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# Recoil nuclear polarization in muon capture—Effect of target thickness and finite range of nuclear recoils

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**Abstract.** The effect of target thickness and finite range of nuclear recoils is investigated in the study of average and longitudinal polarizations of recoiling nuclei resulting from the capture of muons by spin zero target nuclei.

**Keywords.** Muon capture; average nuclear polarization; longitudinal nuclear polarization.

## 1. Introduction

The advantage of studying the recoil nuclear polarization in muon capture has been sufficiently emphasized by Devanathan *et al* (1972) for obtaining unambiguous information on the fundamental muon capture interaction without the uncertainties of nuclear wave functions. In muon capture by spin zero target nucleus, there can be only three independent observables as shown by Bernabeu (1975) and among the several observables that are possible, interesting relations can be obtained (Subramanian *et al* 1976; Subramanian and Devanathan 1975, 1979; Devanathan and Subramanian 1975). The Louvain-Saclay—Zurich experimental group (Possoz *et al* 1974, 1977; Truttmann *et al* 1979) has made valuable contributions by developing the ion-implantation technique to retain the nuclear polarization produced and applying it to the measurement of polarization of the recoil nuclei in muon capture process. In a recent experiment, the average and longitudinal polarizations $P_{av}$ and $P_L$ of the recoiling nucleus $^{12}B$ in the muon capture reaction $^{12}C(\mu^-, \nu)\ ^{12}B\ (1^+)$ have been measured simultaneously by Roesch *et al* (1981) using the method of selective recoil implantation. The theory underlying this experiment has been discussed by Devanathan and Subramanian (1974) earlier and it is the purpose of this paper to consider the effect of target thickness and the finite range of nuclear recoils.

## 2. Average and longitudinal polarizations

In the experimental set-up of Roesch *et al* (1981), muons are stopped in a carbon foil sandwiched between a polarization retaining layer made of Ag and a polarization destroying layer made of Al. The experiment consists of two parts. In the first part, the polarization of nuclei recoiling into the forward hemisphere is retained. In the second part of the experiment, the target is reversed and so the polarization of nuclei recoiling into the backward hemisphere alone is retained. As shown by Devanathan and Subramanian (1974), the polarizations obtained in the two parts of the experiment are respectively

$$P_f = \tfrac{1}{2}\,(P_{\mathrm{av}} + P_L/2), \tag{1}$$

and

$$P_b = \tfrac{1}{2}\,(P_{\mathrm{av}} - P_L/2). \tag{2}$$

This method allows the simultaneous determination of the average polarization $P_{\mathrm{av}}$ and the longitudinal polarization $P_L$ of the recoiling nucleus.

$$P_{\mathrm{av}} = P_f + P_b, \tag{3}$$

$$P_L = 2\,(P_f - P_b). \tag{4}$$

In the above consideration, it is explicitly assumed that the target is sufficiently thin so as to allow all the $^{12}B$ recoils enter the layers of Ag and Al, between which the target is sandwiched.

## 3. Effect of target thickness

If the target is thick and the range of nuclear recoils is finite, then as shown in figures 1a and 1b, polarizations are retained only in a portion of the hemispheres. In figure 1a, the polarization is retained only in the forward segment of the sphere shown by the shaded area and the corresponding $P_f$ can be obtained from expression (23) of

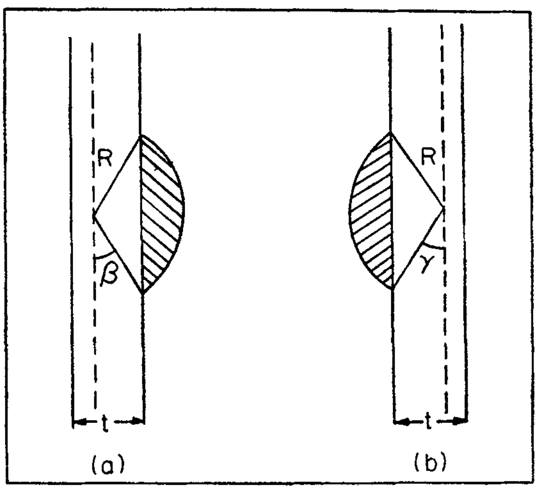


**Figure 1.** In arrangement (a), polarization is retained in the forward segment of the hemisphere (shaded area). In arrangement (b), polarization is retained in the backward segment of the hemisphere (shaded area). The angles $\beta$ and $\gamma$ depend on the range $R$ of nuclear recoils and they can vary from 0 to $\epsilon$ where $\epsilon = \sin^{-1}(t/R)$.

