Pramāṇa, Vol. 16, No. 6, June 1981, pp. 493–510. © Printed in India.

# First and second order hadron mass formulas in internal SU(6)

C P SINGH*
Department of Physics, Panjab University, Chandigarh 160 014, India.
*Permanent address: Department of Physics, VSSD College, Kanpur, India.

MS received 17 January 1981; revised 18 April 1981

**Abstract.** We use broken SU(6) internal symmetry to derive the mass formulas amongst hadrons ($1/2^+$, $3/2^+$, $0^-$, $1^-$) including second order mass contributions from symmetric 405 representation. Some hybrid mass relations are also obtained by relating second order parameters.

**Keywords.** Hadron masses; internal SU(6) symmetry; second order effects.

## 1. Introduction

In the last few years, high energy experiments have provided us with far-reaching developments. On the one side, the discovery of the $J/\psi$ particles (Aubert *et al* 1974; Augustin *et al* 1974) and of charmed mesons (Goldhaber *et al* 1976) and baryons (Cazzoli *et al* 1975; Knapp *et al* 1976) has confirmed the theoretical model of charm (Glashow *et al* 1970). On the other side, the upsilon family $\Upsilon$, $\Upsilon'$ (Herb *et al* 1977; Berger *et al* 1978) with masses 9·46 and 10·06 GeV discovered in the reaction,

$$p + (\mathrm{Cu}, \mathrm{Pt}) \rightarrow \mu^+ + \mu^- + \text{anything},$$

could not be accommodated within the SU(4) framework which compelled people to think beyond charm. The new particles, like $J/\psi$ as $c\bar{c}$, are interpreted as bound states ($b\bar{b}$) of a new quark flavour $b$ (beauty), which suggest that a rich spectrum of new heavier particles might exist. Recently, Thorndike (1980) has reported that $\Upsilon(4S)$ is seen at CESR with mass 10·55 GeV and has a large width, suggesting that the mass of the $b$-quark meson may be between $5{\cdot}16 \leqslant D_b \leqslant 5{\cdot}28$ GeV, further confirming the existence of the $b$-quark hadrons. Next, the discovery of a new heavy lepton $\tau$(Perl *et al* 1975) accompanied by its own neutrino $\nu_\tau$ (Perl *et al* 1977) supports the expectation of a sixth quark $t$ (taste) from the quark-lepton symmetry considerations (Harari 1975) and the unified gauge theories (Kobayashi and Maskawa 1973). Though there is no experimental evidence for the $t$ quark hadrons until now, it is hoped that with improved experimental techniques at PETRA (Wiik 1980) the theoretical predictions for the quark $t$ will become true. Many authors (Boal 1978; Aubrecht and Scott 1979; Camiz *et al* 1979; Misra and Sastry 1979; Singh *et al* 1980; Khanna 1980; Singh 1981) have therefore started theoretical studies on their properties in different frameworks.

In this paper we consider the masses of the *b* and *t* quark hadrons in the internal SU(6) symmetry. Since the higher internal symmetries are badly broken due to the large mass differences amongst the quarks, contributions from the higher order perturbation terms may be significant. The higher order effects on the masses of charmed hadrons (Verma and Khanna 1978; Singh *et al* 1980) have already been studied and found to be important. In § 3 we derive the first order mass relations and then incorporate the second order effects which relate the discrepancies present in the first order mass relations. Higher order symmetry breaking parameters are also evaluated in terms of the masses of particles. § 4 deals with the discussion of the hybrid mass relations between the baryons and mesons.

## 2. Preliminaries

Since baryons are supposed to be made up of three quarks, they are conveniently described by Young diagrams with three boxes generated in the direct product

$$6 \otimes 6 \otimes 6 = 56_s \otimes 20_A \otimes 70_M \otimes 70_M. \tag{1}$$

**Table 1.** $J^P = 1/2^+$ baryons

| SU (6) | SU (5) | SU (4) | SU (3) | States |
|---|---|---|---|---|
| $70_M$ | $40_M$ | $20_M$ | 8 | $(N\, \Sigma\, \Lambda\, \Xi)$ |
| | | | $6_c$ | $(\Sigma_c\, \Xi_c\, \Omega_c)$ |
| | | | $\bar{3}_c$ | $(\Lambda'_c\, \Xi'_c)$ |
| | | | $3_{cc}$ | $(\Xi_{cc}\, \Omega_{cc})$ |
| | | $6_A$ | $\bar{3}_b$ | $(\Lambda'_b\, \Xi'_b)$ |
| | | | $\bar{3}_{cb}$ | $(\Xi'_{cb}\, \Omega'_{cb})$ |
| | | $10_S$ | $6_b$ | $(\Sigma_b\, \Xi_b\, \Omega_b)$ |
| | | | $3_{cb}$ | $(\Xi_{cb}\, \Omega_{cb})$ |
| | | | $1_{ccb}$ | $(\Omega_{ccb})$ |
| | | 4 | $3_{bb}$ | $(\Xi_{bb}\, \Omega_{bb})$ |
| | | | $1_{cbb}$ | $(\Omega_{cbb})$ |
| | $10_A$ | $6_A$ | $\bar{3}_t$ | $(\Lambda'_t\, \Xi'_t)$ |
| | | | $\bar{3}_{ct}$ | $(\Omega'_{ct}\, \Xi'_{ct})$ |
| | | 4 | $\bar{3}_{bt}$ | $(\Xi'_{bt}\, \Omega'_{bt})$ |
| | | | $1_{cbt}$ | $(\Omega'_{cbt})$ |
| | $15_S$ | $10_S$ | $6_t$ | $(\Sigma_t\, \Xi_t\, \Omega_t)$ |
| | | | $3_{ct}$ | $(\Xi_{ct}\, \Omega_{ct})$ |
| | | | $1_{cct}$ | $(\Omega_{cct})$ |
| | | 4 | $3_{bt}$ | $(\Xi_{bt}\, \Omega_{bt})$ |
| | | | $1_{cbt}$ | $(\Omega_{cbt})$ |
| | | 1 | $1_{bbt}$ | $(\Omega_{bbt})$ |
| | 5 | 4 | $3_{tt}$ | $(\Xi_{tt}\, \Omega_{tt})$ |
| | | | $1_{ctt}$ | $(\Omega_{ctt})$ |
| | | 1 | $1_{btt}$ | $(\Omega_{btt})$ |

