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Convergent polynomial expansion, scaling of differential cross-section and computation of scaling functions for elastic diffraction scattering processes

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Abstract. Existing data on the differential cross-section ratio at high energies for pp, $\bar{p}p$, $\pi^{\pm}p$ and $K^{\pm}p$ scattering have been fitted by the proposed convergent polynomial expansion to determine the unknown coefficients in the scaling function. It is found that the data are very well represented within and somewhat outside the peak regions by only four or five terms in the proposed series in terms of Laguerre polynomials.

Keywords. Scaling hypothesis; convergent polynomial expansion; diffraction scattering; scaling function.

1. Introduction

Recently, a new approach to scaling of differential cross-sections has been proposed for two-body hadronic diffractive and nondiffractive processes using Mandelstam analyticity and convergent polynomial expansion (CPE) (Parida 1979a, b; Parida and Giri 1980a b). Using this approach predictions of differential cross-section as a function of $|t|$ have also been made for several higher energies for elastic diffractive processes (Parida 1980a; Giri and Parida 1981). Although it has been proposed in these papers (Parida 1979a; Parida and Giri 1980a, b) that the scaling function is a convergent series in Laguerre polynomials in the proposed variable χ at asymptotic energies, the scaling function has not been computed yet for any of the processes. In this paper we compute the scaling functions for all the elastic diffraction scattering processes by fitting the data on the cross-section ratio by the proposed series. It is found that only four or five terms in the series are needed to represent scaling of the data inside the forward peak region and somewhat outside it. To our knowledge such a computation of the scaling function is the first of its kind being reported.

2. Scaling hypothesis and computation of scaling functions

The proposed scaling hypothesis using CPE has been discussed in detail in earlier papers (Parida and Giri 1980a; Giri and Parida 1981). In this section after defining the necessary variables and formulae very briefly we state our results on computation.

2.1 *The scaling hypothesis*

Using conformal mapping of the $s(x)$ plane onto $\eta(z)$ plane with

$$w = \frac{x_+ - x}{x_- + x} \; \frac{x_- + 1}{x_+ - 1}, \tag{1}$$

$$Z = [\cosh^{-1} \sqrt{w}]^2, \tag{2}$$

$$w_s = \frac{s - s_{th}}{s_{th} - s_1}, \tag{3}$$

$$\eta = \sinh^{-1} \sqrt{w_s}, \tag{4}$$

where s_{th} (x_+) and s_1 $(- x_-)$ are the starting points of the right and the left-hand cuts, respectively, in the $s(x)$ plane, the following scaling variable χ has been proposed,

$$\chi(s, t) = a(s)\, Z(s, t), \tag{5}$$

where $$a(s) = \sum_m d_m\, \eta^m. \tag{6}$$

The parameters d_i's occurring in $a(s)$ have been related to the forward-slope-parameter data

$$b(s) = \frac{a(s)}{t_R}\left(1 + \frac{t_R}{4q^2 + t_L - \Delta/s}\right), \tag{7}$$

where t_R (t_L) stands for the equation to the boundary of spectral function ρ_{st} (ρ_{su}) which coincides with the start of the right-hand cut in the $t(u)$ plane, at asymptotic energies. It has been proposed that at asymptotic energies, the following approximation to the cross-section ratio

$$f(s, t) = \frac{d\sigma}{dt}(s, t) \Big/ \frac{d\sigma}{dt}(s, o) \equiv F(X) = e^{-\chi} \sum_n e_n\, L_n\,(2\chi), \tag{8}$$

as a convergent series in Laguerre polynomials, should represent the scaling function reasonably well.

2.2 *Computation of scaling functions*

It is necessary to compute the values of the parameters e_n's occurring on the right side of (8) to know the scaling function and the rate of convergence of the series. For this purpose, the unknown parameters occurring in the variable χ are first computed for the process of interest by fitting the slope-parameter data (Parida and Giri 1980a;

Giri and Parida 1981). The values of these parameters and the corresponding references where they have been computed are given in table 1. With the knowledge of the variable χ for the process, experimental data on $f(s, t)$ are plotted against χ to obtain the patterns of scaling as shown in figures 1 to 6. Taking the mean positions in an $f(s, t)$ *vs* χ plot as reference data points and the spread around the mean position as the error corridor, we fit the scaling curve by series in (8) by means of a computer. The best fit is decided by minimising the total χ^2 value by varying the parameters, e_n's occurring in (8) using a search programme. To test as to what range of χ the series (8)

Table 1. Values of unknown parameters occurring in the scaling variable for different processes as computed from fits to the slope-parameter data. In the last column are cited references where these parameters have been reported.

