

Rotational excitation of H₂ and N₂ molecules by electron impact

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Abstract. The rotational excitation of the H₂ and N₂ molecules from ($J \rightarrow J+2$, $J=0$) and ($J \rightarrow J+2$, $J=1$) states by electron impact is studied, in the framework of the regional-plane-wave approximation. The empirical cut-off parameter ρ , involved in these calculations, is varied to give the best fit to the experimental data of Srivastava and co-workers.

Keywords. Rotational excitation; homonuclear diatomic molecule; regional-plane-wave approximation; electron impact.

1. Introduction

The study of rotational excitation, in homonuclear diatomic molecules, is important for modelling several different types of plasmas, electron-excitation laser systems, earth's ionosphere and the Jovian atmosphere. It also plays a dominant role in the energy loss mechanism for low-energy electron-molecule scattering processes and has been the subject of considerable theoretical and some experimental investigations (Linder and Schmidt 1969; Takayanagi and Itikawa 1970; Golden *et al* 1971; Takayanagi 1972, 1974; Garrett 1972; Joyez *et al* 1973; Srivastava *et al* 1975; Burke and Williams 1977; Itikawa 1978; Collins and Norcross 1978; Truhlar *et al* 1979; Lane 1980). Most of the theoretical and experimental electron-scattering studies of rotational excitation have been confined to incident energies less than 12 eV. Rotational excitation cross-sections by electrons at intermediate energies were in general assumed to be insignificant relative to the elastic scattering cross sections. A recent experiment (Srivastava *et al* 1975), in which hydrogen molecules were used as the target, has shown that the rotational excitation cross-sections are not insignificant, particularly at large scattering angles. Therefore, the study of rotational excitation of diatomic molecule by intermediate energy electrons is of interest.

Henry and Lane (1969) have used the procedure of Arthurs and Dalgarno (1960), as generalized by Ardill and Davison (1968), and included exchange and polarization effects to study the electron H₂ molecule rotational excitation and elastic scattering and found that these effects are important for energies less than 10 eV. Truhlar and Brandt (1976) performed two-state close-coupling calculations for electron scattering by H₂ molecule. They studied the variation in the form of exchange and polarization potentials and found that such variation in the nature of exchange and polarization potentials had a small effect on the cross-sections.

Recently Bhattacharyya and Ghosh (1976) used the eikonal approach in the framework of the adiabatic approximation to obtain the rotational excitations of homo-

nuclear diatomic molecules by 40 eV electrons. They obtained a qualitative agreement with the recent experimental data (Srivastava *et al* 1975).

In this paper, we study the rotational excitation of H_2 and N_2 molecules by electrons within the framework of the regional-plane-wave approximation (RPWA). The RPWA has been applied previously to molecules having a permanent dipole moment (Rudge 1974). The molecules studied were CsCl, CsF and KI with dipole moment $D > 3.0$ a.u. The energy region studied was 0.5 to 10.0 eV. Here we have extended the above study to the quadrupolar molecules (H_2 and N_2). To compare with the available data, the energy range studied by us is ≥ 40 eV. At these energies we expect some penetration of the electron in the molecular region. The value of the cut-off parameter ρ , which gives the best agreement with the data for H_2 molecule is found to be 0.55 a.u.

2. Theory

Let \mathbf{r} denote the position vector of the scattering electron relative to the centre of mass of the molecule and \mathbf{R} be a vector along the molecular axis. We suppose that in the collision process, the electronic and vibrational states of the molecule are unaltered and therefore we suppress reference to them. The molecule is treated as a rigid rotator. If (J, M_J) are the rotational quantum numbers in the initial state and $(J', M_{J'})$ in the final state then the scattering amplitude for a collision in which the target molecule is excited rotationally from the initial state J to a final state J' , in the Born approximation, is given by (Takayanagi and Itikawa 1970),

$$f(J M_J; J' M_{J'} | K_i K_f) = -\frac{1}{2\pi} \int_0^\infty \exp(i\mathbf{q} \cdot \mathbf{r}) V(\mathbf{r} \cdot \hat{R}) Y_{JM_J}(\hat{R}) Y_{J'M_{J'}}^*(\hat{R}) d\mathbf{r} d\hat{R}, \quad (1)$$

where \mathbf{K}_i and \mathbf{K}_f are the initial and final momenta and $\mathbf{q}(= \mathbf{K}_i - \mathbf{K}_f)$ is the momentum transfer vector. $Y(\hat{R})$'s are the rotational wavefunctions and $V(\mathbf{r}, \hat{R})$ is the electron molecule interaction potential.

