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Microscopic study of odd-A nuclei

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Abstract. The odd-proton nucleus ^{155}Tb and odd-neutron nucleus ^{155}Dy are studied along with doubly-even nucleus ^{156}Dy using microscopic method of variation after projection of angular momentum and conservation of nucleon number in each projected state. The calculated energies of the ground band in ^{156}Dy and the ground and excited bands in ^{155}Dy and ^{155}Tb are in good agreement with the corresponding experimental data. The role played by the $i_{13/2}$ neutron pair in these nuclei is discussed.

Keywords. Nuclear structure; odd-A rare-earth nuclei; microscopic variational calculations; nucleon number conservation in each projected state; energy spectra of ground and excited bands.

1. Introduction

The variational method with angular momentum projection and number conservation has been used successfully (Warke and Gunye 1976; Gunye and Warke 1979; Gunye and Ashok Kumar 1980a) to study the yrast states of a few even-even nuclei in the rare-earth and transition region. It is desirable to extend this microscopic approach to odd-A nuclei, in order to investigate their intrinsic structure and consequently their energy spectra. Such microscopic studies are quite essential to test the validity of the phenomenological models (Mottelson and Valatin 1960; Stephens and Simon 1972) proposed to explain the characteristic features of the yrast states. Moreover, it is essential to explain consistently the rotational bands observed in the energy spectra of many odd-A nuclei. The rotational bands in odd-A nuclei are explained qualitatively by resorting to rotor plus particle model (Stephens 1975; Faessler *et al* 1977) and also by the Hartree-Fock-Bogoliubov (HFB) calculations with cranking constraint for conserving angular momentum (Ring *et al* 1974). However the simplifying approximations regarding angular momentum conservation in the cranked HFB calculations lead to errors which could be of the same order of magnitude as the observed energy differences between the excited states to be explained. It is thus worthwhile to study the rotational bands in odd-A nuclei in the framework of a microscopic theory with exact angular momentum projection.

In this paper, we report the results of our investigations in the odd-A nuclei ^{155}Tb and ^{155}Dy along with their doubly-even neighbour ^{156}Dy, using the microscopic formulation of variation after angular momentum projection (VAP) with nucleon number conservation. Our method differs from that of rotor plus particle model, since we determine the intrinsic state of the doubly-even nucleus ^{156}Dy for each angular momentum state by determining the variational parameters (Gunye and Warke 1979) by employing the VAP method. The intrinsic state of the odd-A nucleus is then obtained by annihilating a particle (proton or neutron) near the Fermi surface of the even $(A+1)$ nucleus. The choice of the particle to be destroyed is suggested by the band quantum number K and the parity π of the band under investigation. We have chosen the doubly-even $(A+1)$ nucleus ^{156}Dy and its neighbouring odd-A nuclei ^{155}Dy and ^{155}Tb in order to study the role played by the neutron pair in the $i_{13/2}$ orbit near the Fermi surface.

The theoretical formulation is outlined in § 2, the results are presented and discussed in § 3 and conclusions are given in § 4.

2. Theoretical formulation

An accurate calculation of the excited states of heavier nuclei from the microscopic many-body theory would require a large number of nucleons to be treated dynamically in a large configuration space. The computational difficulties involved in performing such projected Hartree-Fock-Bogoliubov (HFB) calculations can be somewhat reduced by employing a simpler many-body Hamiltonian. In this paper, we use the quadrupole plus pairing interaction Hamiltonian, whose parameters are determined by Kumar and Baranger (1968) from their study of equilibrium deformations of heavy nuclei.

$$H = \sum \epsilon_\alpha a_\alpha^\dagger a_\alpha - \tfrac{1}{2} \chi \sum q^\mu_{\alpha\gamma} q^\mu_{\delta\beta} a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma - \tfrac{1}{2} G \sum (-1)^{j_\alpha - m_\alpha + j_\gamma - m_\gamma} a_\alpha^\dagger a_{\bar\alpha}^\dagger a_{\bar\gamma} a_\gamma, \tag{1}$$

where q^μ is the quadrupole operator and χ and G are the strengths of the quadrupole and pairing interactions respectively. The subscript α in (1) denotes all the quantum numbers $(n_\alpha, l_\alpha, j_\alpha, m_\alpha)$ necessary for the specification of a spherical single particle state. The state $\bar\alpha$ is connected to the state α by a time reversal operator. The sums in (1) run over the entire configuration space ($N = 4, 5$ proton shells and $N = 5, 6$ neutron shells).