$J^P = 1/2^+$ baryons are assigned in the $70_M$ representation and can be represented by the wave function

$$B_{[\alpha\beta]\,\delta}, \text{ satisfying}$$

$$B_{[\alpha\beta]\,\delta} = -\,B_{[\beta\alpha]\,\delta},$$

and $$B_{[\alpha\beta]\,\delta} + B_{[\beta\delta]\,\alpha} + B_{[\delta\alpha]\,\beta} = 0. \tag{2}$$

The SU(3) decompositions of these is given in table 1. The totally symmetric representation $56_S$ contain $3/2^+$ isobars and represented by the symmetric wave function $D_{\alpha\beta\delta}$. The indices $\alpha$, $\beta$, $\delta$ refer to the flavour of the quarks and run from 1 to 6 representing $u$, $d$, $s$, $c$, $b$, $t$ quarks. The SU(3) decomposition of the $3/2^+$ baryons is given in table 2.

**Table 2.** $J^P = 3/2^+$ baryons

| SU (6) | SU (5) | SU (4) | SU (3) | States |
|---|---|---|---|---|
| $56_S$ | $35_S$ | $20_S$ | 10 | $(\Delta\ \Sigma^*\ \Xi^*\ \Omega^*)$ |
| | | | $6_c$ | $(\Sigma^*_c\ \Xi^*_c\ \Omega^*_c)$ |
| | | | $3_{cc}$ | $(\Xi^*_{cc}\ \Omega^*_{cc})$ |
| | | | $1_{ccc}$ | $(\Omega^*_{ccc})$ |
| | | 10 | $6_b$ | $(\Sigma^*_b\ \Xi^*_b\ \Omega^*_b)$ |
| | | | $3_{cb}$ | $(\Xi^*_{cb}\ \Omega^*_{cb})$ |
| | | | $1_{ccb}$ | $(\Omega^*_{ccb})$ |
| | | 4 | $3_{bb}$ | $(\Xi^*_{bb}\ \Omega^*_{bb})$ |
| | | | $1_{cbb}$ | $(\Omega^*_{cbb})$ |
| | | 1 | $1_{bbb}$ | $(\Omega^*_{bbb})$ |
| | $15_S$ | $10_S$ | $6_t$ | $(\Sigma^*_t\ \Xi^*_t\ \Omega^*_t)$ |
| | | | $3_{ct}$ | $(\Xi^*_{ct}\ \Omega^*_{ct})$ |
| | | | $1_{cct}$ | $(\Omega^*_{cct})$ |
| | | 4 | $3_{bt}$ | $(\Omega^*_{bt}\ \Xi^*_{bt})$ |
| | | | $1_{cbt}$ | $(\Omega^*_{cbt})$ |
| | | 1 | $1_{bbt}$ | $(\Omega^*_{bbt})$ |
| | 5 | 4 | $3_{tt}$ | $(\Xi^*_{tt}\ \Omega^*_{tt})$ |
| | | | $1_{ctt}$ | $(\Omega^*_{ctt})$ |
| | | 1 | $1_{btt}$ | $(\Omega^*_{btt})$ |
| | 1 | 1 | $1_{ttt}$ | $(\Omega^*_{ttt})$ |

**Table 3.** Pseudoscalar mesons

| SU (6) | SU (5) | SU (4) | SU (3) | States |
|---|---|---|---|---|
| 1: | 1 | 1 | 1 | $[P_0]$ |
| 35: | 5 | 4 | 3 | $[D_t^+, D_t^0, F_t^+]$ |
| | | | 1 | $[G_t^0]$ |
| | | 1 | 1 | $[H_t^+]$ |
| | $\bar{5}$ | 1 | 1 | $[H_t^-]$ |
| | | $\bar{4}$ | 1 | $[\bar{G}_t^0]$ |
| | | | $\bar{3}$ | $[D_t^-, \bar{D}_t^0, F_t^-]$ |
| | 1: | 1 | 1 | $[P_{35}]$ |
| | 24: | 4 | 3 | $[D_b^-, D_b^0, F_b^0]$ |
| | | | 1 | $[G_b^-]$ |
| | | $\bar{4}$ | 1 | $[G_b^+]$ |
| | | | $\bar{3}$ | $[D_b^+, \bar{D}_b^0, \bar{F}_b^0]$ |
| | | 1: | 1 | $[P_{24}]$ |
| | | 15: | 3 | $[D_c^+, D_c^0, F_c^+]$ |
| | | | $\bar{3}$ | $[D_c^-, \bar{D}_c^0, F_c^-]$ |
| | | | 1: | $[P_{15}]$ |
| | | | 8: | $[K^-, K^0]$ |
| | | | | $[K^+, \bar{K}^0]$ |
| | | | | $[P_8]$ |
| | | | | $[\pi^+, \pi^0, \pi^-]$ |

There will be 36 mesons generated in the direct product

$$6 \otimes \bar{6} = 1 \oplus 35, \tag{3}$$

which are represented in table 3. The tensor representations for $1/2^+$, $3/2^+$ baryons, and mesons are gvien in appendix 1, 2 and 3 respectively.

