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$^3\vec{He}$ (γ, π°) $^3\vec{He}$ as the nuclear probe

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Abstract. The state of polarization of the recoil nucleus in $\gamma + {}^3\text{He} \rightarrow \pi^\circ + {}^3\text{He}$ as well as the asymmetry in the differential cross-section when the initial ^{3}He is polarized are studied together with the differential cross-section taking into consideration the S, S' and D-state admixtures in the nuclear wavefunctions. In view of the considerable spin dependence in the photoproduction amplitudes these observables are found to be quite sensitive to the small admixtures of S' and D-states in the nuclear wavefunctions.

Keywords. Differential cross section; target asymmetry; recoil nucleus polarization; S, S', D-state admixtures in; ^{3}He nuclear wavefunction; nuclear structure; multipole amplitudes.

1. Introduction

The study of the three-nucleon ground state has been pursued with renewed interest during the last decade. Following the extensive electron scattering studies made earlier as part of the saga of high energy electron scattering experiments, we have now several theoretical solutions of the dynamical equations for the three-nucleon ground state with realistic potentials (see for example Sick 1981; Payne 1981; Drechsel 1980 where extensive references to the literature can be found). However it has been found that (McCarthy *et al* 1977) there is no agreement between experiment and theory especially around $Q^2 = 11\ \text{fm}^{-2}$ for the charge form factor. A recent study of the magnetic form factor (Riska 1980) with simple wavefunction models and taking into account the pion and ρ-meson exchange current effect shows that the single-nucleon current contribution depends strongly on the D-state probability. Since the new data (Arnold *et al* 1978; Riska 1980; Sick 1981) on the ^{3}He form factors at large Q^2 indicate that the existing microscopic calculations of the wavefunctions are missing an important ingredient, it appears quite reasonable to use the wavefunctions for the nuclear systems following the traditional enumeration of Sachs (1953); Schiff (1964); Gibson and Schiff (1965), where free parameters can be adjusted in order to reproduce the experimental data. In fact this approach was used by Lazard and Maric (1973) in their discussions of photoproduction of charged pions on ^{3}He.

Here, the photoproduction of neutral pions on ^{3}He is studied since the reaction offers several advantages as a tool to study the nuclear structure (i) corrections due to Coulomb interaction between the pions and the nucleus in the final state or between the target and the beam in the initial state are absent, (ii) the momentum transfers involved here are high even for comparatively low beam energies as this is a production process, (iii) the spin-dependent amplitudes in the photoproduction reaction are comparable to the spin-independent amplitudes unlike in electron scattering where the spin-independent Coulomb scattering is very dominant and the spin-dependent magnetic scattering is almost zero relatively except at the backward angles, and (iv) in view of the fact that observables like asymmetry and polarization are more sensitive probes to the small admixtures in the amplitudes, it is considered desirable to investigate the effects of S' and D-state admixtures in the wavefunctions by measuring the differential cross-section on polarized ^{3}He nuclei or by measuring the recoil ^{3}He polarization. In § 2 is outlined an elegant theoretical method to calculate the differential cross-section, target asymmetry and recoil nuclear polarization taking into account all possible admixtures of wavefunctions in the nucleus. In § 3 the admixture of states following basically the classification of Sachs (1953), Schiff (1964, 1965) and Gibson and Schiff (1965) is considered specifically; typical numerical estimates for the various observables are presented and the sensitivity of the observables to S' and D-state admixtures is pointed out. For simplicity the Gaussian forms for the radial wave-functions and the well-known photoproduction multipole amplitudes due to Berends and Donnachie (1975) are used taking into account all the four ($l = 0, 1, 2, 3$) partial waves.

2. Method of calculation

The earliest calculations on photoproduction of neutral pions on ^{3}He were made by Ramachandran and Ananthanarayanan (1964), using the CGLN amplitudes and taking into account the S-state in the nuclear wavefunction. We follow essentially the same formalism here and take into account all the admixtures of S', P and D states in addition to the dominant S-state. The amplitude for the reaction may therefore be written using PWIA in the form

$$\langle m' | T | m \rangle = \langle m' | \sum_{j=1}^{3} t_j \exp(i\mathbf{Q}\cdot\mathbf{r}_j) | m \rangle, \tag{1}$$

where m and m' denote the initial and final magnetic quantum numbers of the nuclear spin states and t_j denotes the basic amplitude for $\gamma N \to \pi^\circ N$ in the momentum space on a nucleon whose position coordinates are denoted by $\mathbf{r}_j$. $\mathbf{Q}$ denotes the momentum transfer to the nucleus

$$\mathbf{Q} = \mathbf{k} - \mathbf{q}, \tag{2}$$

where $\mathbf{k}$ is the momentum of the incoming photon and $\mathbf{q}$ denotes the momentum of the outgoing π°. The nuclear wavefunction ψ^m may be written following Sachs (1953); Schiff (1964) and Gibson and Schiff (1965) in the form

$$\psi^m = \sum_{i=1}^{8} a_i \, \psi_i^m, \tag{3}$$

where $|a_i|^2$ denote the relative probabilities of each of the admixture states if ψ^m as well as each of the ψ_i^m are normalized to unity. Connection with the form of the nuclear wave-functions generally used in the context of Faddeev calculations (Brandenburg *et al* 1975, Hajduk *et al* 1980) can readily be established by expanding each of the ψ_i^m in the form

$$\psi_i^m = \sum_a C_i^a \mid [(L_\rho L_r) L (\tfrac{1}{2} S_{23}) S] \tfrac{1}{2} m; (\tfrac{1}{2} T_{23}) \tfrac{1}{2} \tfrac{1}{2}\rangle, \tag{4}$$

in terms of L-S coupling states where a denotes collectively the quantum numbers $a \equiv \{L_\rho, L_r, L, S_{23}, S, T_{23}\}$ and

$$C_i^a = \langle [(L_\rho L_r) L (\tfrac{1}{2} S_{23}) S] \tfrac{1}{2} m; (\tfrac{1}{2} T_{23}) \tfrac{1}{2} \tfrac{1}{2} \mid \psi_i^m\rangle. \tag{5}$$