Subramanian *et al* (1976) by integrating over the angle $\theta$ from 0 to $\pi/2 - \beta$.

$$P_f = \tfrac{1}{4}\,(3\,P_{\mathrm{av}} + P_L\,P_\mu)\,x + \tfrac{1}{2}\,P_L\,(y - P_\mu\,z)\,, \tag{5}$$

where $P_\mu$ denotes the muon polarization and $x$, $y$, $z$ are given by

$$x = \int_0^{\pi/2-\beta} \sin^3\theta\, d\theta = (1 - \sin\beta) - (1/3)\,(1 - \sin^3\beta), \tag{6}$$

$$y = \int_0^{\pi/2-\beta} \cos\theta \sin\theta\, d\theta = \tfrac{1}{2}\cos^2\beta, \tag{7}$$

$$z = \int_0^{\pi/2-\beta} \cos^2\theta \sin\theta\, d\theta = (1/3)\,(1 - \sin^3\beta). \tag{8}$$

It can be easily seen from figure 1a that $\beta$ depends on the point at which the muon capture takes place. It can take place anywhere along the thickness of the target and consequently $\beta$ will vary from 0 to $\epsilon$ where $\epsilon$ is determined by the target thickness $t$ and the range $R$ of nuclear recoils,

$$\epsilon = \sin^{-1}(t/R). \tag{9}$$

Correspondingly, we have to replace $x$, $y$ and $z$ in equation (5) by their expectation values $\langle x \rangle$, $\langle y \rangle$ and $\langle z \rangle$.

$$\langle x \rangle = \frac{1}{\epsilon}\int_0^{\epsilon} x\, d\beta = (2/3) + (\cos 3\epsilon + 27\cos\epsilon - 28)/36\,\epsilon, \tag{10}$$

$$\langle y \rangle = \frac{1}{\epsilon}\int_0^{\epsilon} y\, d\beta = (1 + \sin 2\epsilon/2\epsilon)\,/4, \tag{11}$$

$$\langle z \rangle = \frac{1}{\epsilon}\int_0^{\epsilon} z\, d\beta = (1/3) - (\cos 3\epsilon - 9\cos\epsilon + 8)\,/\,12\epsilon. \tag{12}$$

Now let us turn to figure 1b where the polarization is retained in the backward segment. $P_b$ is obtained by integrating over the angle $\theta$ from $\pi/2 + \gamma$ to $\pi$.

$$P_b = \tfrac{1}{4}\,(3\,P_{\mathrm{av}} + P_L\,P_\mu)\,x' + \tfrac{1}{2}\,P_L\,(y' - P_\mu\,z'), \tag{13}$$

where $$x' = \int_{\pi/2+\gamma}^{\pi} \sin^3\theta \, d\theta = (1-\sin\gamma) - (1/3)(1-\sin^3\gamma), \tag{14}$$

$$y' = \int_{\pi/2+\gamma}^{\pi} \cos\theta \sin\theta \, d\theta = -\tfrac{1}{2}\cos^2\gamma, \tag{15}$$

$$z' = \int_{\pi/2+\gamma}^{\pi} \cos^2\theta \sin\theta \, d\theta = (1/3)(1-\sin^3\gamma). \tag{16}$$

As discussed earlier, the muon capture can take place at any position along the target thickness and hence one has to replace $x'$, $y'$ and $z'$ in (13) by their expectation values $\langle x'\rangle$, $\langle y'\rangle$ and $\langle z'\rangle$.

$$\langle x'\rangle = \frac{1}{\epsilon}\int_0^{\epsilon} x' \, d\gamma = \tfrac{2}{3} + (\cos 3\epsilon + 27\cos\epsilon - 28)/36\epsilon, \tag{17}$$

$$\langle y'\rangle = \frac{1}{\epsilon}\int_0^{\epsilon} y' \, d\gamma = -\tfrac{1}{4}\left(1+\frac{\sin 2\epsilon}{2\epsilon}\right), \tag{18}$$

$$\langle z'\rangle = \frac{1}{\epsilon}\int_0^{\epsilon} z' \, d\gamma = (1/3) - (\cos 3\epsilon - 9\cos\epsilon + 8)/12\epsilon. \tag{19}$$

From (5) and (13), we obtain

$$P_f + P_b = \tfrac{1}{4}(3P_{\text{av}} + P_L P_\mu)(\langle x\rangle + \langle x'\rangle) - \tfrac{1}{2}P_L P_\mu(\langle z\rangle + \langle z'\rangle). \tag{20}$$

Substituting the values of $\langle x\rangle$, $\langle x'\rangle$, $\langle z\rangle$ and $\langle z'\rangle$,

we obtain

$$P_f + P_b = P_{\text{av}} + \frac{1}{24\epsilon}\left\{P_{\text{av}}(\cos 3\epsilon + 27\cos\epsilon - 28) + \frac{P_L P_\mu}{3}(7\cos 3\epsilon - 27\cos\epsilon + 20)\right\}. \tag{21}$$

It can be seen that as $\epsilon \to 0$, the quantity within the curly bracket vanishes and we retrieve the result (3) obtained in the case of a thin target. The effect of the target thickness is felt when

$$P_f + P_b \neq P_{\text{av}}. \tag{22}$$

This inequality can be taken as a test to find whether the target thickness has any effect on the experimental measurement. Also

$$P_f - P_b = (1/4)\, P_L \left(1 + \frac{\sin 2\epsilon}{2\epsilon}\right). \tag{23}$$

In the limit $\epsilon \to 0$, we once again retrieve the old result (4).

In conclusion, we wish to reiterate that the above results (21) and (23) are exact and involve no approximations and they can be used to calculate $P_{av}$ and $P_L$ taking into account the effect of target thickness and the finite range of nuclear recoils. Although, in this paper, we have considered the specific muon capture reaction $^{12}C(\mu^-, \nu)\, ^{12}B(1^+)$, the results obtained are more general and true for any muon capture reaction by spin zero target nucleus leading to a well-defined final nuclear state with spin $J_f$.

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