We consider the intrinsic variational wavefunction to be axially symmetric in view of the fact that the even-even nuclei in the rare-earth region are found (Das Gupta and Preston 1963; Gunye *et al* 1964; Kumar and Baranger 1968) to prefer axially symmetric equilibrium deformation. The intrinsic wavefunction Φ_0 of the even-even nucleus with $(A+1)$ nucleons is given by

$$\Phi_0 = \Pi\, u_i \exp \Sigma\, (v_i/u_i)\, b_{i+}^\dagger b_{i-}^\dagger \,|\, 0 \rangle, \tag{2}$$

where b_i^+ is the fermion operator corresponding to the deformed single particle state i with occupation probability v_i [$u_i = (1 - v_i^2)^{1/2}$]. The fermion operator $b_i^\dagger$ is obtained from the spherical state operators a_α^+ by the transformation

$$b_{i\pm}^\dagger = \sum_\alpha \langle \alpha \pm | C | i \pm \rangle a_{\alpha\pm}^\dagger. \tag{3}$$

It should be noted that the basis states are divided into two subsets ($\alpha \pm$) connected by the time-reversal operator. The transformation matrix in (3) satisfies the relation

$$C\tilde{C} = \tilde{C}C = 1,$$

where $\tilde{C}$ is the transpose of the matrix C. The wavefunction for the odd-A nucleus can be obtained from the intrinsic wavefunction Φ_0 by annihilating a particle in the deformed $K \pm$ single-particle state in Φ_0. Thus

$$\Phi_K = b_{K-}\, \Phi_0$$

$$\Phi_{-K} = b_{K+}\, \Phi_0. \tag{4}$$

The projected wavefunction in the JM state of the odd-A nucleus is then given by

$$\psi^J_M = N^J (P^J_{M,K}\Phi_K + (-1)^{J-K}\, P^J_{M,-K}\Phi_{-K}), \tag{5}$$

where P^J_{MK} is the angular momentum projection operator. The normalization condition gives

$$N^J = \{2\,[\langle \Phi_K | P^J_{K,K} | \Phi_K \rangle + (-1)^{J-K} \langle \Phi_K | P^J_{K,-K} | \Phi_{-K} \rangle]\}^{1/2}. \tag{6}$$

The expectation value of the Hamiltonian of (1) in the state ψ^J_M of the odd-A nucleus is given by

$$\langle \psi^J_M | H | \psi^J_M \rangle = 2\,(N^J)^2\,[\langle \Phi_K | HP^J_{K,K} | \Phi_K \rangle + (-1)^{J-K} \langle \Phi_K | HP^J_{K,-K} | \Phi_{-K} \rangle]. \tag{7}$$

To compute N^J and $\langle \psi^J_M | H | \psi^J_M \rangle$ as given in (6) and (7) one has to evaluate the matrix elements $\langle \Phi_K | R | \Phi_{\pm K} \rangle$ and $\langle \Phi_K | HR | \Phi_{\pm K} \rangle$ where R is the rotation operator. Neglecting the interaction of the single odd-particle in the K state with the rest of the nucleons, one obtains

$$\langle \Phi_K | H\,R | \Phi_{\pm K} \rangle = \langle H\,R \rangle \langle \Phi_0 | R | \Phi_0 \rangle\, Y_{K\mp}, \tag{8}$$

$$\langle \Phi_K | R | \Phi_{\pm K} \rangle = \langle \Phi_0 | R | \Phi_0 \rangle\, Y_{K\mp}, \tag{9}$$

$$Y_{K\mp} = \sum_{K'} \langle K\mp | \tilde{C}\, \tilde{D}^*\, C | K' \rangle\, \rho_{K'\,K-}, \tag{10}$$

where C is the transformation matrix in (3) and D is the rotation matrix. The expressions for $\langle HR \rangle$ and $\langle \phi_0 | R | \phi_0 \rangle$ are derived explicitly by Warke and Gunye (1976):

$$\langle HR \rangle = 2 \sum \varepsilon_i \rho_{i+i+} - G \left(\sum \sigma_{i+i-}\right)^2$$

$$- \tfrac{1}{2} \chi \left(Q^2_{0+} + 2\, Q^2_{1-} + 2\, Q^2_{2+}\right), \tag{11}$$

$$\langle \Phi_0 | R | \Phi_0 \rangle = (\det W)^{1/2}. \tag{12}$$

The generalized density matrices ρ and σ in (10) and (11) and the matrix W in (12) are given in terms of the transformation matrix C, the rotation matrix D and the occupation probabilities (Warke and Gunye 1976). The quantity $Q_{\mu\pm}$ in (11) is given by

$$Q_{\mu\pm} = \sum \left(q^{\mu}_{i\pm j+} + q^{\mu}_{j+i\pm}\right) \rho_{j+i\pm}.$$