Alternatively one can also expand each of the ψ_i^m in the form

$$\psi_i^m = \sum_\beta B_i^\beta \mid [(L_\rho \tfrac{1}{2}) J_\rho (L_r S_{23}) J_r] \tfrac{1}{2} m; (\tfrac{1}{2} T_{23}) \tfrac{1}{2} \tfrac{1}{2}\rangle, \tag{6}$$

in terms of the j-j coupling states, where β denotes collectively the quantum numbers $\beta \equiv \{L_\rho, J_\rho, L_r, S_{23}, J_r, T_{23}\}$ and

$$B_i^\beta = \langle ((L_\rho \tfrac{1}{2}) J_\rho (L_r S_{23}) J_r) \tfrac{1}{2} m; (\tfrac{1}{2} T_{23}) \tfrac{1}{2} \tfrac{1}{2} \mid \psi_i^m\rangle. \tag{7}$$

An explicit evaluation of the coefficients C_i^a has recently been carried out by Keshavamurthy and Ramachandran (1981) and B_i^β are related to these coefficients through

$$B_i^\beta = \sum_a \begin{Bmatrix} L_\rho & \tfrac{1}{2} & J_\rho \\ L_r & S_{23} & J_r \\ L & S & \tfrac{1}{2} \end{Bmatrix} [J_\rho] [J_r] [L] [S] C_i^a, \tag{8}$$

where $[j] = (2j+1)^{1/2}$ and $\{\ \}$ denotes the standard Wigner $9j$ symbol. The Jacobi coordinates $\vec{\rho}$ and $\mathbf{r}$ or the equivalent $\mathbf{R}_1$ and $\mathbf{R}_2$ of Gibson and Schiff (1965) are related to the nucleon coordinates $\mathbf{r}_j$ through

$$\vec{\rho} = \mathbf{r}_1 - \frac{\mathbf{r}_2 + \mathbf{r}_3}{2} = \frac{\sqrt{3}}{2} \mathbf{R}_1, \tag{9a}$$

$$\mathbf{r} = \mathbf{r}_3 - \mathbf{r}_2 = - \mathbf{R}_2, \tag{9b}$$

in terms of which the amplitude (1) may be rewritten as

$$\langle m' \mid T \mid m\rangle = 3 \langle \psi_{m'} \mid t_1 \exp (i\mathbf{Q} \cdot \mathbf{R}_1/3^{1/2}) \mid \psi_m\rangle, \tag{10}$$

after eliminating the over all momentum conservation factors in the usual way so that the differential cross-section may be written as

$$\frac{d\sigma}{d\Omega} = \frac{q}{k} \frac{1}{4} \sum_{\epsilon, m, m'} \mid\langle m' \mid T \mid m\rangle\mid^2, \tag{11}$$

where ϵ denotes photon polarization.

2.1 *Evaluation of the matrix element*

The matrix element $\langle \psi_{m'} \,|\, t_1 \exp(i\mathbf{Q}\cdot\mathbf{R}_1/3^{1/2}) \,|\, \psi_m\rangle$ may be evaluated in either of the two alternative ways corresponding to the expansion of the states following (4) or (6) respectively. Using (4), for example, we have

$$\langle m' \,|\, T \,|\, m\rangle = 3 \sum_{i,j,\,a,\,a'} a_j^* a_i \int (\tfrac{3}{4})^{3/2} R_1^2\, dR_1 \int R_2^2\, dR_2\, C_j^{a'*} C_i^a$$

$$\times \langle [(L'_\rho L'_r) L' (\tfrac{1}{2} S'_{23}) S'] \tfrac{1}{2} m'; (\tfrac{1}{2} T'_{23}) \tfrac{1}{2} \tfrac{1}{2} \,|$$

$$t_1 \exp(i\mathbf{Q}\cdot\mathbf{R}_1/3^{1/2}) \,|\, [(L_\rho L_r) L(\tfrac{1}{2} S_{23}) S] \tfrac{1}{2} m; (\tfrac{1}{2} T_{23}) \tfrac{1}{2} \tfrac{1}{2}\rangle. \tag{12}$$

Noting that the operator acts only on the nucleon labelled 1, we see readily that

$$L'_r = L_r,\ S'_{23} = S_{23},\ T'_{23} = T_{23}. \tag{13}$$

Moreover, if we are interested in calculating only those terms that are connected to the dominant S-state, the P states ψ_3, ψ_4, ψ_5 get eliminated from our considerations. Further $L_\rho = L$ since $L_r = 0$. In this approximation the states that contribute to the summations in (12) are listed in table 1. The summation over T_{23} and S_{23} are not independent since antisymmetry requires that T_{23} be zero if $S_{23} = 1$ and $T_{23} = 1$ if $S_{23} = 0$. It is very important to note here that while the quartet spin states are not connected to the dominant S-states in the context of the charge form factor, they contribute considerably to (12), since t_1 has the general structure

$$t_1 = i\sigma_1\cdot\mathbf{K}(I=0) + L(I=0) + \tau_{1z}\,[i\sigma_1\cdot\mathbf{K}(I=1) + L(I=1)], \tag{14}$$

and the spin dependent amplitudes $\mathbf{K}$ are comparable to the spin independent amplitudes L. The isoscalar amplitudes are denoted by putting $I = 0$ within brackets while the isovector amplitudes are denoted by $I = 1$. We shall not consider here the possible existence of isotensor amplitudes (Dombey and Kabir 1966; Donnachie and

Table 1. The quantum numbers of the dominant S-state and states connected to the S-state

ψ_i	L_ρ	L_r	L	S_{23}	S	T_{23}
ψ_1	0	0	0	1	$\frac{1}{2}$	0
	0	0	0	0	$\frac{1}{2}$	1
ψ_2	0	0	0	1	$\frac{1}{2}$	0
	0	0	0	0	$\frac{1}{2}$	1
ψ_6	2	0	2	1	$\frac{3}{2}$	0
ψ_7	2	0	2	1	$\frac{3}{2}$	0
ψ_8	2	0	2	1	$\frac{3}{2}$	0