Process	d_0	d_1	References
pp	0·659	0·050	Giri (1980)
$\bar{p}p$	1·045	0	Parida and Giri (1980a)
π^+p	0·383	0·082	Giri and Parida (1981)
π^-p	0·487	0·074	Giri and Parida (1981)
K^+p	0·307	0·100	Giri and Parida (1981)
K^-p	0·593	0·032	Parida and Giri (1980a)

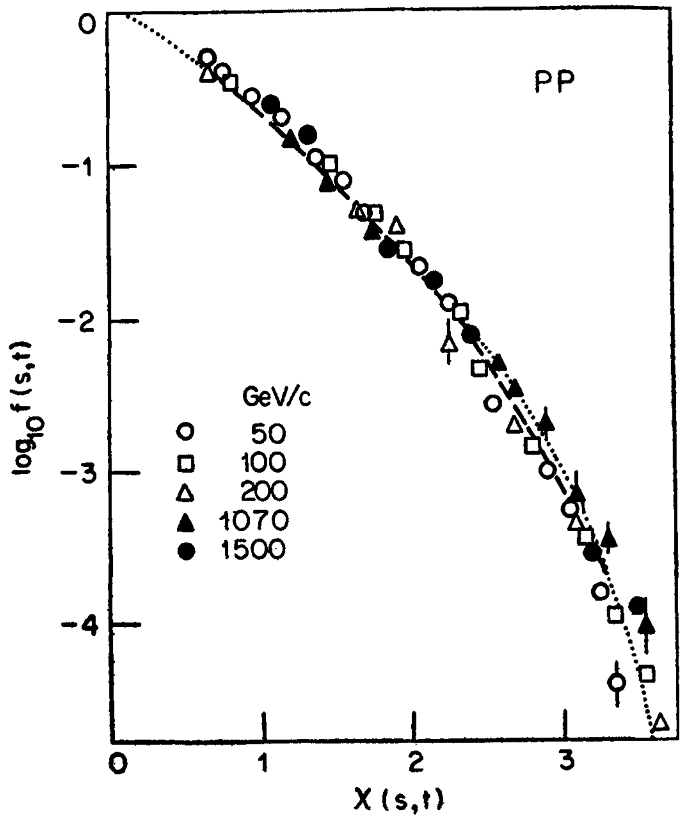

Figure 1. Scaling of the cross-section-ratio data and fits by the proposed series in (8) for pp scattering. The broken (dotted) line represents the fit with the first three (five) terms in the series as given by the first (second) row in table 2.

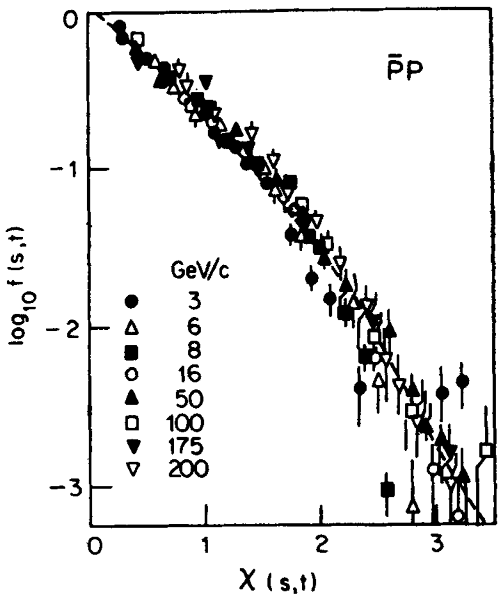

Figure 2. Scaling of the cross-section-ratio data and the fit by the proposed series for $\bar{p}p$ scattering with the first four terms of the series as shown by the broken line.