To break the SU(6) symmetry, we extend the idea originally introduced by Gell-Mann (1962) and Okubo (1962) for SU(3) and later employed by Mathur *et al* (1975) for SU(4). We divide the Hamiltonian into two parts

$$H = H_0 + H_{\text{S.B.}} \tag{4}$$

where $H_0$ is invariant under SU(6) transformations and $H_{\text{S.B.}}$ breaks the symmetry. The first order mass breaking operator (excluding electromagnetism) is taken to transform like

$$T_a^\beta \sim a\, T_3^3 + b\, T_4^4 + c\, T_5^5 + d\, T_6^6, \tag{5}$$

component of 35. The second order mass-breaking Hamiltonian then would have an SU(6) transformation property dictated by the direct product

$$35 \otimes 35 = 1 \oplus 35 \oplus 35' \oplus 189 \oplus 280 \oplus \overline{280} \oplus 405. \tag{6}$$

The representation 35 has already been taken in the first order breaking (equation (5)). We wish to consider the effects of totally symmetric 405 representation as second order mass contributions (Machacek and Tomozawa 1976). The general mass operator in SU(6) can be written as

$$\begin{aligned} &\sim m_0 + a^{D/F} T_3^3 + b^{D/F} T_4^4 + c^{D/F} T_5^5 + d^{D/F} T_6^6, \\ &+ eT^{(33)}_{(33)} + fT^{(44)}_{(44)} + gT^{(55)}_{(55)} + hT^{(66)}_{(66)}, \\ &+ iT^{(34)}_{(34)} + jT^{(35)}_{(35)} + kT^{(36)}_{(36)} + lT^{(45)}_{(45)}, \\ &+ mT^{(46)}_{(46)} + nT^{(56)}_{(56)}. \end{aligned} \tag{7}$$

## 3. Hadron mass relations

In this section we first derive the first order mass relations. The second order contributions which relate the discrepancies present in the first order mass relations are then included. We use the particle symbol to denote its mass.

### 3.1 $J^P = 1/2^+$ *baryons*

The first and the second order mass contributions are obtained from the contraction

$$(\tfrac{1}{2}\bar{B}^{[\delta\eta]\alpha} B_{[\delta\eta]\beta} \pm \bar{B}^{[\delta\alpha]\eta} B_{[\delta\beta]\eta})\, T^{\beta}_{\alpha}, \tag{8}$$

and

$$(\bar{B}^{[\alpha m]\beta} B_{[\delta m]\eta})\, T^{(\delta\eta)}_{(\alpha\beta)}, \tag{9}$$

respectively, where $T^{\beta}_{\alpha}$ and $T^{(\delta\eta)}_{(\alpha\beta)}$ represent the first and the second order mass breaking spurion.

A. First order mass relations are

$$3\Lambda + \Sigma = 2\,\Xi + 2\,N$$

$$(4539{\cdot}3 \text{ MeV}) \qquad (4508{\cdot}9 \text{ MeV}), \tag{10}$$

$$3\Lambda_c + \Sigma_c = 2\,\Xi_{cc} + 2N, \tag{11}$$

$$3\Lambda_b + \Sigma_b = 2\,\Xi_{bb} + 2N, \tag{12}$$

$$3\Lambda_t + \Sigma_t = 2\,\Xi_{tt} + 2N, \tag{13}$$

$$(\Omega_{tt} - \Xi_{tt}) = (\Omega_{bb} - \Xi_{bb}) = (\Omega_{cc} - \Xi_{cc}) = (\Sigma - N)\,(252{\cdot}9\text{ MeV}), \tag{14}$$

$$(\Omega_{bt} - \Xi_{bt}) = (\Omega_{ct} - \Xi_{ct}) = (\Xi_t - \Sigma_t) = (\Omega_t - \Xi_t)$$
$$= (\Omega_{cb} - \Xi_{cb}) = (\Omega_b - \Xi_b) = (\Xi_b - \Sigma_b) = (\Omega_c - \Xi_c)$$
$$= (\Xi_c - \Sigma_c) = (\tfrac{1}{2}\,(\Xi - N)\,(187{\cdot}7\text{ MeV}), \tag{15}$$

$$(\Omega_{ctt} - \Xi_{tt}) = (\Omega_{cbb} - \Xi_{bb}) = (\Sigma_c - N)\,(1560{\cdot}5\text{ MeV}), \tag{16}$$

$$(\Omega_{btt} - \Xi_{tt}) = (\Sigma_b - N) \tag{17}$$

$$(\Omega_{cbt} - \Xi_{bt}) = (\,\Xi_{ct} - \Sigma_t) = (\Omega_{cct} - \Xi_{ct}) = (\Omega_{ccb} - \Xi_{cb})$$
$$= (\Xi_{cb} - \Sigma_b) = \tfrac{1}{2}\,(\Xi_{cc} - N), \tag{18}$$

$$(\Omega_{bbt} - \Xi_{bt}) = (\Xi_{bt} - \Sigma_t) = \tfrac{1}{2}\,(\,\Xi_{bb} - N), \tag{19}$$

$$2\Xi_{cc} + \Xi_c - 3\Xi'_c = 2(\Sigma_c - \Sigma)\,(2615\text{ MeV}), \tag{20}$$