Shaw 1972), since the isotensor amplitudes can contribute to the process only through the negligible isospin $T = 3/2$ admixture of states in the nuclear wavefunction. Using spherical tensor notation we write the operator in the form

$$t_1 \exp(i\mathbf{Q}\cdot\mathbf{R}_1/3^{1/2}) = 4\pi \sum_l \sum_{\substack{I=0,1\\ n=0,1}} \sum_\lambda (i)^{l+n} j_l\left(\frac{QR_1}{\sqrt{3}}\right)(-1)^{l+n-\lambda}$$

$$\times\ [(Y_l(\hat{R}_1)\otimes\sigma^n)^\lambda \cdot (Y_l(\hat{Q})\otimes K^n(I))^\lambda]\, \tau_0^I, \tag{15}$$

where $\tau_0^0 = 1$, $\tau_0^1 = \tau_{1z}$, $\sigma_0^0 = 1$, $\sigma_0^1 = \sigma_{1z}$, $\sigma_{\pm 1}^1 = \mp \dfrac{(\sigma_{1x} \pm i\,\sigma_{1y})}{\sqrt{2}}$, $K_0^0(I) = L(I)$, $K^1(I) = \mathbf{K}(I)$. Using standard Racah techniques the matrix element is now evaluated to give

$$\langle m' | T | m\rangle = 24\sqrt{\pi} \sum_{i,j,\alpha,\alpha'} a_j^* a_i \sum_{l,n,\lambda,I,\nu} C(\tfrac{1}{2}\, I\, \tfrac{1}{2};\ \tfrac{1}{2}\, 0\, \tfrac{1}{2})\,[I]$$

$$\times\ \delta_{T_{23}T'_{23}}\, W(T_{23}\, \tfrac{1}{2}\, \tfrac{1}{2}\, I;\ \tfrac{1}{2}\, \tfrac{1}{2})\, C(\tfrac{1}{2}\, \lambda\, \tfrac{1}{2};\ m\, \nu\, m')\,(i)^{l+n}(-1)^{l+n-\lambda}$$

$$\times\ (-1)^\nu (-1)^{S-S'} [L'] [l] [S] [S'] [\lambda] [n] [L] [L_\rho] (-1)^{L_\rho - L'_\rho - L + L'}$$

$$\times\ \delta_{S_{23}S'_{23}}\, \delta_{L_r L'_r}\, W(L_r L_\rho L' l;\ L L'_\rho)\, W(S_{23}\, \tfrac{1}{2}\, S'\, n;\ S\, \tfrac{1}{2})$$

$$\times \begin{Bmatrix} L & S & \tfrac{1}{2} \\ l & n & \lambda \\ L' & S' & \tfrac{1}{2} \end{Bmatrix} C(L_\rho\, l\, L'_\rho;\ 000)\,(Y_l(\hat{Q})\otimes K^n(I))^\lambda_{-\nu} F_l, \tag{16}$$

where the radial integrals F_l are given by

$$F_l = \left(\frac{3}{4}\right)^{3/2} \int R_1^2\, dR_1 \int R_2^2\, dR_2\, C_{\alpha'}^{j*}\, j_l\left(\frac{QR_1}{\sqrt{3}}\right) C_\alpha^i. \tag{17}$$

If we restrict ourselves in (16) now to those transitions envisaged in table 1 the radial integrals that are required are only

$$F_0^{SS} = \left(\frac{3}{4}\right)^{3/2} a_1^2\, 16\pi^2 \int f_1^2\, j_0\left(\frac{QR_1}{\sqrt{3}}\right) R_1^2 R_2^2\, dR_1\, dR_2, \tag{18}$$

$$F_0^{SS'} = a_1\, a_2 \left(\frac{3}{4}\right)^{3/2} 16\pi^2 \int f_1\, v_1\, j_0\left(\frac{QR_1}{\sqrt{3}}\right) R_1^2 R_2^2\, dR_1\, dR_2, \tag{19}$$

$$F_2^{SD} = \left(\frac{3}{4}\right)^{3/2} 16\pi^2 a_1 \int f_1 \left[S_s\, a_7 f_7 - \frac{4}{3} R_2^2 a_8 f_8 - S_1 (a_6 f_6 + a_8 f_8) \right]$$

$$\times j_2\left(\frac{QR_1}{\sqrt{3}}\right) R_1^4 R_2^2\, dR_1\, dR_2, \tag{20}$$

where $S_s = R_1^2 + R_2^2$ and $S_1 = R_2^2 - R_1^2$. The radial functions f_i, v_1 can be chosen to have tractable analytic forms containing some adjustable parameters as has been done by Gibson and Schiff (1965) and Lazard and Maric (1973).

Noting further that the ^{3}He nucleus has spin $\frac{1}{2}$ and introducing the nuclear spin operator, $\mathbf{J}$ we can write $\langle m' \mid T \mid m\rangle$ in the form

$$\langle m' \mid T \mid m\rangle = \langle m' \mid 2i\, \mathbf{J} \cdot \mathscr{K} + \mathscr{L} \mid m\rangle, \tag{21}$$

where $\mathscr{L}$ and $\mathscr{K}$ denote the spin independent and spin dependent amplitudes with respect to the nuclear spin, and are given by

$$\mathscr{L} = 12\sqrt{2\pi} \sum_{i,j,\alpha,\alpha'} a_j^* a_i \sum_{l,I} C(\tfrac{1}{2}\, I\, \tfrac{1}{2};\, \tfrac{1}{2}\, 0\, \tfrac{1}{2})\, [I]\, \delta_{T_{23} T'_{23}}$$

$$\times\ W(T_{23}\, \tfrac{1}{2}\, \tfrac{1}{2}\, I;\, \tfrac{1}{2}\, \tfrac{1}{2})\, (i)^{2l}\, (-1)^{L_\rho - L'_\rho - l + L}\, (-1)^{2S + S' + 1/2}$$

$$\times\ W(LSL'S';\, \tfrac{1}{2}\, l)\, \delta_{ln}\, \delta_{S_{23} S'_{23}}\, \delta_{L_r L'_r}\, W(L_r\, L_\rho\, L'\, l;\, L L'_\rho)$$