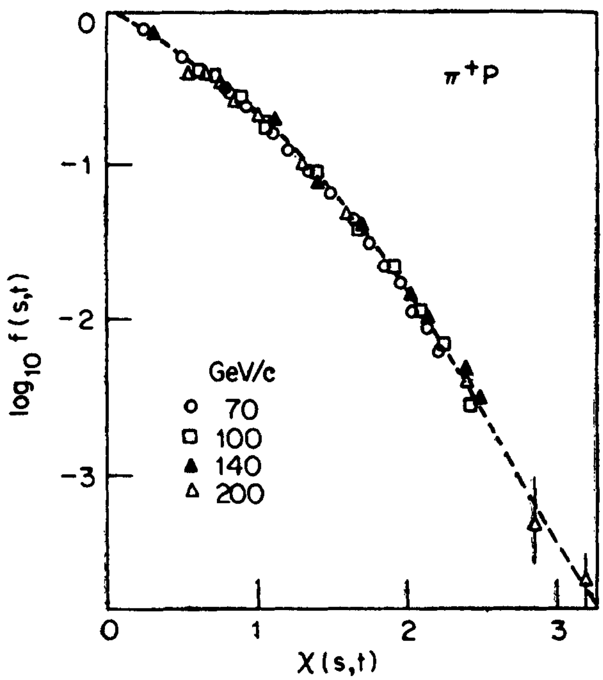

Figure 3. Scaling of the cross-section-ratio data and the fit by the first five terms of the proposed series for π^+p scattering as shown by the broken line.

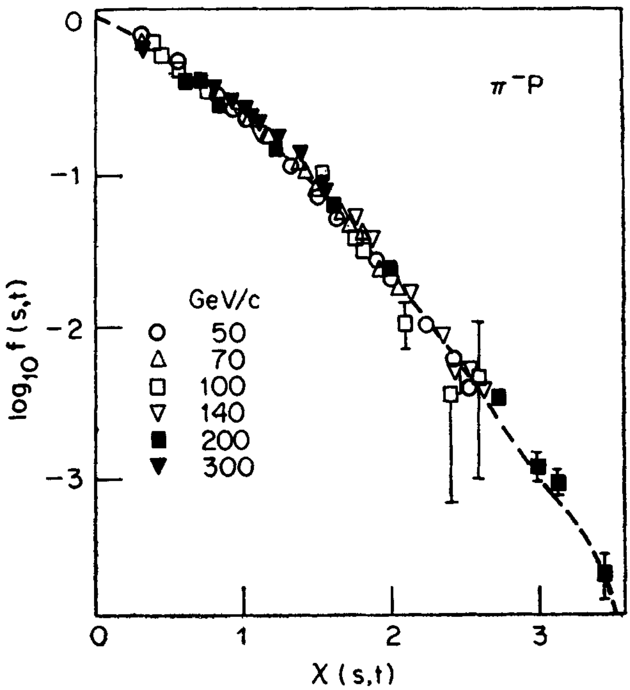

Figure 4. Scaling of the cross-section-ratio data and the fit by the first five terms of the proposed series for $\pi^- p$ scattering as shown by the broken line.

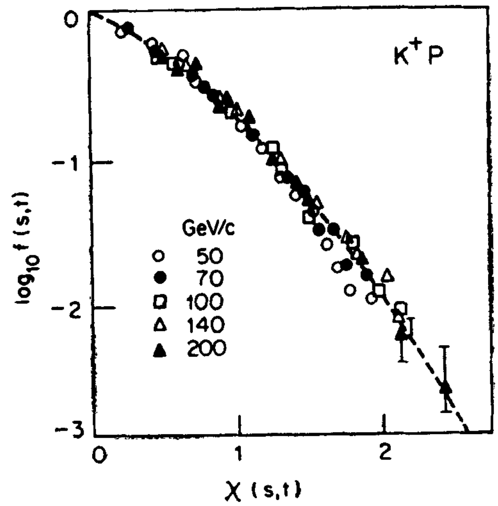

Figure 5. Scaling of the cross-section-ratio data and the fit by the first five terms of the series for K^+p scattering as shown by the broken line.

can fit the data, a segment of a scaling curve between $\chi = 0$ and some $\chi = \chi_{\max}$ was chosen in the beginning. Sufficiently large number of points with values of $f(s, t)$ corresponding to the mean positions and their errors, as decided by the spread in the scaling curve, were fed to the computer and the coefficients e_n's were determined by means of the search programme. The value of $\chi_{\max}$ and also the number of points were gradually increased and the fit was obtained taking more number of terms in the series whenever required. The results of the fit with the values of the coefficients and the maximum range of X within which the scaling curves can be fitted are presented in table 2. The value of $F(X_{max})$ upto and above which the data have been fitted by the series (8) has been shown in the last column of table 2.