$$2\Xi_{bb} + \Xi_b - 3\Xi'_b = 2(\Sigma_b - \Sigma), \tag{21}$$

$$2\Xi_{bb} + \Xi_{cb} - 3\Xi'_{cb} = 2(\Sigma_b - \Sigma_c), \tag{22}$$

$$2\Xi_{tt} + \Xi_t - 3\Xi'_t = 2(\Sigma_t - \Sigma), \tag{23}$$

$$2\Xi_{tt} + \Xi_{ct} - 3\Xi'_{ct} = 2(\Sigma_t - \Sigma_c), \tag{24}$$

$$2\Xi_{tt} + \Xi_{bt} - 3\Xi'_{bt} = 2(\Sigma_t - \Sigma_b), \tag{25}$$

$$\Omega_{cb} - 3\Omega'_{cb} + 2\Xi_{bb} + 2(\Sigma + \Sigma_c - \Sigma_b) = 0, \tag{26}$$

$$\Omega_{ct} - 3\Omega'_{ct} + 2\Xi_{bb} + 2(\Sigma + \Sigma_c - \Sigma_b) = 0, \tag{27}$$

$$\Omega_{bt} - 3\Omega'_{bt} + 2\Xi_{tt} + 2(\Sigma + \Sigma_b - \Sigma_t) = 0, \tag{28}$$

$$\Omega_{cbt} - 3\Omega'_{cbt} + 2\Xi_{tt} + 2(\Sigma_c + \Sigma_b - \Sigma_t) = 0 \tag{29}$$

The mass relations (10) to (13) are the Gell-Mann-Okubo mass formulas in different quark sectors.

B. Inclusion of the second order effects give following mass relations and the values of the higher order parameters:

$$4(\Xi_{cc} - \Omega_{cc}) - 3(\,\Lambda'_c - \Xi'_c) = (\Xi_c - \Sigma_c) - 2(\Sigma - N), \tag{30}$$

$$4(\Xi_{bb} - \Omega_{bb}) - 3(\Lambda'_b - \Xi'_b) = (\Xi_b - \Sigma_b) - 2(\Sigma - N), \tag{31}$$

$$4(\Xi_{bb} - \Omega_{cbb}) - 3(\Lambda_b' - \Xi_b') = (\Xi_{cb} - \Sigma_b) - 2(\Sigma_c - N), \tag{32}$$

$$4(\Xi_{tt} - \Omega_{tt}) - 3(\Lambda_t' - \Xi_t') = (\Xi_t - \Sigma_t) - 2(\Sigma - N), \tag{33}$$

$$4(\Xi_{tt} - \Omega_{ctt}) - 3(\Lambda_t' - \Xi_t') = (\Xi_{ct} - \Sigma_t) - 2(\Sigma_c - N), \tag{34}$$

$$4(\Xi_{tt} - \Omega_{btt}) - 3(\Lambda_t' - \Xi_t') = (\Xi_{bt} - \Sigma_t) - 2(\Sigma_b - N) \tag{35}$$

$$3(\Lambda_b' - \Omega_{cb}') + (\Omega_{cb} - \Sigma_b)$$
$$= 4(2\Xi_{bb} - \Omega_{bb} - \Omega_{cbb}) + 2(\Omega_{cc} + \Sigma_c - \Xi_{cc} - 2N) \tag{36}$$

$$3(\Lambda'^t - \Omega_{ct}') + (\Omega_{ct} - \Sigma_t)$$
$$= 4(2\Xi_{tt} - \Omega_{tt} - \Omega_{ctt}) + 2(\Omega_{cc} + \Sigma_c - \Xi_{cc} - 2N) \tag{37}$$

$$3\,(\Lambda_t' - \Omega_{bt}') + (\Omega_{bt} - \Sigma_t)$$
$$= 4\,(2\,\Xi_{tt} - \Omega_{tt} - \Omega_{btt}) + 2\,(\Omega_{bb} + \Sigma_b - \Xi_{bb} - 2\,N) \tag{38}$$

$$3\,(\Lambda_t' - \Omega_{cbt}') + (\Omega_{cbt} - \Sigma_t)$$
$$= 4\,(2\,\Xi_{tt} - \Omega_{ctt} - \Omega_{btt}) + 2\,(\,\Omega_{cbb} + \Sigma_b - \Xi_{bb} - 2\,N) \tag{39}$$

$$e_{1/2} = (\Omega_t + \Sigma_t - 2\,\Xi_t) = (\Omega_b + \Sigma_b - 2\,\Xi_b) = (\Omega_c + \Sigma_c - 2\,\Xi_c)$$
$$= \tfrac{1}{2}\,[2\,\Xi + 2\,N - \Sigma - 3\,\Lambda]\;(-\,15{\cdot}15\text{ MeV}) \tag{40}$$

$$f_{1/2} = (\Omega_{cct} + \Sigma_t - 2\,\Xi_{ct}) = (\Omega_{ccb} + \Sigma_b - 2\,\Xi_{cb}) \tag{41}$$
$$= \tfrac{1}{2}\,[\,2\,\Xi_{cc} + 2\,N - \Sigma_c + -\,3\,\Lambda_c]$$

$$g_{1/2} = (\Omega_{bbt} + \Sigma_t - 2\,\Xi_{bt}) = \tfrac{1}{2}\,[2\,\Xi_{bb} + 2\,N - \Sigma_b - 3\Lambda_b], \tag{42}$$

$$h_{1/2} = \tfrac{1}{2}\,[2\,\Xi_{tt} + 2\,N - \Sigma_t - 3\,\Lambda_t], \tag{43}$$