The final expression for the energy E^J_K in the projected state ψ^J_M in (5) of the odd-A nucleus is given by

$$E^J_K = \langle \psi^J_M | H | \psi^J_M \rangle / \langle \psi^J_M | \psi^J_M \rangle$$

$$= \frac{\langle \Phi_K | H P^J_{K,K} | \Phi_K \rangle + (-1)^{J-K} \langle \Phi_K | H P^J_{K,-K} | \Phi_{-K} \rangle}{\langle \Phi_K | P^J_{K,K} | \Phi_K \rangle + (-1)^{J-K} \langle \Phi_K | P^J_{K,-K} | \Phi_{-K} \rangle}$$

$$= \frac{\int_0^{\pi} \langle HR \rangle (\det W)^{1/2}\, Y_K(\theta) \sin\theta\, d\theta}{\int_0^{\pi} (\det W)^{1/2}\, Y_K(\theta) \sin\theta\, d\theta}, \tag{13}$$

where

$$Y_K(\theta) = Y_{K-}(\theta)\, d^J_{KK}(\theta) + (-1)^{J-K}\, Y_{K+}(\theta)\, d^J_{K,-K}(\theta). \tag{14}$$

The expressions for the energy E^J of the $(A+1)$ doubly-even nucleus is given by (Warke and Gunye 1976)

$$E^J = \frac{\int_0^{\pi/2} \langle HR \rangle (\det W)^{1/2}\, d^J_{00}(\theta) \sin\theta\, d\theta}{\int_0^{\pi/2} (\det W)^{1/2}\, d^J_{00}(\theta) \sin\theta\, d\theta}. \tag{15}$$

2.1 *Renormalization procedure*

The nuclear energies are calculated by minimizing E^J of (15) in the case of doubly-even $(A+1)$ nucleus and E^J_K of (13) in the case of odd-A nucleus, by varying the deformation (β), pairing gaps (Δ_p, Δ_n) and chemical potentials (λ_p, λ_n) for each angular momentum state J. The constraint of nucleon number conservation determines the chemical potentials in each projected state. The parameters χ and G of the Hamiltonian H in (1), employed in the present calculations are determined by Kumar and Baranger (1968). The values of these parameters are estimated by the configuration space of two major shells and assuming an inert core with 40 protons and 70 neutrons. The assumption of inert core necessitates the modification of energies of the states projected from the intrinsic state. The simplest way to incorporate the effect of the neglected core on the projected energies is by renormalizing them. To do this we introduce a parameter, the moment of inertia $\mathcal{I}_{\text{core}}$ of the core. We assume that the moment of inertia $\mathcal{I}^J_{\text{total}}$ of a nucleus in a state J is the sum of the moment of inertia $\mathcal{I}_{\text{core}}$ of the core and the moment of inertia $\mathcal{I}^J_{\text{calc}}$ of the outer nucleons. It is reasonable to assume that $\mathcal{I}_{\text{core}}$ is independent of J for at least a set of states. The moment of inertia $\mathcal{I}^J_{\text{calc}}$ of the outer nucleons, however, depends on J, as deduced from the computed energies E^J_{calc}. Thus for even-even nucleus

$$E^J_{\text{calc}} = \frac{\hbar^2}{2\,\mathcal{I}^J_{\text{calc}}} J(J+1), \tag{16a}$$

and for odd-A nucleus

$$E^J_{\text{calc}} = \frac{\hbar^2}{2\,\mathcal{I}^J_{\text{calc}}} [J(J+1) - K(K+1)]. \tag{16b}$$

The corrected or normalized energy is then given as

$$E^J_{\text{norm}} = E^J_{\text{calc}} \left(1 + \frac{\mathcal{I}_{\text{core}}}{\mathcal{I}^J_{\text{calc}}}\right)^{-1} = E^J_{\text{calc}} \left(\frac{\mathcal{I}^J_{\text{calc}}}{\mathcal{I}^J_{\text{total}}}\right). \tag{17}$$

A few remarks about the parameter $\mathcal{I}_{\text{core}}$ are relevant here. This parameter is introduced to incorporate the effect of the neglected core on the energies computed by considering only the valence nucleons. This parameter will depend not only on the core but also on the valence nucleons and the deformed orbits they occupy in the intrinsic state. Thus $\mathcal{I}_{\text{core}}$ can be different for different nuclei. It can be different for two bands of the same nucleus if the intrinsic nature of the states on which the bands are built, is different. Furthermore, it can be different within a band of a nucleus for two sets of states, which differ in their intrinsic structure as pointed out by Moholkar *et al* (1980). Moreover in the case of odd-A nucleus, the $\mathcal{I}_{\text{core}}$ value required to yield a reasonably good agreement with corresponding experimental data can be markedly larger than that used for neighbouring doubly-even nucleus. This is consistent with the experimental observation (Bohr and Mottleson 1975) that the values of moment of inertia in odd-A nuclei are larger than those in the neighbouring even-A nuclei.