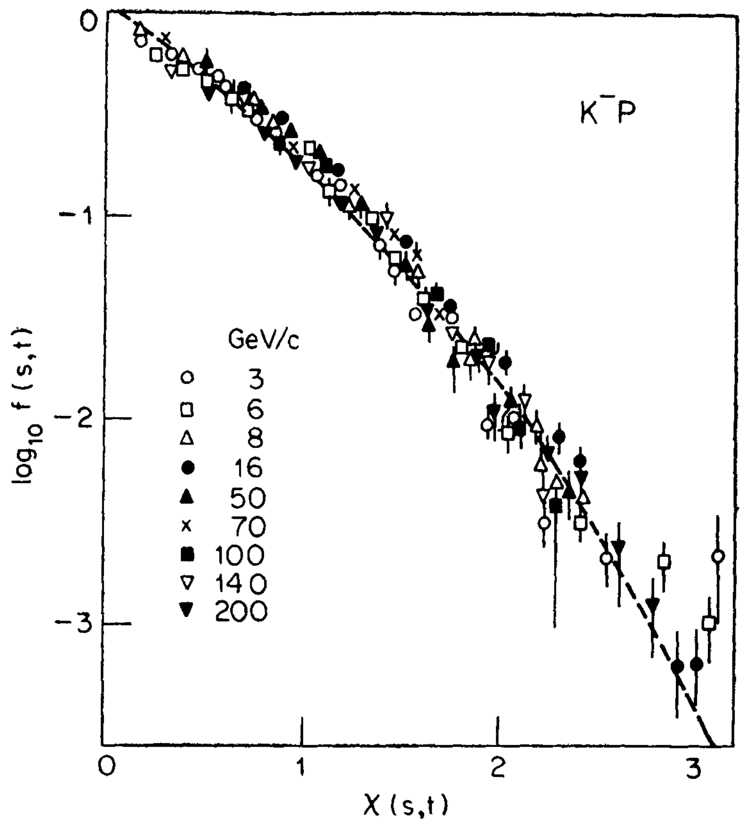

Figure 6. Scaling of the cross-section-ratio data and the fit by the first four terms of the series for K^-p scattering as shown by the broken line.

Table 2. Results of computation of the scaling function expressing the values of the coefficients in the proposed series, the range of X and the minimum value of the cross-section ratio upto and above which the data have been used for the fit.

Scattering process	e_0	e_1	e_2	e_3	e_4	Range of scaling variable ($0 - X_{max}$)	Values of $F(X_{max})$
pp	0·8718	0·2538	0·0587	...	...	0 − 3·0	0·0008
pp	0·8692	0·2598	0·0632	0·0097	0·0386	0 − 3·5	0·00008
$\bar{p}p$	0·8386	0·2154	0·0393	−0·0064	...	0 − 3·4	0·00047
π^+p	0·8079	0·2346	0·0255	0·0079	−0·0384	0 − 3·2	0·00015
π^-p	0·8586	0·2139	0·0051	−0·0066	−0·0411	0 − 3·5	0·0002
K^+p	0·7924	0·2383	0·0334	0·0051	−0·043	0 − 2·5	0·0018
K^-p	0·8036	0·2656	0·0739	0·0137	...	0 − 3·3	0·00017