$$i_{1/2} = (\Omega_{cc} - \Xi_{cc}) - (\Sigma - N), \tag{44}$$

$$j_{1/2} = (\Omega_{bb} - \Xi_{bb}) - (\Sigma - N), \tag{45}$$

$$k_{1/2} = (\Omega_{tt} - \Xi_{tt}) - (\Sigma - N), \tag{46}$$

$$l_{1/2} = (\Omega_{cbb} - \Xi_{bb}) - (\Sigma_c - N), \tag{47}$$

$$m_{1/2} = (\Omega_{ctt} - \Xi_{tt}) - (\Sigma_c - N), \tag{48}$$

$$n_{1/2} = (\Omega_{btt} - \Xi_{tt}) - (\Sigma_b - N). \tag{49}$$

### 3.2 $J^P = 3/2^+$ baryons

The first order mass contributions come from the contraction

$$(\bar{D}^{\alpha mn} D_{\beta mn}) T^{\beta}_{\alpha}, \quad (50)$$

whereas the second order from

$$(\bar{D}^{\alpha\beta m} D_{\delta\eta m}) T^{(\delta\eta)}_{(\alpha\beta)}. \quad (51)$$

A. The first order contribution gives the equal spacing rules as follows:

$$(\Sigma^* - \Delta) = (\Xi^* - \Sigma^*) = (\Omega^* - \Xi^*) = (\Xi^*_c - \Sigma^*_c)$$

(152 MeV) (149 MeV) (139 MeV)

$$= (\Omega^*_c - \Xi^*_c) = (\Omega^*_{cc} - \Xi^*_{cc}) = (\Xi^*_b - \Sigma^*_b) = (\Omega^*_b - \Xi^*_b)$$

$$= (\Omega^*_{cb} - \Xi^*_{cb}) = (\Omega^*_{bb} - \Xi^*_{bb}) = (\Xi^*_t - \Sigma^*_t) = (\Omega^*_t - \Xi^*_t)$$

$$= (\Omega^*_{ct} - \Xi^*_{ct}) = (\Omega^*_{tt} - \Xi^*_{tt}) = (\Omega^*_{bt} - \Xi^*_{bt}) \quad (52)$$

$$(\Omega^*_c - \Omega^*) = (\Omega^*_{cc} - \Omega^*_c) = (\Omega^*_{cb} - \Omega^*_b) = (\Omega^*_{ccb} - \Omega^*_{cb})$$

$$= (\Omega^*_{cbb} - \Omega^*_{bb}) = (\Omega^*_{ct} - \Omega^*_t) = (\Omega^*_{ctt} - \Omega^*_{tt}) = (\Omega^*_{cbt} - \Omega^*_{bt})$$

$$(\Sigma^*_c - \Delta) = (\Omega^*_{ccc} - \Xi^*_{cc}) = (\Xi^*_{cc} - \Sigma^*_c) \quad (53)$$

$$= (\Xi^*_{cb} - \overset{*}{\Sigma}_b) = (\Xi^*_{ct} - \Sigma^*_t)$$

$$(\Sigma^*_b - \Delta) = (\Xi^*_{bb} - \Sigma^*_b) = (\Omega^*_{bb} - \Xi^*_b) \quad (54)$$

$$= (\Xi^*_{bt} - \Sigma^*_t)$$

$$(\Sigma^*_t - \Delta) = (\Omega^*_{tt} - \Xi^*_t) = (\Omega^*_{ctt} - \Xi^*_{cf}) \quad (55)$$

$$= (\Omega^*_{btt} - \Xi^*_{bt}) = (\Omega^*_{ttt} - \Xi^*_{tt}) \quad (56)$$

$$(\Sigma^*_t - \Sigma^*) = (\Omega^*_{tt} - \Omega^*_t) = (\Omega^*_{ctt} - \Omega^*_{ct})$$

$$= (\Omega^*_{ttt} - \Omega^*_{tt}) = (\Omega^*_{btt} - \Omega^*_{bt}) \quad (57)$$

$$(\Sigma^*_t - \Sigma^*_b) = (\Omega^*_{btt} - \Omega^*_{bbt}) = (\Omega^*_{ctt} - \Omega^*_{cbt}) \quad (58)$$

5. **Second order effects relates the discrepancies in the equal spacing rules and give**

$$3(\Xi^* - \Sigma^*) = (\Omega^* - \Delta) \qquad (59)$$
(447 MeV) (443 MeV)

$$3\,(\Xi^*_{cc} - \Sigma^*_c) = (\Omega^*_{ccc} - \Delta) \qquad (60)$$

$$3\,(\Xi^*_{bb} - \Sigma^*_b) = (\Omega^*_{bbb} - \Delta) \qquad (61)$$

$$3\,(\Xi^*_{tt} - \Sigma^*_t) = (\Omega^*_{ttt} - \Delta), \qquad (62)$$

$$(\Omega^*_{ccc} - \Omega^*) = 3\,(\Omega^*_{cc} - \Omega^*_c), \qquad (63)$$

$$(\Omega^*_{bbb} - \Omega^*) = 3\,(\Omega^*_{bb} - \Omega^*_b), \qquad (64)$$

$$(\Omega^*_{ttt} - \Omega^*) = 3\,(\Omega^*_{tt} - \Omega^*_t), \qquad (65)$$

$$(\Omega^*_{bbb} - \Omega^*_{ccc}) = 3\,(\Omega^*_{cbb} - \Omega^*_{ccb}), \qquad (66)$$