The interaction potential $V(\mathbf{r}, \hat{R})$ may be expanded in terms of the Legendre polynomial as

$$V(\mathbf{r}, \hat{R}) = \sum_{L=0}^{\infty} V_L(r) P_L(\cos \theta), \quad (2)$$

and is usually approximated by the first few terms. θ is the angle between the vectors \hat{r} and \hat{R} .

Several model potentials have been used to approximate the interaction potential of equation (2). We write for $V(\mathbf{r}, \hat{R})$ an effective potential of the following form

$$V(\mathbf{r}, \hat{R}) = V_s(\mathbf{r}, \hat{R}) + V_p(\mathbf{r}, \hat{R}), \quad (3)$$

where $V_s(\mathbf{r}, \hat{R})$ is the static potential and $V_p(\mathbf{r}, \hat{R})$ is the polarization potential.

In the above, we have neglected exchange, as for the energies considered (≥ 40 eV) in the present paper, its contribution would be small.

Using the molecular wavefunctions of Hara (1967), the static potential $V_s(\mathbf{r}, \hat{R})$ is given by (Bhattacharyya and Ghosh 1975)

$$V_s(\mathbf{r}, \hat{R}) = V_s^{(0)}(r) + [V_s^{(2)}(r) + V_q^{(2)}(r)]P_2(\hat{r} \cdot \hat{R}), \quad (4)$$

$$\begin{aligned} \text{with } V_s^{(2)}(r) &= -5.01637 \left(\frac{1}{r^3}\right) [e^{tr} - e^{-tr}] + 11.69816 \left(\frac{1}{r^2}\right) [e^{tr} + e^{-tr}] \\ &\quad - 10.27985 \left(\frac{1}{r}\right) [e^{tr} - e^{-tr}] + 2.76688 [e^{tr} + e^{-tr}], \quad r \leq Re/2 \\ &= -0.96504 \left(\frac{1}{r^3}\right) e^{-tr} - 2.25047 \left(\frac{1}{r^2}\right) e^{-tr} - 2.81971 \left(\frac{1}{r}\right) e^{-tr} \\ &\quad - 2.49604 e^{-tr}, \quad \geq Re/2, \end{aligned} \quad (5)$$

$$\text{and } V_q^{(2)}(r) = -Q/r^3, \quad (6)$$

where Q is the quadrupole moment of the molecule, $t = 2.332 a_0^{-1}$ and Re is the internuclear separation of the molecule. The contribution of $V_s^{(0)}(r)$ to equation (1) would be zero due to the orthogonality of the wavefunction.

Since rotational excitation can only be affected by anisotropic potentials, we use for $V_p(\mathbf{r}, \hat{R})$, in the second term of equation (3), the anisotropic polarization potential used by Dalgarno and Moffet (1963)

$$V_p(\mathbf{r}, \hat{R}) = V_p(r) P_2(\hat{r} \cdot \hat{R}), \quad (7)$$

$$\text{with } V_p(r) = -\frac{\alpha'}{2r^4}, \quad (8)$$

where α' is the anisotropic polarizability of the molecule. Thus, the total effective potential which contributes to the rotational excitation is

$$V(\mathbf{r}, \hat{R}) = v(r) P_2(\hat{r} \cdot \hat{R}), \quad (9)$$

$$\text{where, } v(r) = v_s^{(2)}(r) + v_1(r), \quad (10)$$

$$\text{with } v_1(r) = v_a^{(2)}(r) + V_p(r). \quad (11)$$

In the Born approximation [equation (1)], one assumes the motion of the incident electron to be described by plane waves in the entire region of space. When the incident electron is within the molecule, this assumption is not correct. An accurate representation of the wavefunction in the inner region would require a treatment which takes into account the complex structure of the molecule. In the study of the rotational excitation of the molecules having permanent moments, the cross-section is usually dominated by the long range interaction, and it is possible that the cross-sections are not strongly dependent upon the detailed nature of the wavefunction in the molecular region. In the regional-plane-wave approximation (Rudge 1974; Rudge *et al* 1976), one assumes the incident electron to be represented by plane wave in the region ($r > \rho$) and nothing in the region ($r < \rho$), where ρ is a cut-off parameter. One usually relates ρ to the equilibrium separation Re . We therefore define $\rho (= \gamma Re)$ where γ is a variable parameter.