For pp scattering only the first three terms fit the data well for $0 \leqslant X \leqslant 3{\cdot}0$. This fit has been shown by a broken line in figure 1. But to fit the scaling curve for a larger range with $0 \leqslant X \leqslant 3{\cdot}5$, the first five terms are necessary for this process, although the quality of the fit worsens to some extent. The fit with five terms has been shown by the dotted line in figure 1. Both the fits of figure 1 coincide for $X \leqslant 0{\cdot}5$. For $\bar{p}p$ the first four terms fit the data of figure 2 well in the range $0 \leqslant X \leqslant 3{\cdot}4$ as shown by the broken line. For π^+p, π^-p and K^+p, the first five terms are sufficient to fit the data well in the range $0 \leqslant X \leqslant 3{\cdot}2$, $0 \leqslant X \leqslant 3{\cdot}5$ and $0 \leqslant X \leqslant 2{\cdot}5$, respectively. These fits have been shown by the broken curves in figures 3, 4 and 5. For K^-p only the first four terms are sufficient to yield a good fit in the range $0 \leqslant X \leqslant 3{\cdot}3$ as shown by the dotted line in figure 6.

It may be noted that for pp all the terms are positive, for π^+p and K^+p, e_4 is negative and for $\bar{p}p$, e_3 is negative, whereas for π^-p, e_3 and e_4 both appear to be negative. But we have noted that if we confine to lower ranges of X, only the first three terms for $\bar{p}p$ and π^-p and first four terms for π^+p and K^+p with positive coefficients would be sufficient to fit the data. It has been proved (Cornille 1976) that for diffraction scattering $\tau = tb(s)$ is a scaling variable and the scaling function, can be a series in orthogonal polynomials with positive coefficients including Laguerre, but excluding Hermite. In our approach, the proposed scaling function is a series in Laguerre polynomials in X, modulated by the exponential weight function. For small $|t| \ll 4m_\pi^2$, $X \sim tb(s)$, but for larger $|t|$ values, X is different from the scaling variable proposed by Cornille (1976). With the ranges and the values of the coefficients specified in table 2 if we include next higher order terms in the fit, the corresponding coefficients are found to be negligible.

Before ending this section we comment briefly on the observed scaling region as compared to other known cases. According to Leader (1963) it is reasonable to say that the diffraction-peak width is that interval of momentum transfer where the differential cross-section is larger than a finite fraction of its forward value. Using the popular exponential fits to the cross-section data in the forward peak region, the diffraction peak may be extended upto those values of $|t|$ for which

$$f(s, t) = 1/e. \tag{9}$$

where $e = 2{\cdot}71828$. According to Cornille and Martin (1972) and Martin (1975) their results on scaling are true within the diffraction peak for which

$$\lim_{s \to \infty} f(s, t) \equiv G(\tau) \leqslant \tfrac{1}{2}, \tag{10}$$

where $\tau = tb(s)$. Model-independent results due to Auberson *et al* (1971) in the variable $t(\ln s)^2$ are applicable to those values of $|t| < 4m_\pi^2 \simeq 0{\cdot}078$ GeV^2 at asymptotic energies. If (9) or (10) is taken to be the definition of the diffraction peak region, we find from a comparison of the values of $F(X_{\max})$ given in the last column of table 2 with the right side of (9) or (10) that our proposed scaling function represents the existing data beyond the peak region for every process.

3. Discussion

Our computation of the scaling function is the first of its kind being reported. In the earlier paper (Giri and Parida 1981) predictions of differential-cross-section ratio as a function of $|t|$ and for higher energies were carried out for $\pi^{\pm}p$ and $K^{+}p$ scattering using scaling curves obtained by joining mean positions of data points in the $f(s, t)$ *vs* X plots. The scaling curves generated by the coefficients in table 2 are very close to those used earlier (Giri and Parida 1981) in the region of X investigated here. For pp scattering, whereas the variable proposed by Parida (1979a) possesses $t \longleftrightarrow u$ symmetry, the variable used here (Parida and Giri 1980a) does not possess such a symmetry. Several other limitations of this approach have been discussed in earlier papers (Parida 1979a; Parida and Giri 1980a; Giri and Parida 1981). So far predictions of differential cross-sections and computations of scaling functions have been reported only for elastic diffraction scattering processes. Such a type of analysis is in progress for inelastic nondiffractive processes like $\pi^{-}p \rightarrow \pi^{0}n$ and $\pi^{-}p \rightarrow \eta\, n$ and will be reported when complete.

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