$$(\Omega^*_{ttt} - \Omega^*_{ccc}) = 3\,(\Omega^*_{ctt} - \Omega^*_{cct}), \qquad (67)$$

$$(\Omega^*_{ttt} - \Omega^*_{bbb}) = 3\,(\Omega^*_{btt} - \Omega^*_{bbt}), \qquad (68)$$

$$\tfrac{1}{3}\, e_{3/2} = \Omega^*_t + \Sigma^*_t - 2\,\Xi^*_t = \Omega^*_b + \Sigma^*_b - 2\,\Xi^*_b$$
$$= \Omega^*_c + \Sigma^*_c - 2\,\Xi^*_c = \Omega^* + \Sigma^* - 2\,\Xi^* \text{ (3 MeV)}, \qquad (69)$$

$$\tfrac{1}{3} f_{3/2} = \Omega^*_{cct} + \Sigma^*_t - 2\Xi^*_{ct} = \Omega^*_{ccb} + \Sigma^*_b - 2\Xi^*_{cb}$$
$$= \Omega^*_{ccc} + \Sigma^*_c - 2\Xi^*_{cc} = \Omega^*_{cc} + \Sigma^* - 2\Xi^*_c\,, \qquad (70)$$

$$\tfrac{1}{3}\, g_{3/2} = \Omega^*_{bbt} + \Sigma^*_t - 2\Xi^*_{bt} = \Omega^*_{bbb} + \Sigma^*_b - 2\Xi^*_{bb}, \qquad (71)$$

$$\tfrac{1}{3}\, h_{3/2} = \Omega^*_{fft} + \Sigma^*_t - 2\Xi^*_{tt} = \Omega^*_{btt} + \Sigma^*_b - 2\Xi^*_{bt} \qquad (72)$$

$$\tfrac{1}{6} i_{3/2} = (\Xi^*_c - \Sigma^*_c) - (\Sigma^* - \Delta) = (\Omega^*_c - \Xi^*_c) - (\Xi^* - \Sigma^*), \qquad (73)$$

$$\tfrac{1}{6}\, j_{3/2} = (\Xi^*_b - \Sigma^*_b) - (\Sigma^* - \Delta) = (\Omega^*_b - \Xi^*_b) - (\Xi^* - \Sigma^*), \qquad (74)$$

$$\tfrac{1}{6} k_{3/2} = (\Xi^*_t - \Sigma^*_t) - (\Sigma^* - \Delta) = (\Omega^*_t - \Xi^*_t) - (\Xi^* - \Sigma^*), \qquad (75)$$

$$\tfrac{1}{6} l_{3/2} = (\Xi^*_{cb} - \Sigma^*_b) - (\Sigma^*_c - \Delta) = (\Omega^*_{cbb} - \Xi^*_{cb}) - (\Xi^*_{bb} - \Sigma^*_b) \qquad (76)$$

$$\tfrac{1}{6}m_{3/2} = (\Xi^*_{ct} - \Sigma^*_t) - (\Sigma^*_c - \Delta) = (\Omega^*_{ctt} - \Xi^*_{ct}) - (\Xi^*_{tt} - \Sigma^*_t) \tag{77}$$

$$\tfrac{1}{6}n_{3/2} = (\Xi^*_{bt} - \Sigma^*_t) - (\Sigma^*_b - \Delta) = (\Omega^*_{btt} - \Xi^*_{bt}) - (\Xi^*_{tt} - \Sigma^*_t) \tag{78}$$

### 3.3 *Mesons*

Now there are 36 mesons ($0^-$ and $1^-$ both) belonging to the $35 \oplus 1$ irreducible representation of SU(6). The first order mass contribution to these comes from the contraction

$$(\bar{P}^a_\delta P^\delta_\beta + \bar{P}^\delta_\beta P^a_\delta)\, T^\beta_a, \tag{79}$$

whereas the second order contribution comes from

$$(\bar{P}^\delta_a P^\eta_\beta)\, T^{(a\,\beta)}_{(\delta\,\eta)}. \tag{80}$$

A. The following relations are obtained in the first order considerations.

$$(F_t - D_t) = (F_b - D_b) = (F_c - D_c) \quad = (K - \pi) \tag{81}$$

(174·5 MeV) (357 MeV)

(677·6 MeV²) (226·6 MeV²)

$$(G_t - D_t) = (G_b - D_b) = (F_c - K) = (D_c - \pi) \tag{82}$$

(1533 MeV) (1733 MeV)

$$3G_t - 2D_t - F_t = 3G_b - 2D_b - F_b = F_c + 2D_c - \pi - 2K \tag{83}$$

$$4H_t - (G_t + F_t + 2D_t)$$

$$= (G_b + F_b + 2D_b) - (F_c + D_c + K + \pi) \tag{84}$$

$$3(P_8 - \pi) \quad = \quad 4(K - \pi) \tag{85}$$

(1230 MeV) (1432 MeV)

2 2

(849 MeV) (917 MeV)

$$6(P_{15} - \pi) = (K - \pi) + 9(D_c - \pi) \tag{86}$$

$$10(P_{24} - \pi) = (K - \pi) + (D_c - \pi) + 16\,(D_b - \pi) \tag{87}$$

$$15(P_{35} - \pi) = (K - \pi) + (D_c - \pi) + (D_b - \pi) + 25\,(D_t - \pi) \tag{88}$$

$$3(P_0 - \pi) = (K - \pi) + (D_c - \pi) + (D_b - \pi) + (D_t - \pi) \tag{89}$$

The available mass values do not seem to satisfy these relations in the linear as well as in the quadratic form, indicating a large second order mass contributions.