$$\times\ [L']\, [l]\, [S]\, [S']\, [L]\, [L_\rho]\, W(S_{23}\, \tfrac{1}{2}\, S'\, n;\, S\, \tfrac{1}{2})$$

$$\times\ C(L_\rho\, l\, L'_\rho;\, 000)\, (Y_l\, (\hat{Q}) \otimes K^n\, (I))_0^0\, F_l, \tag{22}$$

$$\mathscr{K}_{-\nu}^1 = 24\sqrt{\pi} \sum_{i,j,\alpha,\alpha'} a_j^* a_i \sum_{l,n,I} C(\tfrac{1}{2}\, I\, \tfrac{1}{2};\, \tfrac{1}{2}\, 0\, \tfrac{1}{2})\, [I]\, \delta_{T_{23} T'_{23}}$$

$$\times\ W(T_{23}\, \tfrac{1}{2}\, \tfrac{1}{2}\, I;\, \tfrac{1}{2}\, \tfrac{1}{2})\, (i)^{l+n-1}\, (-1)^{l+n-1}\, (-1)^{S-S'}\, [L']\, [l]\, [S]\, [S']$$

$$\times\ [n]\, [L]\, [L_\rho]\ (-1)^{L_\rho - L'_\rho - L + L'}\, W(L_r\, L_\rho\, L'\, l;\, L\, L'_\rho)$$

$$\times\ \delta_{S_{23} S'_{23}}\, \delta_{L_r L'_r}\, W(S_{23}\, \tfrac{1}{2}\, S'\, n;\, S\, \tfrac{1}{2})\, C(L_\rho\, l\, L'_\rho;\, 000)$$

$$\times \begin{Bmatrix} L & S & \tfrac{1}{2} \\ l & n & 1 \\ L' & S' & \tfrac{1}{2} \end{Bmatrix} (Y_l\, (\hat{Q}) \otimes K^n\, (I))_{-\nu}^1\, F_l. \tag{23}$$

If we confine ourselves to the transitions listed in table 1, these amplitudes have the following elegant form

$$\mathscr{L} = 2\, L_p\, (F_0^{SS} + F_0^{SS'}) + L_n\, (F_0^{SS} - 2\, F_0^{SS'}), \tag{24}$$

$$\mathcal{K}^1_\nu = -2\,(K_p)^1_\nu F_0^{SS'} + (K_n)^1_\nu (F_0^{SS} - 2\,F_0^{SS'})$$

$$-\left(\frac{16\sqrt{\pi}}{\sqrt{3}}\right)\ [Y_2(\hat{Q}) \otimes K^1_p]^1_\nu F_2^{SD}, \tag{25}$$

where L_p, $\mathbf{K}_p$ and L_n, $\mathbf{K}_n$ denote the spin-independent and spin-dependent amplitudes for neutral pion photoproduction on protons and neutrons respectively and are related to the $I = 0$ and $I = 1$ amplitudes in the usual way.

In the case of $j - j$ coupling expansion for the wavefunctions using (6) we can write

$$\langle m' | T | m\rangle = 3 \sum_{i, j, \beta', \beta} a_j^*\, a_i\, (\tfrac{3}{4})^{3/2} \int R_1^2\, dR_1 \int R_2^2\, dR_2\, B_i^\beta\, B_j^{\beta' *}$$

$$\times \Big\langle ((L'_\rho\, \tfrac{1}{2})\, J'_\rho\, (L'_r\, S'_{23})\, J'_r)\, \tfrac{1}{2}\, m'; (\tfrac{1}{2}\, T'_{23})\, \tfrac{1}{2}\, \tfrac{1}{2} \Big| t_1 \exp\left(\frac{i\,\mathbf{Q}\cdot\mathbf{R}_1}{\sqrt{3}}\right)\Big|$$

$$\times ((L_\rho\, \tfrac{1}{2})\, J_\rho\, (L_r\, S_{23})\, J_r)\, \tfrac{1}{2}\, m; (\tfrac{1}{2}\, T_{23})\, \tfrac{1}{2}\, \tfrac{1}{2}\Big\rangle, \tag{26}$$

which may be simplified using (15) and standard Racah techniques to give

$$\langle m' | T | m\rangle = 24\sqrt{\pi} \sum_{i, j, \beta, \beta'} a_j^*\, a_i \sum_{l, n, \lambda, I} C(\tfrac{1}{2}\, I\, \tfrac{1}{2}; \tfrac{1}{2}\, 0\, \tfrac{1}{2})\, [I]$$

$$\times\ \delta_{T_{23}\, T'_{23}}\, W(T_{23}\, \tfrac{1}{2}\, \tfrac{1}{2}\, I; \tfrac{1}{2}\, \tfrac{1}{2})\, C(\tfrac{1}{2}\, \lambda\, \tfrac{1}{2}; m\, \nu\, m')\, (-1)^\nu$$

$$\times\ (i)^{l+n}\, (-1)^{l+n-\lambda}\, (-1)^{J_\rho - J'_\rho}\, \delta_{J_r J'_r}\, \delta_{L_r L'_r}\, \delta_{S_{23} S'_{23}}$$

$$\times\ W(J_r\, J_\rho\, \tfrac{1}{2}\, \lambda; \tfrac{1}{2}\, J'_\rho)\, [J_\rho]\, [J'_\rho]\, [\lambda]\, [L_\rho]\, [l]\, [n]\, C(L_\rho\, l\, L'_\rho; 000)$$

$$\times \begin{Bmatrix} L_\rho & \tfrac{1}{2} & J_\rho \\ l & n & \lambda \\ L'_\rho & \tfrac{1}{2} & J'_\rho \end{Bmatrix} (Y_l(\hat{Q}) \otimes K^n(I))^\lambda_{-\nu}\, G_l, \tag{27}$$

where the radial integrals G_l are given by

$$G_l = (\tfrac{3}{4})^{3/2} \int R_1^2\, dR_1 \int R_2^2\, dR_2\, B_j^{\beta' *}\, j_l\left(\frac{QR_1}{\sqrt{3}}\right) B_i^\beta. \tag{28}$$