The trial wavefunctions, representing the initial and final states of the total system, for the electron molecule collision process are chosen to be

$$\begin{aligned} \Psi_i(\mathbf{r}, \hat{R}) &= Y_{JM_J}(\hat{R}) \exp(i\mathbf{K}_i \cdot \mathbf{r}), & r > \rho \\ &= 0, & r < \rho \end{aligned} \quad (12)$$

$$\begin{aligned} \Psi_f^*(\mathbf{r}, \hat{R}) &= Y_{J'M_{J'}}^*(\hat{R}) \exp(-i\mathbf{K}_f \cdot \mathbf{r}), & r > \rho \\ &= 0. & r < \rho \end{aligned} \quad (13)$$

The scattering amplitude, in the RPWA which corresponds to the choice of functions (12) and (13), is then given by

$$\begin{aligned} f(J M_J; J' M_{J'} | K_i K_f) &= -\frac{1}{2\pi} \int_{r>\rho} \exp(i\mathbf{q} \cdot \mathbf{r}) V(\mathbf{r}, \hat{R}) Y_{JM_J}(\hat{R}) \\ &Y_{J'M_{J'}}^*(\hat{R}) d\mathbf{r} d\hat{R}. \end{aligned} \quad (14)$$

Using (9) and (14) and performing the angular integrations, we obtain

$$\begin{aligned} f(J M_J; J' M_{J'} | K_i K_f) &= \frac{8\pi}{(2l'+1)} \sum_{m'} I(\rho) (-1)^{J+M_J} \\ &\left\{ \frac{(2J+1)}{4\pi} \right\}^{1/2} C_{M_{J'} - M_J m'}^{J' J} C_{0 0 0}^{J l' J'} Y_{l' m'}^*(\hat{q}) \end{aligned} \quad (15)$$

with $l' = 2$

where C denotes a Clebsch-Gordon coefficient

$$\text{and } I(\rho) = \int_{\rho}^{\infty} r^2 v(r) j_{l'}(qr) dr, \quad (16)$$

$j_{l'}$ is a spherical Bessel function.

The integral of (16), in the Born approximation (BA) as a function of cut-off parameter ρ can be written as

$$\begin{aligned} I(\rho) = & \int_0^{\infty} r^2 V_s^{(2)}(r) j_{l'}(qr) dr + v_1(\rho) \int_0^{\rho} r^2 j_{l'}(qr) dr \\ & + \int_{\rho}^{\infty} r^2 v_1(r) j_{l'}(qr) dr, \end{aligned} \quad (17)$$

where in the region $r < \rho$, we have replaced $v_1(r)$ by $v_1(\rho)$. Using equations (15) and (16), the differential cross-section in the RPWA may be written as

$$\frac{d\sigma}{d\Omega} = \frac{K_f}{K_i(2J+1)} \sum_{M_J M_{J'}} |f|^2. \quad (18)$$

which on carrying out the summation is written as

$$\frac{d\sigma}{d\Omega} (JJ'/K_i K_f) = \frac{4K_f}{5K_i} I^2(\rho) (C_0^J \begin{smallmatrix} J & 2 & J' \\ 0 & 0 & 0 \end{smallmatrix})^2. \quad (19)$$

We have evaluated the differential cross-sections using the above equation. In our calculations, like Rudge (1974), we have chosen γ as a free parameter. For the ($J \rightarrow J+2, J=1$) excitation of the H_2 molecule, we find that for incident electron energies ≥ 40 eV, results in best agreement with the data of Srivastava *et al* (1975) are obtained with a value of γ equal to 0.55. This conclusion is obtained on the basis of data at large scattering angles.