It should be emphasized here that the procedure of renormalizing the N-N interaction to incorporate the effects of core-polarization is very involved particularly in the case where there are many valence nucleons outside the core. Consequently one has to resort to a simplified prescription to incorporate the effects of core-polarization on the energy spectra. We have found (Gunye and Warke 1979; Gunye and Ashok Kumar 1980b; Moholkar *et al* 1980) that the introduction of the parameter $\mathcal{I}_{core}$ gives a good account of the observed energy spectra in a number of doubly-even nuclei in the rare-earth region. It is quite gratifying to note that the value of $\mathcal{I}_{core}$ required to explain the energy spectra of many doubly-even nuclei studied by us, is about 10 $\hbar^2$/MeV. Moreover a single value of $\mathcal{I}_{core}$ gives a good agreement between the calculated and experimental energies for all the levels in a band, for which the intrinsic state is the same. It is interesting to find out the values of parameter $\mathcal{I}_{core}$ required to obtain a reasonable agreement in energy spectra of different bands built on different intrinsic states in the odd-A nuclei.

3. Results and discussion

We have studied the yrast band in ^{156}Dy, the $3/2^+$ and $5/2^-$ bands in ^{155}Tb and $3/2^-$ and $11/2^-$ bands in ^{155}Dy using the theoretical formulation outlined in §2. The Hamiltonian and the configuration space used is that of Kumar and Baranger (1968). For each angular momentum J, the variational parameters β, Δ_p, Δ_n, λ_p and λ_n are determined by minimizing the projected energy and conserving nucleon numbers (Gunye and Warke 1979) in each band. The results are presented and discussed below.

3.1 ^{156}*Dy*

The calculated renormalized energy E^J_{norm} and the experimental energy E^J_{expt} for each angular momentum state J of the yrast band of ^{156}Dy is shown in table 1. The values of the variational parameters β, Δ_p and Δ_n corresponding to the minimum of energy of each state are also shown in table 1. It is seen from table 1 that the deformation β

Table 1. The calculated and experimental energies (in MeV) E^J_{norm} and E^J_{expt} are tabulated for each angular momentum state J in the ground band of ^{156}Dy. The deformation parameter β, the proton pairing gap Δ_p, the neutron pairing gap Δ_n and the total moment of inertia $\mathcal{I}^J_{total}$ are shown in columns 2, 3, 4 and 5 respectively.

J^π	β	Δ_p	Δ_n	$\mathcal{I}^J_{total}$	E^J_{expt}*	E^J_{norm}
0^+	0·26	1·08	0·87	—	0·00	0·00
2^+	0·26	1·08	0·87	24·5	0·14	0·14
4^+	0·29	1·08	0·87	25·2	0·40	0·42
6^+	0·29	1·08	0·87	26·7	0·77	0·83
8^+	0·33	0·99	0·52	29·4	1·22	1·29
10^+	0·33	0·84	0·30	32·5	1·72	1·77
12^+	0·33	0·84	0·00	31·7	2·29	2·22
14^+	0·33	0·84	0·00	35·4	2·89	2·96
16^+	0·33	0·84	0·00	38·7	3·52	3·51
18^+	0·33	0·84	0·00	41·5	4·18	4·12
20^+	0·33	0·84	0·00	44·2	4·86	4·80

*Lieder *et al* (1974)

goes on gradually increasing from 0·26 to 0·33 as angular momentum J increases from 0 to 8 and then remains constant for all the states with $J > 8$. The pairing gap Δ_p for protons decreases slowly with increasing J from the value 1·08 MeV for $J = 0$ to the value 0·84 MeV for $J = 20$. The neutron pairing gap Δ_n first decreases gradually from the value 0·87 MeV for $J = 0$ to 0·52 MeV for $J = 8$ and then starts decreasing drastically. The neutron pairing gap Δ_n vanishes at $J = 12$ and remains zero for all higher angular momentum states with $J \geqslant 12$. The calculated energies of the yrast band are renormalized as discussed in § 2 [equation (17)] by using the value $\mathcal{I}_{\text{core}} = 13\hbar^2/\text{MeV}$ for $J < 12$ and $\mathcal{I}_{\text{core}} = 8\hbar^2/\text{MeV}$ for $J \geqslant 12$. It is seen from table 1 that the calculated renormalized energies are in good agreement with the corresponding experimental (Lieder *et al* 1974) energies. This agreement is obtained with essentially two values of the parameter $\mathcal{I}_{\text{core}}$.