We may once again express (27) in the convenient form (21) with $\mathcal{L}$ and $\mathcal{K}$ now given by

$$\mathcal{L} = 12\sqrt{2\pi} \sum_{i, j, \beta, \beta'} a_j^* a_i \sum_{l, I} C(\tfrac{1}{2}\, I\, \tfrac{1}{2}; \tfrac{1}{2}\, 0\, \tfrac{1}{2})\, [I]\, (i)^{2l}$$

$$\times\ \delta_{T_{23}\, T'_{23}}\, \delta_{ln}\, \delta_{S_{23}\, S'_{23}}\, \delta_{J_r\, J'_r}\, \delta_{L_r\, L'_r}\, (-1)^{J_\rho + L'_\rho - l + 3/2}\, [J_\rho]\, [L_\rho]\, [l]\, [n]$$

$$\times\ W(T_{23}\, \tfrac{1}{2}\, \tfrac{1}{2}\, I; \tfrac{1}{2}\, \tfrac{1}{2})\, W(L_\rho\, \tfrac{1}{2}\, L'_\rho\, \tfrac{1}{2}; J_\rho\, l)\, C(L_\rho\, l\, L'_\rho; 000)$$

$$\times\ (Y_l(\hat{Q}) \otimes K^n(I))^0_0\, G_l, \tag{29}$$

$$\mathscr{K}^1_{-\nu} = 24\sqrt{\pi} \sum_{i,j,\beta',\beta} a_j^* a_i \sum_{l,n,I} C(\tfrac{1}{2}\, I\, \tfrac{1}{2};\, \tfrac{1}{2}\, 0\, \tfrac{1}{2})\, [I]\, \delta_{T_{23}\, T'_{23}}$$

$$\times\, \delta_{J_r\, J'_r}\, \delta_{L_r\, L'_r}\, \delta_{S_{23}\, S'_{23}}\, (i)^{l+n-1}\, (-1)^{l+n-1}\, (-1)^{J_\rho - J'_\rho}\, [J_\rho]\, [J'_\rho]\, [L_\rho]$$

$$\times\, [l]\, [n]\, W(T_{23}\, \tfrac{1}{2}\, \tfrac{1}{2}\, I;\, \tfrac{1}{2}\, \tfrac{1}{2})\, W(J_r\, J_\rho\, \tfrac{1}{2}\, 1;\, \tfrac{1}{2}\, J'_\rho)\, C(L_\rho\, l\, L'_\rho;\, 000)$$

$$\times \begin{Bmatrix} L_\rho & \tfrac{1}{2} & J_\rho \\ l & n & 1 \\ L'_\rho & \tfrac{1}{2} & J'_\rho \end{Bmatrix} (Y_l(\hat{Q}) \otimes K^n(I))^1_{-\nu}\, G_i. \tag{30}$$

2.2 *Differential cross-section*

The elegant form (21) into which we have reduced the matrix element in either case of $L-S$ or $j-j$ coupling expansions enables us immediately to write down the differential cross section as

$$\frac{d\sigma}{d\Omega} = \frac{q}{k}\frac{1}{2}\sum_{\epsilon} [\mathscr{L}\, \mathscr{L}^* + \mathscr{K}\cdot\mathscr{K}^*]. \tag{31}$$

2.3 *Target asymmetry and recoil nucleus polarization*

If the target nucleus is polarized, the state of polarization is characterized by the density matrix ρ^i which has the form

$$\rho^i = \tfrac{1}{2}\, [1 + 2\mathbf{J} \cdot \mathbf{P}^i], \tag{32}$$

where $\mathbf{P}^i$ denotes the nuclear polarization initially. The final spin state of the nucleus is then characterized by the density matrix ρ^f given by

$$\rho^f = [i\, 2\mathbf{J} \cdot \mathscr{K} + \mathscr{L}]\, \rho^i\, [-2\,\mathbf{J} \cdot \mathscr{K}^* + \mathscr{L}^*]$$
$$= \frac{\mathrm{Tr}\, \rho^f}{2}\, [1 + 2\mathbf{J} \cdot \mathbf{P}^f], \tag{33}$$

where $\mathbf{P}^f$ denotes the polarization of the recoil nucleus.

The cross-section with polarized targets is simply given by

$$\frac{d\sigma}{d\Omega}(\mathbf{P}^i) = \frac{q}{k}\, \tfrac{1}{2} \sum_{\epsilon} \mathrm{Tr}\, \rho^f. \tag{34}$$

Denoting by $\hat{n}$ the unit vector perpendicular to the reaction plane

$$\hat{n} = \frac{\mathbf{k} \times \mathbf{q}}{|\mathbf{k} \times \mathbf{q}|}, \tag{35}$$

the target asymmetry A is defined through

$$A = \mathbf{A}\cdot\hat{n} = \frac{\dfrac{d\sigma}{d\Omega}(\hat{n}) - \dfrac{d\sigma}{d\Omega}(-\hat{n})}{\dfrac{d\sigma}{d\Omega}(\hat{n}) + \dfrac{d\sigma}{d\Omega}(-\hat{n})}, \tag{36}$$

where $\mathbf{A}$ is given by

$$\mathbf{A} = \frac{i\sum\limits_{\epsilon}[(\mathscr{K}\mathscr{L}^* - \mathscr{L}\mathscr{K}^*) - (\mathscr{K}\times\mathscr{K}^*)]}{\sum\limits_{\epsilon}(|\mathscr{L}|^2 + |\mathscr{K}|^2)}. \tag{37}$$

With initially unpolarized targets the polarization $\mathbf{P}^f$ of the recoil nucleus is also readily calculated to give

$$\mathbf{P}^f = \frac{i\sum\limits_{\epsilon}(\mathscr{L}^*\mathscr{K} - \mathscr{L}\mathscr{K}^*) + (\mathscr{K}\times\mathscr{K}^*)}{\sum\limits_{\epsilon}|\mathscr{L}|^2 + |\mathscr{K}|^2}. \tag{38}$$