3. Results and discussion

Figures 1 to 3 show our results for the differential cross-section, for the ($J \rightarrow J+2, J=1$) and ($J \rightarrow J+2, J=0$) excitation of the H_2 molecule for incident electron energies (≥ 40 eV). Table 1 gives the value of molecular constants used in the calculation.

Figure 1 shows our results for the ($J \rightarrow J+2, J=1$) transition at 40 eV electron energy. The curve RPWA, based on the regional-plane-wave approximation, is obtained using (19), (16) and (10); and curve BA is the corresponding result in the

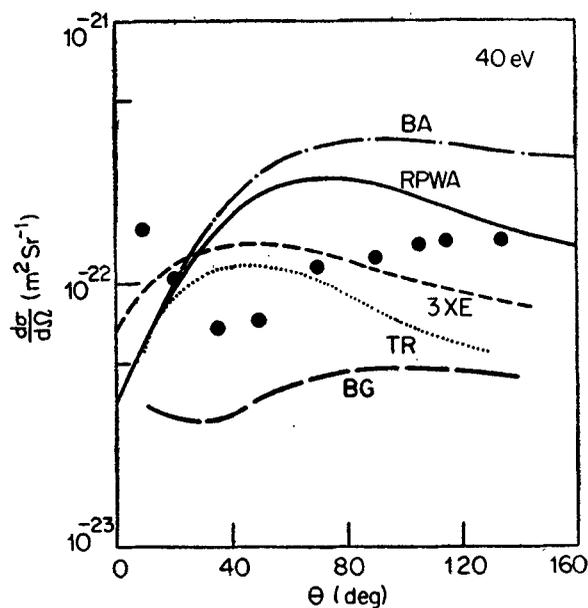


Figure 1. Differential cross-section for the $(J \rightarrow J+2, J=1)$ excitation of the H_2 molecule at incident electron energy of 40 eV.—Present calculations in the RPWA; —. —. —Present calculations in the BA; - - - - Calculations based on two-state close-coupling approximation with exchange neglected (Truhlar and Brandt 1976, curve 3 XE); Calculations of Henry and Lane at 45 eV as reported by Srivastava *et al* (1975, curve TR); — — —Calculations based on the eikonal approximation (Bhattacharyya and Ghosh 1976, curve BG); ●, experimental data (Srivastava *et al* 1975).

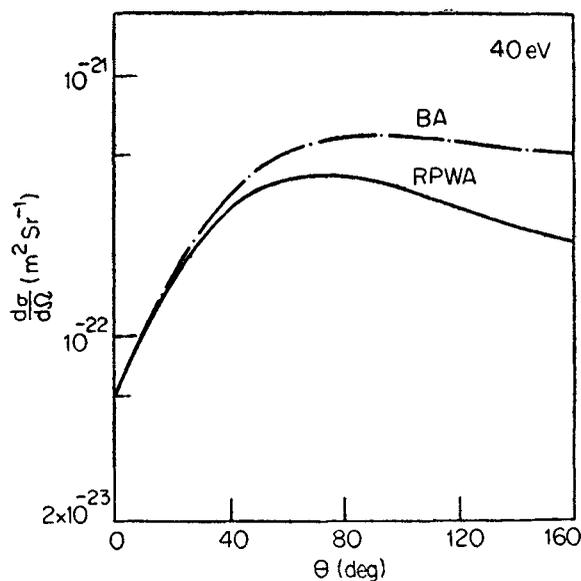


Figure 2. Differential cross-section for the $(J \rightarrow J+2, J=0)$ excitation of the H_2 molecule at incident electron energy of 40 eV. Other details as in figure 1.