A characteristic feature brought out in the present calculations regarding the occupancies of the neutron orbitals should be pointed out here. The neutron occupancies vary with deformation β, which gradually changes with J. It is found that for $J \leqslant 8$, the $\Omega = \pm 11/2$ of $0h_{11/2}$ orbit is occupied by the last neutron pair, whereas for $J > 8$, the last neutron pair occupies the $\Omega = \pm 3/2$ of $0i_{13/2}$ orbit. Another orbit $\Omega = \pm 1/2$ of $0i_{13/2}$ close to the Fermi surface remains occupied for all J. It is worthwhile to point out that the neutron pairing gap Δ_n reduces from 0·87 MeV to 0·52 MeV at $J = 8$ and becomes zero for $J > 12$. Thus the intrinsic structure drastically changes for higher J states leading to a different value of $\mathcal{I}_{\text{core}}$ as discussed in § 2.

The band structure of the odd-proton nucleus ^{155}Tb and odd-neutron nucleus ^{155}Dy can be understood from the intrinsic structure of the doubly-even nucleus ^{156}Dy. The bands in the two odd-A nuclei under investigation here are constructed by annihilating a nucleon from the single-particle orbitals near the Fermi surface of neutrons and protons in the doubly-even $(A + 1)$ nucleus as described in § 2. In the range of deformations relevant to the nuclei considered in this paper, there are three energetically close single-particle orbits near the proton Fermi surface. The first one at the proton Fermi surface is characterized by $\Omega^\pi = 3/2^+$ from a predominantly ($\sim$ 80%) $1d_{5/2}$ state. The other two orbits just below the proton Fermi surface are $\Omega^\pi = 5/2^-$ from a predominantly ($\sim$ 96%) $0h_{11/2}$ and $\Omega^\pi = 5/2^+$ from a predominantly (92%) $0g_{7/2}$ state. These single-particle orbits near the proton Fermi surface in the intrinsic structure of ^{156}Dy suggest that the ground state band in the odd-proton nucleus ^{155}Tb would be characterized by $K^\pi = 3/2^+$. Moreover, the excited bands with $K^\pi = 5/2^-$ and $K^\pi = 5/2^+$ would be energetically quite close to the ground band. It is gratifying to note that the experimental energy spectra in ^{155}Tb confirm these inferences deduced from the single-particle orbits near the proton Fermi surface in the intrinsic structure of ^{156}Dy. The bands observed in the odd-neutron nucleus ^{155}Dy are, however, not so obvious from the intrinsic structure. In the relevant range of deformation, the single-particle orbit near the neutron Fermi surface of ^{156}Dy is characterized by $\Omega^\pi = 1/2^+$ from predominantly ($\sim$85%) $0i_{13/2}$. The next orbit below the neutron Fermi surface is $\Omega^\pi = 11/2^-$ from solely $0h_{11/2}$ state and this is followed by $\Omega^\pi = 3/2^-$ orbit which has large admixtures from various single-particle states. This single-particle structure near the neutron Fermi surface of ^{156}Dy suggests that the ground band and the first excited bands in the odd-neutron nucleus ^{155}Dy would be characterized by $K^\pi = 1/2^+$ and $K^\pi = 11/2^-$ respectively. The experimental energy spectra in ^{155}Dy corroborate this conclusion as far as $K^\pi = 11/2^-$ excited band is concerned. The observed ground band in ^{155}Dy, however, is $K^\pi = 3/2^-$ and not $K^\pi = 1/2^+$ as indicated by the intrinsic structure of ^{156}Dy. To facilitate comparison between the theoretical and experimental energy spectra, we have performed calculations for the experimentally observed ground and first excited bands in the odd-A nuclei ^{155}Tb and ^{155}Dy. The calculations are carried out in the framework of microscopic variational formulation outlined in § 2. The results of these calculations for the $K^\pi = 3/2^+$ and $5/2^-$ bands in ^{155}Tb and $K^\pi = 3/2^-$ and $11/2^-$ bands in ^{155}Dy are discussed below.

