 3^+ band occurs at 218 keV below the experimental band for ^{234}U and at 268 keV above the experimental one for ^{184}W. However, we should like to make an important comment here.



(2)



(3)

Figure 2. Spectrum of ^{234}U.
Figure 3. Spectrum of ^{184}W.

We have used $\eta = 6$ (i-boson) in computing the $B(E2;\ 2^+ \to 0^+)$ values of ^{234}U but the spectrum is generated only with $\eta = 4$ (g-boson). Inclusion of i boson does not change level spacings or band positions but only increases the multiplicity with which a particular band occurs.

3. Concluding remarks

In conclusion we emphasize that the GIBM has exactly the same simple features of IBM. In addition, it predicts the existence of bands with $K = 1$ and 3 which are named as the V and the W bands respectively. There is experimental evidence for the occurrcnce of the V band also. In a recent paper Goldfarb (1981) has wrongly concluded that including high spin bosons will not give rise to bands with $K \rangle 2$. In ^{158}Gd the ($K = 1$) V band starts at 2·494 MeV and in ^{162}Dy it starts at 1·7455 MeV (Lederer *et al* 1978). The model also provides an explanation for the systematic rise in the $B(E2)$ values of even-even nuclei as we go from the low mass region to heavy mass regions of the nuclear periodic table. The largest angular momentum state that can occur in the yrast bands is pushed up to ηN. We have also demonstrated that in GIBM it is possible to describe the $B(E2;\ 2^+ \rightarrow 0^+)$ values of all rotational nuclei with a single value for the effective charge *viz.* $\alpha_2 = 2/15\ eb$ or with a very tiny variation in it.

By suitably modifying the computer code of Kota (1978) one can generate the SU(3) representations contained in any partition $[N]$ of the group $U((\eta + 1)(\eta + 2)/2)$ for arbitrarily large values of η. The only limitation is on the capability of the computer.

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