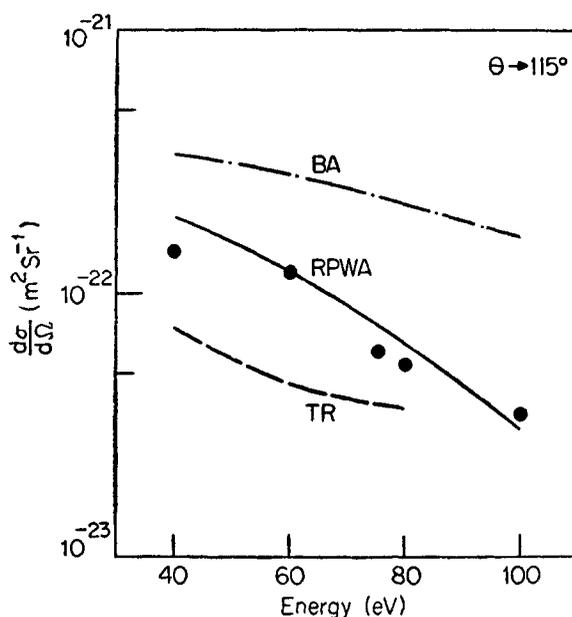


Figure 3. Differential cross section for the ($J \rightarrow J + 2$, $J = 1$) excitation of the H_2 molecule at a scattering angle of 115° and for energies from 40 eV to 100 eV. — — — TR. Other details as in figure 1.

Table 1. Molecular constants used in the calculations

Target molecule	Re (a_0)	Q (a.u.)	α' (a_0^3)
H_2	1.406 ^a	0.49 ^b	1.4 ^c
N_2	2.068 ^a	-1.1308 ^b	4.2 ^d

^aHerzberg (1950)

^bKocher *et al* (1972)

^cChang E S (1974)

^dLandolt and Börnstein (1951)

Born approximation obtained by using (19), (17), and (10). The other theoretical calculations shown are (i) the calculations of Truhlar and Brandt (1976) based on the two-state close-coupling approximation with exchange neglected (3XE) (ii) the calculations of Henry and Lane (as plotted by Srivastava *et al* (1975), Curve TR), (iii) the calculations of Bhattacharyya and Ghosh (1976) based on the eikonal approximation (BG). All the above theoretical calculations are compared with the recent experimental data of Srivastava *et al* (1975). From the figure it is evident that at large angles the quantitative difference between our calculations and the experimental data is smaller than that between other calculations and the experimental data. Our calculations, however, do not give the correct variation of the differential cross-section with scattering angles. When compared with the Born approximation, we notice that the present calculation provides a considerable improvement over it. The TR calculation disagrees with the data in the entire angular range. The calculation BG reproduces the shape of the experimental data very well and amongst

Table 2. Rotational excitation ($J \rightarrow J + 2, J = 0$) differential cross sections for electron nitrogen molecule at incident electron energy of 40 eV, in unit of ($10^{-22} \text{ m}^2/\text{Sr}$)

Angle in deg θ	$\gamma = 0.5$		$\gamma = 1.0$		$\gamma = 1.5$		$\gamma = 2.0$	
	BA	RPWA	BA	RPWA	BA	RPWA	BA	RPWA
10	1.53 (-0)	1.55 (-0)	1.58 (-0)	1.57 (-0)	1.57 (-0)	1.52 (-0)	1.52 (-0)	1.39 (-0)
20	6.00 (-1)	6.33 (-1)	7.13 (-1)	6.97 (-1)	7.01 (-1)	5.70 (-1)	5.89 (-1)	3.53 (-1)
30	1.51 (-1)	1.89 (-1)	2.88 (-1)	2.66 (-1)	2.75 (-1)	1.44 (-1)	1.70 (-1)	2.47 (-2)
40	6.19 (-3)	2.44 (-2)	1.02 (-1)	8.33 (-2)	9.38 (-2)	1.46 (-2)	3.35 (-2)	3.47 (-3)
50	2.66 (-2)	2.44 (-3)	3.12 (-2)	1.85 (-2)	2.72 (-2)	4.74 (-4)	4.14 (-3)	7.79 (-3)
60	1.19 (-1)	3.70 (-2)	7.79 (-3)	1.75 (-3)	6.75 (-3)	5.17 (-3)	4.40 (-4)	1.01 (-3)
70	2.25 (-1)	8.10 (-2)	1.46 (-3)	1.07 (-4)	1.53 (-3)	4.18 (-3)	2.04 (-4)	4.68 (-4)
80	3.16 (-1)	1.14 (-1)	1.66 (-4)	1.17 (-3)	4.07 (-4)	1.13 (-3)	3.05 (-4)	1.33 (-3)
90	3.81 (-1)	1.30 (-1)	5.40 (-6)	1.66 (-3)	1.91 (-4)	5.35 (-6)	3.46 (-4)	4.66 (-4)
100	4.20 (-1)	1.32 (-1)	1.71 (-7)	1.44 (-3)	1.64 (-4)	3.42 (-4)	2.52 (-4)	7.02 (-7)
110	4.37 (-1)	1.25 (-1)	1.83 (-7)	9.62 (-4)	1.77 (-4)	7.40 (-4)	1.23 (-4)	2.56 (-4)
120	4.38 (-1)	1.14 (-1)	6.12 (-6)	5.38 (-4)	1.88 (-4)	7.30 (-4)	4.89 (-5)	4.07 (-4)
130	4.31 (-1)	1.01 (-1)	2.09 (-5)	2.58 (-4)	1.86 (-4)	4.91 (-4)	1.12 (-5)	3.01 (-4)
140	4.19 (-1)	8.96 (-2)	3.91 (-5)	1.06 (-4)	1.73 (-4)	2.53 (-4)	9.39 (-7)	1.37 (-4)
150	4.07 (-1)	8.01 (-2)	5.52 (-5)	3.66 (-5)	1.56 (-4)	1.05 (-4)	4.89 (-7)	3.84 (-5)
160	3.97 (-1)	7.32 (-2)	6.69 (-5)	1.03 (-5)	1.42 (-4)	3.69 (-5)	4.32 (-6)	6.84 (-6)
170	3.90 (-1)	6.91 (-2)	7.36 (-5)	2.51 (-6)	1.33 (-4)	1.33 (-5)	6.71 (-6)	1.16 (-7)
180	3.88 (-1)	6.77 (-2)	7.58 (-5)	1.12 (-6)	1.30 (-4)	8.04 (-6)	9.13 (-6)	3.83 (-9)