B. The second order effects will give the following values of higher order parameters

$$i_0 = (F_c - D_c) - (K - \pi)$$

$$(-183 \text{ MeV}) \tag{90}$$

$$j_0 = (F_b - D_b) - (K - \pi), \tag{91}$$

$$k_0 = (F_t - D_t) - (K - \pi), \tag{92}$$

$$l_0 = (G_b - D_b) - (D_c - \pi), \tag{93}$$

$$m_0 = (G_t - D_t) - (D_c - \pi), \tag{94}$$

$$n_0 = (H_t - D_t) - (D_b - \pi). \tag{95}$$

Similar relations follow for vector mesons by replacing corresponding vector particles for each pseudo-scalar mesons.

As the mass of the one $b$-quark meson ($D_b$) is now known (Thorndike 1980), the masses of the others can be estimated and they come out to be

$D_b$ : $\underline{5{\cdot}25}$ GeV,

$F_b$ : $(5.61 + j)$ GeV,

$G_b$ : $(6.97 + l)$ GeV,

where $j$ and $l$ are the higher order parameters.

## 4. Hybrid mass formulas

Many authors (Eliezer and Singer 1973; Verma and Khanna 1978; Singh *et al* 1980) have studied the hybrid mass relations among the baryons and mesons upto SU(4), using different considerations. Assuming the universality of the ratio of higher order parameters (which are obtained in terms of the masses of the particles in §§ 3.1, 3.2 and 3.3 for $1/2^+$, $3/2^+$ and mesons respectively) various hybrid mass formulas can be obtained, such as

$$\frac{(\Omega_{bb} - \Xi_{bb}) - (\Sigma - N)}{(\Omega_{btt} - \Xi_{tt}) - (\Sigma_b - N)} = \frac{(\Xi_b^* - \Sigma_b^*) - (\Sigma^* - N)}{(\Xi_{bt}^* - \Sigma_t^*) - (\Sigma_b^* - \Delta)}$$

$$= \frac{(F_b - D_b) - (K - \pi)}{(H_t - D_t) - (D_b - \pi)} \tag{96}$$

## 5. Conclusions

In this paper we have extended the SU(3) broken symmetry scheme of Gell-Mann and Okubo to an SU(6) broken internal symmetry. First we have derived the first order mass relations and then by incorporating the second order effects, the discrepancies present are related. The SU(6) symmetry will be badly broken so that mass contribution from the higher order will be significant as it is seen in charm sector (Verma and Khanna 1978; Singh *et al* 1980). We see that for $1/2^{+}$ baryon the analogous Gell Mann-Okubo mass formulas are found for all the sectors in the first order but the second order effects do not maintain these relations. For $3/2^{+}$ baryons the first order gives equal spacing rules which are not maintained under the second order considerations. At present because of the nonavailability of the experimental values of the masses of heavier hadrons our relation cannot be tested. Our studies give the definite test for the strength and pattern of symmetry-breaking mechanism when the masses are available.


## Acknowledgements

The author is grateful to Dr M P Khanna and Dr R C Verma for helpful discussions and acknowledges the financial assistance from the University Grants Commission, New Delhi.

## Appendix 1. Tensor representation for 1/2⁺ baryons

$$B_{[12]1} = p$$
$$B_{[13]1} = \Sigma^+$$
$$B_{[14]1} = \Sigma_c^{++}$$
$$B_{[15]1} = \Sigma_b^+$$
$$B_{[16]1} = \Sigma_t^{++}$$
$$B_{[23]1} = \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0$$
$$B_{[24]1} = \frac{1}{\sqrt{6}}\Lambda_c'^+ + \frac{1}{\sqrt{2}}\Sigma_c^+$$
$$B_{[25]1} = \frac{1}{\sqrt{6}}\Lambda_b'^0 + \frac{1}{\sqrt{2}}\Sigma_b^0$$
$$B_{[26]1} = \frac{1}{\sqrt{6}}\Lambda_t'^+ + \frac{1}{\sqrt{2}}\Sigma_t^+$$
$$B_{[34]1} = \frac{1}{\sqrt{6}}\Xi_c'^+ + \frac{1}{\sqrt{2}}\Xi_c^+$$
$$B_{[35]1} = \frac{1}{\sqrt{6}}\Xi_b'^0 + \frac{1}{\sqrt{2}}\Xi_b^0$$

$$B_{[12]2} = n$$
$$B_{[13]2} = -\left(\frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0\right)$$
$$B_{[14]2} = -\left(\frac{1}{\sqrt{6}}\Lambda_c'^+ - \frac{1}{\sqrt{2}}\Sigma_c^+\right)$$
$$B_{[15]2} = -\left(\frac{1}{\sqrt{6}}\Lambda_b'^0 - \frac{1}{\sqrt{2}}\Sigma_b^0\right)$$
$$B_{[16]2} = -\left(\frac{1}{\sqrt{6}}\Lambda_t'^+ - \frac{1}{\sqrt{2}}\Sigma_t^+\right)$$
$$B_{[23]2} = \Sigma^-$$
$$B_{[24]2} = \Sigma_c^0$$
$$B_{[25]2} = \Sigma_b^-$$
$$B_{[26]2} = \Sigma_t^0$$
$$B_{[34]2} = \frac{1}{\sqrt{6}}\Xi_c'^0 + \frac{1}{\sqrt{2}}\Xi_c^0$$
$$B_{[35]2} = \frac{1}{\sqrt{6}}\Xi_b'^- + \frac{1}{\sqrt{2}}\Xi_b^-$$