3. Numerical results and discussion

For purposes of illustration we have estimated here the differential cross section $d\sigma/d\Omega$, the target asymmetry A and the recoil nuclear polarization $\mathbf{P}^f$ numerically at laboratory photon energies 280, 340 and 400 MeV with $x-z$ plane as the reaction plane. The spin-dependent and spin-independent amplitudes $\mathbf{K}_p$, $\mathbf{K}_n$ and L_p, L_n have been calculated in the standard way (Donnachie 1970) using the multipole amplitudes of Berends and Donnachie (1975). Since the purpose of this paper is mainly to show the sensitivity of the observables $d\sigma/d\Omega$, A and $\mathbf{P}^f$ to the S' and D-state admixtures and not to explain theoretically any experimental data (which is not available at present) we choose for simplicity the Gaussian form for the radial wavefunctions. Moreover, since ψ_7 is believed to provide the dominant D-state admixture of about 8% (Gibson and Schiff 1965) we set

$$a_1^2 = 0{\cdot}9, \quad a_2^2 = 0{\cdot}02, \quad a_7^2 = 0{\cdot}08; \quad a_6 = a_8 = 0. \tag{39}$$

The radial wavefunctions used in this computation are

$$f_1 = N_s \exp\left[-\tfrac{3}{4}\,\alpha^2 (R_1^2 + R_2^2)\right], \tag{40}$$

for the dominant S-state, with $N_s^2 = (3^{3/2}\,\alpha^6/\pi^3)$,

$$v_{1,2} = N_{s'}\, S_{1,2} \exp\left[-\tfrac{3}{4}\,\alpha^2 (R_1^2 + R_2^2)\right], \tag{41}$$

for the S' state, where $S_2 = 2\,\mathbf{R}_1\cdot\mathbf{R}_2$, $N_{s'}^2 = (3^{5/2}\,\alpha^{10}/8\,\pi^3)$ with the normalization

$$\frac{3^{3/2}}{8}\int (v_1^2 + v_2^2)\, d^3R_1\, d^3R_2 = 1, \tag{42}$$

and
$$f_7(S_s) = N_D \exp\left[-\tfrac{3}{4}\,\alpha^2\,(R_1^2 + R_2^2)\right], \tag{43}$$
for the D state where $N_D^2 = (3^{9/2}\,\alpha^{14}/25\,\pi^3\,2^7)$. The normalization factor N_D in (43) has been obtained using the Gibson and Schiff normalization for ψ_7^m, *i. e.*,
$$\int \psi_7^*\,\psi_7\,d^3 r_i = \left(\frac{3^{1/2}\,5\,\pi^3}{8}\right)\int f_7^2\,R^{13}\,dR, \tag{44}$$
where $R^2 = R_1^2 + R_2^2$. The parameter α has been chosen to be $\alpha = 0{\cdot}384\ \text{fm}^{-1}$.

We have chosen the expansion (4) for the ψ_i^m and used in these calculations the expansion coefficients C_i^α tabulated explicitly by Keshavamurthy and Ramachandran (1981). The radial integrals have been evaluated analytically using the tables of integrals (Dwight 1961; Gradshteyn and Ryzhik 1965) to give
$$F_0^{SS} = a_1^2 \exp\left(-\frac{Q^2}{18\,\alpha^2}\right), \tag{45}$$
$$F_0^{SS'} = a_1\,a_2\,N_s\,N_{s'}\,\frac{\pi^3 \exp(-Q^2/18\,\alpha^2)}{81\,\sqrt{3}\,\alpha^{10}}, \tag{46}$$
$$F_2^{SD} = \frac{\pi^3\,Q^2\,N_s\,N_D\,a_1\,a_7}{\alpha^{12}\,3^6}\left[10 - \frac{Q^2}{9\alpha^2}\right]\exp\left(-\frac{Q^2}{18\alpha^2}\right)\sqrt{3}. \tag{47}$$

The differential cross-section $d\sigma/d\Omega$ is plotted as a function of the production angle θ in figures 1a, 1b, 1c for different energies. The target asymmetry is given likewise

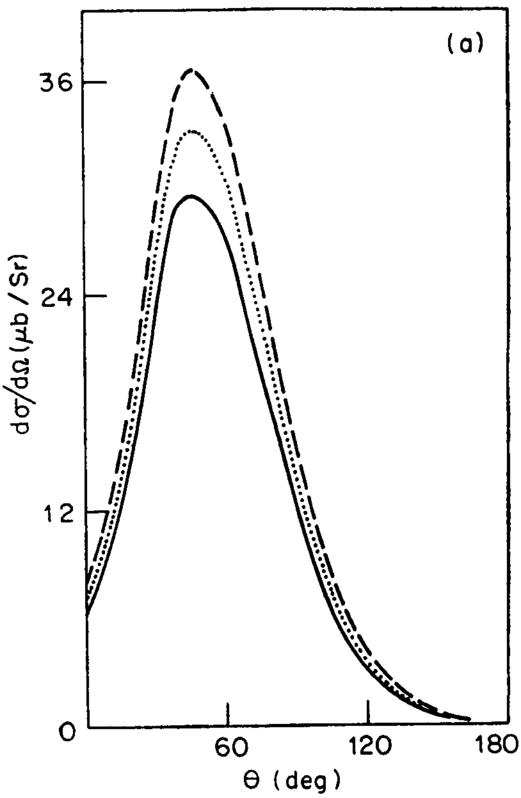
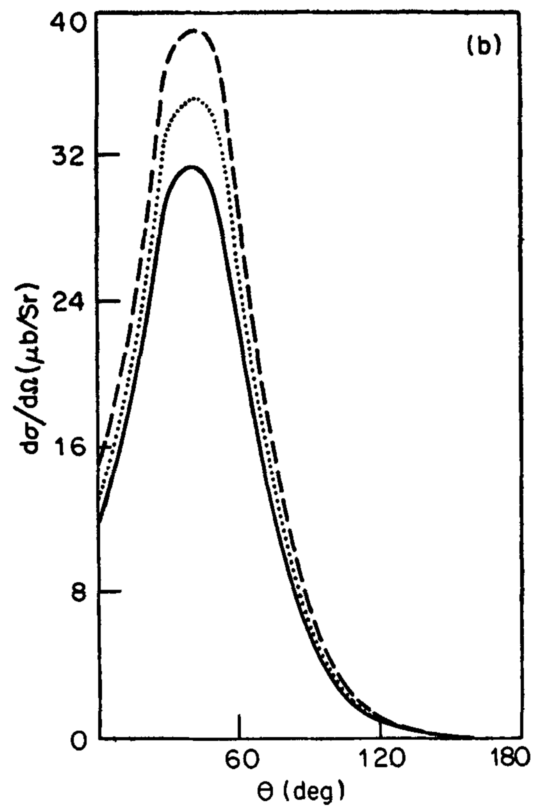
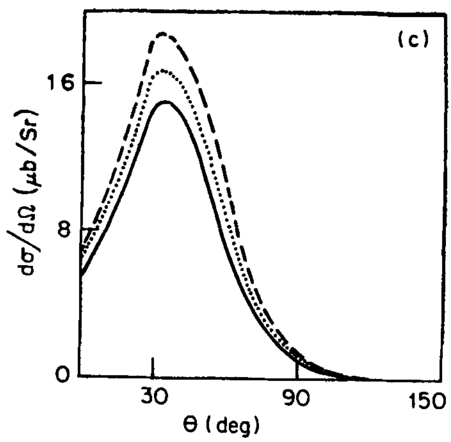