3.2 *The ground and first excited band in ^{155}Tb*

The energy of each angular momentum state J in the band K is calculated by minimizing E_K^J of (13) by varying the deformation β, pairing gaps Δ_p and Δ_n and chemical potentials λ_p and λ_n. The constraint of conservation of the nucleon numbers determines the chemical potentials in each projected state. The calculated renormalized energy spectrum is compared with the corresponding experimental spectrum (Winter *et al* 1971) in table 2 for the $3/2^+$ ground band and in table 3 for the $5/2^-$ excited band in ^{155}Tb. The value of the variational parameters β, Δ_p and Δ_n corresponding to the minimum of energy in each angular momentum state are also shown in tables 2 and 3. It should be pointed out at the very outset that the present calculations yield an energy separation of 0·10 MeV, which is in very good agreement with the corresponding observed separation of 0·23 MeV between the $5/2^-$ and $3/2^+$ bands. The calculated energies are renormalized as discussed in § 2 by employing a single $\mathcal{I}_{\text{core}}$ value for all the states in a band. It should be mentioned here that the values of the total moment of inertia for both the bands in ^{155}Tb are larger than those in the neighbouring doubly-even nucleus ^{156}Dy and consequently the $\mathcal{I}_{\text{core}}$ value required in the renormalization procedure in ^{155}Tb is larger than that required in ^{156}Dy as discussed in § 2. It is seen from tables 2 and 3 that the renormalized energies are in good agreement with the corresponding experimental energies (Winter *et al* 1971) in the $3/2^+$ ground band and $5/2^-$ excited band in ^{155}Tb. It should be stressed that this agreement is obtained by employing just one value for the parameter $\mathcal{I}_{\text{core}}$ in each band as seen from tables 2 and 3.

It can be seen from table 2 for the ground band that the deformation β goes on increasing gradually from 0·25 to 0·35 as angular momentum increases from $J = 3/2^+$ to $17/2^+$. This variation is similar to that in ^{156}Dy. The proton pairing gap Δ_p remains constant whereas the neutron pairing gap Δ_n varies with angular momentum. The Δ_n value decreases slowly from 0·89 MeV at $J = 3/2^+$ to 0·62 MeV at $J = 11/2^+$ and then decreases drastically, becoming zero for the high spin states with $J > 17/2^+$. This similar behaviour of the variation of Δ_n with J in ^{155}Tb and ^{156}Dy can be understood in view of the fact that the intrinsic state of the neutrons is essentially the same in both the nuclei. In this connection, it should also be emphasized that the characteristic feature of the change of occupancy of a neutron pair from $Oh_{11/2}$ to $Oi_{13/2}$ orbit noticed for high spin states ($J > 8$) in ^{156}Dy is also exhibited for high spin states ($J > 15/2$) in the ground band in ^{155}Tb.

It is seen from table 3 for the $5/2^-$ excited band that the deformation β goes on increasing gradually from 0·23 to 0·31 as angular momentum increases from $5/2^-$

Table 2. The calculated and experimental energies (in MeV) E^J_{norm} and E^J_{expt} are tabulated for each angular momentum state J in the $3/2^+$ ground band of ^{155}Tb. The employed $\mathcal{J}_{\text{core}}$ value is 25 $\hbar^2$/MeV. The deformation parameter β, the proton pairing gap Δ_p and the neutron pairing gap Δ_n and $\mathcal{J}^J_{\text{total}}$ are shown in columns 2, 3, 4 and 5 respectively.

J	β	Δ_p	Δ_n	$\mathcal{J}^J_{\text{total}}$	E^J_{expt} *	E^J_{norm}
$3/2^+$	0·25	1·19	0·89	—	0·00	0·00
$5/2^+$	0·25	1·19	0·89	35·6	0·07	0·07
$7/2^+$	0·27	1·19	0·89	34·8	0·16	0·17
$9/2^+$	0·29	1·19	0·89	35·1	0·27	0·30
$11/2^+$	0·31	1·19	0·62	35·9	0·41	0·45
$13/2^+$	0·31	1·19	0·50	37·3	0·58	0·60
$15/2^+$	0·33	1·19	0·36	38·3	0·75	0·78
$17/2^+$	0·35	1·19	0·00	40·4	0·96	0·95
$19/2^+$	0·35	1·19	0·00	42·7	1·17	1·12
$21/2^+$	0·35	1·19	0·00	44·9	1·41	1·32

*Winter *et al* (1971)

Table 3. The calculated and experimental energies (in MeV) E^J_{norm} and E^J_{expt} are tabulated for each angular momentum state J in the excited $5/2^-$ band of ^{155}Tb. The employed value of $\mathcal{J}_{\text{core}}$ is 45 $\hbar^2$/MeV. The deformation parameter β, the proton pairing gap Δ_p and the neutron pairing gap Δ_n and $\mathcal{J}^J_{\text{total}}$ are shown in columns 2, 3, 4 and 5 respectively. The energies are with respect to the band head energy of the $5/2^-$ state.