Notation $a(b) = a \times 10^b$

Table 3. Rotational excitation ($J \rightarrow J + 2, J = 1$) differential cross sections for electron nitrogen molecule at incident electron energy of 40 eV in unit of $(10^{-18} \text{ m}^2/\text{Sr})$

Angle in deg θ	$\gamma = 0.5$			$\gamma = 1.0$			$\gamma = 1.5$			$\gamma = 2.0$		
	BA	RPWA	BA	BA	RPWA	BA	BA	RPWA	BA	RPWA	BA	
10	9.21	9.29	9.48	9.44	9.45	9.10	9.13	8.33	9.13	8.33	8.33	
20	3.60	3.80	4.28	4.17	4.21	3.42	3.53	2.12	3.53	2.12	2.12	
30	9.08 (-1)	1.13	1.73	1.60	1.65	8.63 (-1)	1.02	1.48 (-1)	1.02	1.48 (-1)	1.48 (-1)	
40	3.71 (-2)	1.46 (-1)	6.15 (-1)	5.00 (-1)	5.63 (-1)	8.76 (-2)	2.01 (-1)	2.08 (-2)	2.01 (-1)	2.08 (-2)	2.08 (-2)	
50	1.59 (-1)	1.46 (-2)	1.87 (-1)	1.11 (-1)	1.63 (-1)	2.84 (-3)	2.49 (-2)	4.68 (-2)	2.49 (-2)	4.68 (-2)	4.68 (-2)	
60	7.11 (-1)	2.22 (-1)	4.67 (-2)	1.05 (-2)	4.05 (-2)	3.10 (-2)	2.64 (-3)	6.06 (-3)	2.64 (-3)	6.06 (-3)	6.06 (-3)	
70	1.35	4.86 (-1)	8.75 (-3)	6.44 (-4)	9.19 (-3)	2.51 (-2)	1.23 (-3)	2.81 (-3)	1.23 (-3)	2.81 (-3)	2.81 (-3)	
80	1.90	6.82 (-1)	9.99 (-4)	7.02 (-3)	2.44 (-3)	6.80 (-3)	1.83 (-3)	8.00 (-3)	1.83 (-3)	8.00 (-3)	8.00 (-3)	
90	2.29	7.80 (-1)	3.25 (-5)	9.95 (-3)	1.15 (-3)	3.21 (-5)	2.07 (-3)	2.80 (-3)	2.07 (-3)	2.80 (-3)	2.80 (-3)	
100	2.52	7.94 (-1)	1.02 (-6)	8.62 (-3)	9.81 (-4)	2.05 (-3)	1.52 (-3)	3.77 (-6)	1.52 (-3)	3.77 (-6)	3.77 (-6)	
110	2.62	7.52 (-1)	1.09 (-6)	5.77 (-3)	1.06 (-3)	4.44 (-3)	7.50 (-4)	1.52 (-3)	7.50 (-4)	1.52 (-3)	1.52 (-3)	
120	2.63	6.83 (-1)	3.67 (-5)	3.23 (-3)	1.13 (-3)	4.38 (-3)	2.95 (-4)	2.42 (-3)	2.95 (-4)	2.42 (-3)	2.42 (-3)	
130	2.58	6.07 (-1)	1.25 (-4)	1.55 (-3)	1.11 (-3)	2.94 (-3)	7.21 (-5)	1.75 (-3)	7.21 (-5)	1.75 (-3)	1.75 (-3)	
140	2.52	5.37 (-1)	2.34 (-4)	6.36 (-4)	1.03 (-3)	1.51 (-3)	7.23 (-6)	8.42 (-4)	7.23 (-6)	8.42 (-4)	8.42 (-4)	
150	2.44	4.80 (-1)	3.31 (-4)	2.20 (-4)	9.50 (-4)	6.40 (-4)	6.65 (-6)	2.51 (-4)	6.65 (-6)	2.51 (-4)	2.51 (-4)	
160	2.38	4.39 (-1)	4.01 (-4)	6.15 (-5)	8.46 (-4)	2.22 (-4)	1.98 (-5)	4.81 (-5)	1.98 (-5)	4.81 (-5)	4.81 (-5)	
170	2.34	4.14 (-1)	4.42 (-4)	1.51 (-5)	7.98 (-4)	7.98 (-5)	4.77 (-5)	1.62 (-6)	4.77 (-5)	1.62 (-6)	1.62 (-6)	
180	2.33	4.06 (-1)	4.54 (-4)	6.78 (-6)	7.85 (-4)	4.80 (-5)	4.70 (-5)	3.16 (-9)	4.70 (-5)	3.16 (-9)	3.16 (-9)	

 Notation $a(b) = a \times 10^b$