Figure 1a.





Figures 1a, b, c. The differential cross section $d\sigma/d\Omega$ at laboratory photon energies (a) 280, (b) 340 and (c) 400 MeV as a function of the pion photoproduction angle (θ) (—) including S, S' and D states with 90%, 2% and 8% respectively (...) S and S' states with 96% and 4% respectively (— — —) pure S-state with 100%,

in figures 2a, 2b, 2c and the y component of the recoil nuclear polarization P_y^f in figures 3a, 3b, 3c. An examination of these figures shows that the differential cross-section $d\sigma/d\Omega$, the target asymmetry and recoil nuclear polarization are quite sensitive to the small admixtures of S' and D states. We therefore advocate planning of experiments to measure these observables as a means to study the nuclear wavefunctions

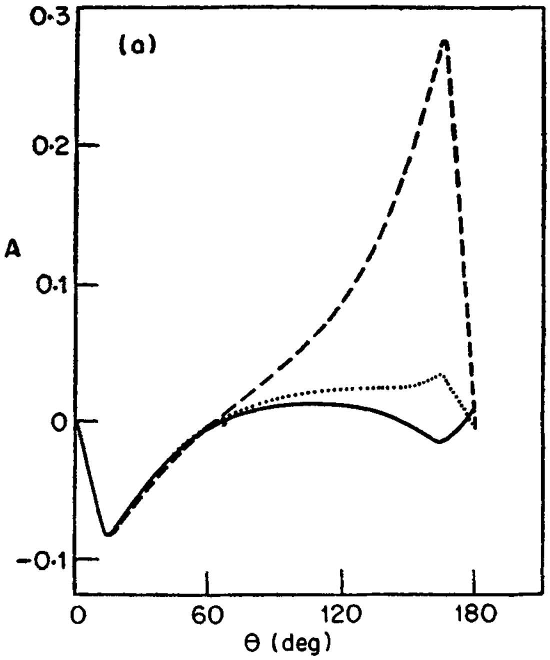


Figure 2a

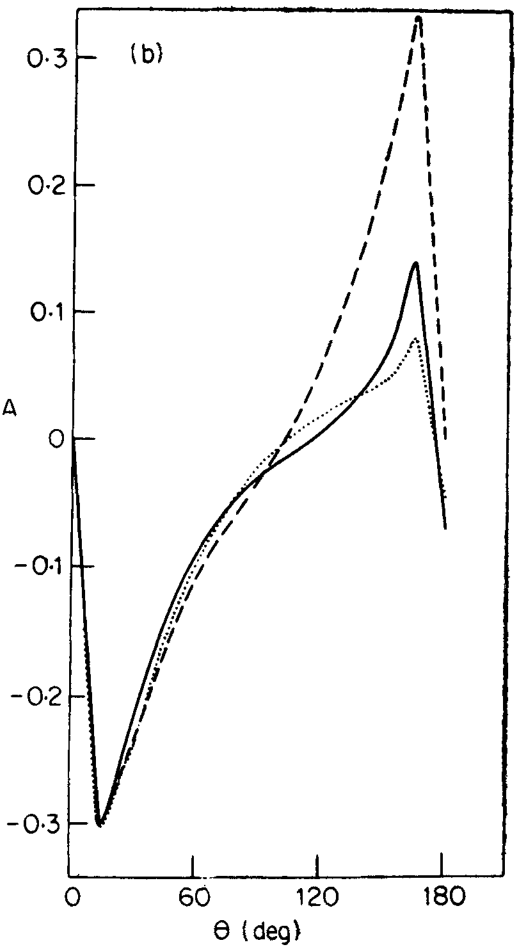
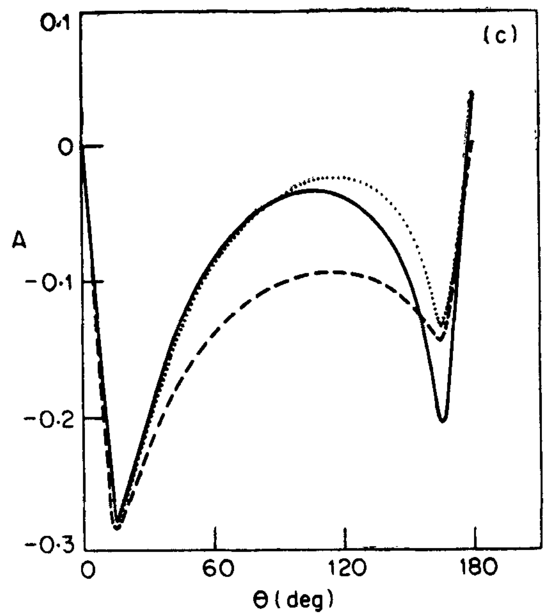


Figure 2b

particularly in view of the active interest in developing polarized targets and in detecting polarization by several groups (Haeberli 1974; Catillon 1974; de Boer 1974).