J^π	β	Δ_p	Δ_n	$\mathcal{J}^J_{\text{total}}$	E^J_{expt} *	E^J_{norm}
$5/2^-$	0·23	1·26	0·91	—	0·00	0·00
$7/2^-$	0·23	1·26	0·91	63·4	0·02	0·06
$9/2^-$	0·23	1·26	0·91	57·7	0·09	0·14
$11/2^-$	0·25	1·26	0·91	56·7	0·17	0·24
$13/2^-$	0·27	1·26	0·91	56·6	0·33	0·35
$15/2^-$	0·29	1·26	0·91	57·4	0·45	0·48
$17/2^-$	0·29	1·26	0·91	58·9	0·69	0·61
$19/2^-$	0·29	1·26	0·91	61·0	0·83	0·75
$21/2^-$	0·31	1·19	0·62	62·9	1·05	0·90
$23/2^-$	0·31	1·19	0·62	65·3	1·30	1·03
$25/2^-$	0·31	1·19	0·36	65·0	—	1·20

*Winter *et al* (1971)

to $21/2^-$. The proton pairing gap Δ_p remains nearly constant. These variational patterns are similar to those found in ^{156}Dy and the $3/2^+$ band in ^{155}Tb. The behaviour of neutron pairing gap Δ_n is also similar in the sense that it decreases substantially from 0·91 MeV at $J = 5/2^-$ to 0·36 MeV at $J = 25/2^-$. It does not, however, vanish completely as in ^{156}Dy and the ground band in ^{155}Tb. It is significant, however, to note that the salient feature of the change of occupancy of a neutron pair from $0h_{11/2}$ orbit to $0i_{13/2}$ orbit noticed in ^{156}Dy and $3/2^+$ band in ^{155}Tb, is also conspicuous in the intrinsic structure of the high spin states in this $5/2^-$ excited band. Thus the change in neutron occupancy manifested at high spin states seems to be a result of the change in deformation rather than that of the vanishing of neutron pairing gap.

3.3 *The ground and first excited band in* ^{155}*Dy*

The renormalized energy spectrum computed from the present microscopic variational formulation is compared with the corresponding experimental spectrum (Krien *et al* 1973) in table 4 for $3/2^-$ ground band and in table 5 for the $11/2^-$ excited band, in ^{155}Dy. The values of the variational parameters β, Δ_p and Δ_n corresponding to the minimum in energy for each angular momentum state are also displayed in tables 4 and 5. The experimentally observed energy separation of 0·23 MeV between the $3/2^-$ ground band and the $11/2^-$ excited band is reproduced fairly well by the present calculations. The corresponding computed energy separation is 0·37 MeV. The relative energy separations within the $3/2^-$ ground band and the $11/2^-$ excited band are also reproduced quite well by the present microscopic calculations. The calculated energies are renormalised by employing a single $\mathcal{J}_{\text{core}}$ value for each band as seen from tables 4 and 5. It is seen from tables 4 and 5 that the renormalized energies are in good agreement with the corresponding experimental energies (Krien *et al* 1973) in the $3/2^-$ ground band and the $11/2^-$ excited band in ^{155}Dy.

It is seen from tables 4 and 5 that the deformation β and the pairing gaps Δ_p and Δ_n do not change with the angular momentum J. This behaviour is in marked contrast with the corresponding situation in ^{155}Tb and ^{156}Dy. This is related to the fact that the intrinsic structure of the neutron part in ^{155}Dy is quite different from that in ^{155}Tb and ^{156}Dy. In particular, the characteristic feature of the change of occupancy of a

Table 4. The calculated and experimental energies (in MeV) E^J_{norm} and E^J_{expt} are tabulated for each angular momentum state J in the ground band ($3/2^-$) of ^{155}Dy. The employed value of $\mathcal{J}_{\text{core}}$ is 50 $\hbar^2$/MeV. The deformation parameter β, the proton pairing gap Δ_p and the neutron pairing gap Δ_n and $\mathcal{J}^J_{\text{total}}$ are shown in columns 2, 3, 4 and 5 respectively.

J^π	β	Δ_p	Δ_n	$\mathcal{J}^J_{\text{total}}$	E^J_{expt}*	E^J_{norm}
$3/2^-$	0·25	1·19	0·91	—	0·00	0·00
$5/2^-$	0·25	1·19	0·91	67·0	0·04	0·04
$7/2^-$	0·25	1·19	0·91	64·4	0·09	0·10
$9/2^-$	0·25	1·19	0·91	64·6	0·15	0·17
$11/2^-$	0·25	1·19	0·91	63·9	—	0·29

*Krien *et al* (1973)

Table 5. The calculated and experimental energies (in MeV) E^J_{norm} and E^J_{expt} are tabulated for each angular momentum state J in the $11/2^-$ band of ^{155}Dy. The employed value of $\mathcal{J}_{\text{core}}$ is 20 $\hbar^2$/MeV. The deformation parameter β, the proton pairing gap Δ_p, the neutron pairing gap Δ_n and $\mathcal{J}^J_{\text{total}}$ are shown in columns 2, 3, 4 and 5 respectively. The energies are with respect to the band head energy of the $11/2^-$ state.