all calculations gives the best qualitative agreement with the data. However, its quantitative agreement with the data of this calculation is very poor. We notice that none of the calculations agree with the data at lower angles.

Figure 2 shows our results at 40 eV of the ($J \rightarrow J + 2, J = 0$) transition. There is no data to compare with the above calculations at present.

Figure 3 shows our results for the differential cross-section for ($J \rightarrow J + 2, J = 1$) transition at an angle of 115° and for energies from 40 to 100 eV. From the figure, we note that our results show a good agreement with the experimental data of Srivastava *et al* (1975) in the entire energy range. The TR calculation underestimates the cross-section in the whole region whereas the Born approximation yields higher values of the cross-sections.

In tables 2 and 3, we give our results based on the regional-plane-wave approximation (RPWA) and the Born approximation (BA) (without $V_s^{(2)}(r)$ potential in (10)) for the ($J \rightarrow J + 2, J = 0$) and ($J \rightarrow J + 2, J = 1$) transitions of the N_2 molecule at the different values of γ . There is no data to compare with the above results.

The major contribution to the rotational excitation comes from the long range interactions. In the case of N_2 molecule, the quadrupole moment is higher than H_2 and so the long range contribution will increase. Also the short range $e - N_2$ static interaction will be stronger and highly asymmetric. It has been shown by Geltman and Takayanagi (1966) that for molecules for which the quadrupole moment is small (*e.g.* O_2) the short range interaction contribution is very important and significant in certain energy regions. In the case of N_2 , they have obtained little change in the cross-section by the inclusion of short range forces. We do not expect that the RPWA will provide a more reasonable model for the N_2 molecule than the H_2 molecule.

It is concluded that in the range of energies studied in this paper, the regional-plane-wave approximation leads to a considerable improvement over the usual Born approximation at large scattering angles. Predictions become better as the energy increases. Further we note that a good agreement requires varying ρ to give a fit to the experimental data.

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