The theoretical formulae derived in § 2 are sufficiently simple to be used by the experimentalists. They are also sufficiently general so that one can use any given set of nuclear wavefunctions. If in particular one employs the analytical forms (Hajduk *et al* 1980) which are given in the $j - j$ coupling basis, or the numerical solutions given in $L - S$ coupling form (Brandenburg *et al* 1975) rather than the Sachs, Gibson



Figures 2a, b, c. The target asymmetry (A) at laboratory photon energies, (a) 280, (b) 340 and (c) 400 MeV as a function of the pion photoproduction angle (θ) (——) including S, S' and D states with 90%, 2% and 8% respectively.
(...) S and S' states with 96% and 4% respectively.
(— — —) pure S-state with 100%.

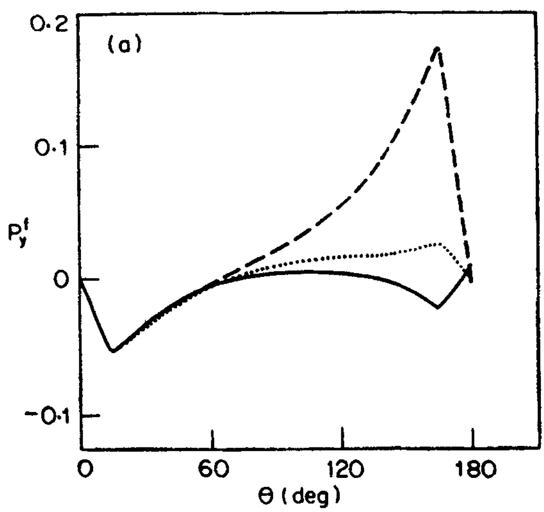


Figure 3a

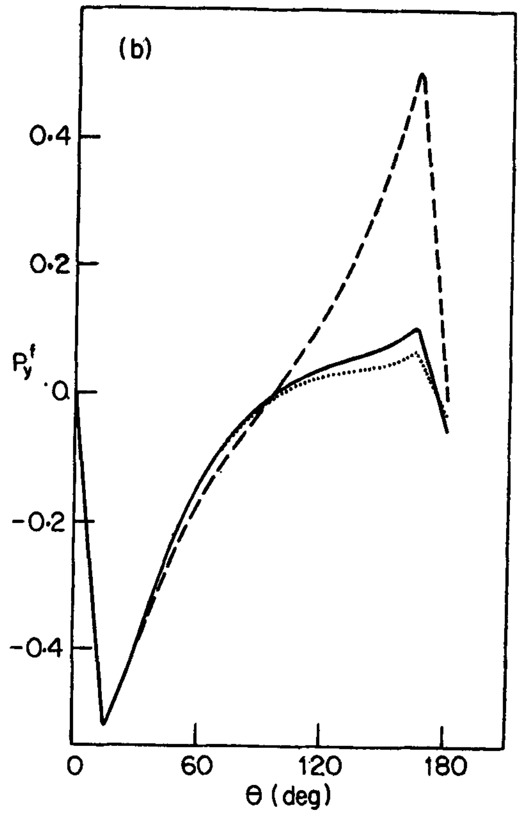


Figure 3b

and Schiff forms employed in this paper, the calculation is simplified further in that there are now no independent summations over i and β or i and α i.e. $a_i\ C_i^{\alpha}$ are simply replaced by $a_\alpha\ C^\alpha$ with only a summation over α. Likewise $a_i\ B_i^{\beta}$ are replaced by $a_\beta\ B^\beta$ with a summation over β but with no summation over i. Thus the differential cross-section, target asymmetry and recoil nuclear polarization could be estimated corresponding to any given experimental situation, using a whole set of theoretical models using the formulae derived here. This facilitates to check the validity of the various microscopic calculations of the nuclear wavefunctions (see for example refs. 3-11 listed in Arnold *et al* 1978; Sick 1981) much more incisively than is possible at present using only the electron scattering data which itself has already led to a disagreement between experiment and theory. It is hoped that the information from the polarization and asymmetry studies advocated here could pinpoint the cause for this disagreement, since they present a perspective on the nuclear structure which is complementary to that obtainable from cross section studies.

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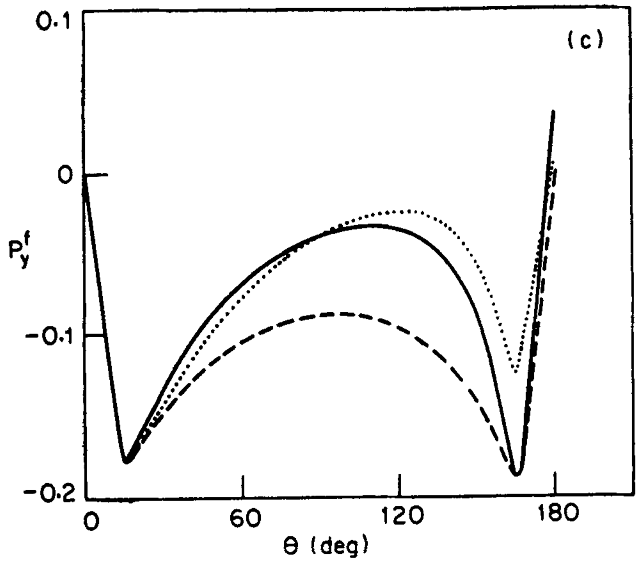

Figures 3a, b, c. The Y component of the recoil nuclear polarization (P_y^f) at laboratory photon energies, (a) 280, (b) 340 and (c) 400 MeV as a function of the pion photo production angle (θ)
(—) including S, S' and D states with 90%, 2% and 8% respectively
(...) S and S' states with 96% and 4% respectively
(— — —) pure S-state with 100%.

performed on the DEC 1090 Computer at the Indian Institute of Science, Bangalore. The authors are indebted to Prof. G N Ramachandran for making available the computer time for this purpose. Two of the authors (RSK and KV) wish to thank Prof. B Sanjeevaiah, for providing research facilities. RSK and KV are thankful to the Department of Atomic Energy (India) and the Council of Scientific and Industrial Research (India) for financial assistance.

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