J^π	β	Δ_p	Δ_n	$\mathcal{J}^J_{\text{total}}$	E^J_{expt}*	E^J_{norm}
$11/2^-$	0·26	1·08	0·87	—	0·00	0·00
$13/2^-$	0·26	1·08	0·87	35·1	0·20	0·19
$15/2^-$	0·26	1·08	0·87	33·5	0·43	0·42
$17/2^-$	0·26	1·08	0·87	32·6	0·67	0·69
$19/2^-$	0·26	1·08	0·87	32·4	0·92	0·99
$21/2^-$	0·26	1·08	0·87	32·6	1·19	1·28

*Krien *et al* (1973)

neutron pair from $0h_{11/2}$ to $0i_{13/2}$ orbit manifested in ^{155}Tb and ^{156}Dy is ruled out in this odd-neutron nucleus ^{155}Dy.

It may be of some interest to point out here that the present calculations yield a relative separation of 1·9 MeV between the $J^\pi = 1/2^+$ state of the $K^\pi = 1/2^+$ band and the $J^\pi = 3/2^-$ state of the $K^\pi = 3/2^-$ ground band. The relative excitation energies (in keV) of the states with $J^\pi = 5/2^+, 3/2^+$ and $7/2^+$ in the $K^\pi = 1/2^+$ band as predicted by our calculations are -70, $+210$ and $+430$ respectively with respect to the $J^\pi = 1/2^+$ state.

The present investigation thus indicates that the calculated energy spectra renormalized by employing the parameter $\mathscr{I}_{\text{core}}$ to incorporate the effect of the neglected core, are in good agreement with the corresponding experimental spectra of the yrast band ($K^\pi = 0^+$) in ^{156}Dy, the ground band ($K^\pi = 3/2^+$) and the first excited band ($K^\pi = 5/2^-$) in ^{155}Tb and the ground band ($K^\pi = 3/2^-$), the first excited ($K^\pi = 11/2^-$) band in ^{155}Dy. The values (in $\hbar^2$/MeV) of $\mathscr{I}_{\text{core}}$ used in the renormalization procedure are 25 for $3/2^+$ band and 45 for $5/2^-$ band in ^{155}Tb and 20 for $11/2^-$ band and 50 for $3/2^-$ band in ^{155}Dy. As pointed out earlier in § 2, the $\mathscr{I}_{\text{core}}$ value depends on the deformed orbits occupied in the intrinsic state. The different bands in the two nuclei are characterized by different deformed orbits occupied by the odd-proton or odd-neutron near the Fermi surface as discussed in the beginning of this section. In the case of $11/2^-$ band in ^{155}Dy and the $3/2^+$ band in ^{155}Tb, the odd-nucleon orbit is close to the Fermi surface. This is reflected in the near equal values of the $\mathscr{I}_{\text{core}}$ for the corresponding bands in these nuclei. Similarly the $\mathscr{I}_{\text{core}}$ values for the $3/2^-$ band in ^{155}Dy and $5/2^-$ band in ^{155}Tb are nearly equal and they are larger than those for the bands mentioned above in view of the fact that the relevant deformed orbits for the odd-nucleons lie slightly below the Fermi surface.

4. Conclusions

The microscopic formulation of variation with number conserved projected states is applied to study the ground and the first excited bands in the two odd-*A* nuclei ^{155}Tb and ^{155}Dy by employing the Hamiltonian with quadrupole plus pairing interactions. The bands in these two odd-*A* nuclei are constructed by annihilating a nucleon from the single-particle orbits near the Fermi surface of the doubly-even nucleus ^{156}Dy. The calculated energy spectra of the yrast band in ^{156}Dy and the ground and excited bands in ^{155}Tb and ^{155}Dy are in good agreement with the corresponding experimental data. The rotational bands in odd-*A* nuclei can also be explained by performing cranked HFB calculations. In particular the three bands in ^{159}Dy have been studied in this approach (Ring *et al* 1974; Mang 1975). The simplifying approximations regarding angular momentum conservation in the cranked HFB calculations can lead to errors which could be of the same order of magnitude as the energy differences between the excited states to be explained. Our approach with exact angular momentum projection seems to be better. The present results for different bands in the two nuclei indicate that our approach with exact angular momentum projection can also be successfully used to study the bands in odd-*A* nuclei.

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