$$B_{[12]3} = -\sqrt{2/3}\,\Lambda^0$$
$$B_{[13]3} = \Xi^\circ$$
$$B_{[14]3} = -\left(\frac{1}{\sqrt{6}}\Xi_c'^+ - \frac{1}{\sqrt{2}}\Xi_c^+\right)$$
$$B_{[15]3} = -\left(\frac{1}{\sqrt{6}}\Xi_b'^0 - \frac{1}{\sqrt{2}}\Xi_b^0\right)$$
$$B_{[16]3} = -\left(\frac{1}{\sqrt{6}}\Xi_t'^+ - \frac{1}{\sqrt{2}}\Xi_t^+\right)$$
$$B_{[23]3} = \Xi^-$$
$$B_{[24]3} = -\left(\frac{1}{\sqrt{6}}\Xi_c'^0 - \frac{1}{\sqrt{2}}\Xi_c^0\right)$$
$$B_{[25]3} = -\left(\frac{1}{\sqrt{6}}\Xi_b'^- - \frac{1}{\sqrt{2}}\Xi_b^-\right)$$
$$B_{[26]3} = -\left(\frac{1}{\sqrt{6}}\Xi_t'^0 - \frac{1}{\sqrt{2}}\Xi_t^0\right)$$
$$B_{[34]3} = \Omega_c^0$$
$$B_{[35]3} = \Omega_b^-$$

$$B_{[36]1} = \frac{1}{\sqrt{6}}\Xi_t'^{+} + \frac{1}{\sqrt{2}}\Xi_t^{+}$$

$$B_{[45]1} = \frac{1}{\sqrt{6}}\Xi_{cb}'^{+} + \frac{1}{\sqrt{2}}\Xi_{cb}^{+}$$

$$B_{[46]1} = \frac{1}{\sqrt{6}}\Xi_{ct}'^{++} + \frac{1}{\sqrt{2}}\Xi_{ct}^{++}$$

$$B_{[56]1} = \frac{1}{\sqrt{6}}\Xi_{bt}'^{+} + \frac{1}{\sqrt{2}}\Xi_{bt}^{+}$$

$$B_{[12]4} = -\sqrt{2/3}\,\Lambda_c'^{+}$$

$$B_{[13]4} = \sqrt{2/3}\,\Xi_c'^{+}$$

$$B_{[14]4} = \Xi_{cc}^{++}$$

$$B_{[15]4} = -\left(\frac{1}{\sqrt{6}}\Xi_{cb}'^{+} - \frac{1}{\sqrt{2}}\Xi_{cb}^{+}\right)$$

$$B_{[16]4} = -\left(\frac{1}{\sqrt{6}}\Xi_{ct}'^{++} - \frac{1}{\sqrt{2}}\Xi_{ct}^{++}\right)$$

$$B_{[23]4} = -\sqrt{2/3}\,\Xi_c'^{0}$$

$$B_{[24]4} = \Xi_{cc}^{+}$$

$$B_{[25]4} = -\left(\frac{1}{\sqrt{6}}\Xi_{cb}'^{0} - \frac{1}{\sqrt{2}}\Xi_{cb}\right)$$

$$B_{[36]2} = \frac{1}{\sqrt{6}}\Xi_t'^{0} + \frac{1}{\sqrt{2}}\Xi_t^{0}$$

$$B_{[45]2} = \frac{1}{\sqrt{6}}\Xi_{cb}'^{0} + \frac{1}{\sqrt{2}}\Xi_{cb}^{0}$$

$$B_{[46]2} = \frac{1}{\sqrt{6}}\Xi_{ct}'^{+} + \frac{1}{\sqrt{2}}\Xi_{ct}^{+}$$

$$B_{[56]2} = \frac{1}{\sqrt{6}}\Xi_{bt}'^{0} + \frac{1}{\sqrt{2}}\Xi_{bt}^{0}$$

$$B_{[12]5} = -\sqrt{2/3}\,\Lambda_b'^{0}$$

$$B_{[13]5} = -\sqrt{2/3}\,\Xi_b'^{0}$$

$$B_{[14]5} = -\sqrt{2/3}\,\Xi_{cb}'^{+}$$

$$B_{[15]5} = \Xi_{bb}^{0}$$

$$B_{[16]5} = -\left(\frac{1}{\sqrt{6}}\Xi_{bt}'^{+} - \frac{1}{\sqrt{2}}\Xi_{bt}^{+}\right)$$

$$B_{[23]5} = -\sqrt{2/3}\,\Xi_b'^{-}$$

$$B_{[24]5} = -\sqrt{2/3}\,\Xi_{cb}'^{0}$$

$$B_{[25]5} = \Xi_{bb}^{-}$$

$$B_{[36]3} = \Omega_t^{0}$$

$$B_{[45]3} = \frac{1}{\sqrt{6}}\Omega_{cb}'^{0} + \frac{1}{\sqrt{2}}\Omega_{cb}^{+}$$

$$B_{[46]3} = \frac{1}{\sqrt{6}}\Omega_{ct}'^{0} + \frac{1}{\sqrt{2}}\Omega_{ct}^{0}$$

$$B_{[56]3} = \frac{1}{\sqrt{6}}\Omega_{bt}'^{0} + \frac{1}{\sqrt{2}}\Omega_{bt}^{0}$$

$$B_{[12]6} = -\sqrt{2/3}\,\Lambda_t'^{+}$$

$$B_{[13]6} = -\sqrt{2/3}\,\Xi_t'^{+}$$

$$B_{[14]6} = -\sqrt{2/3}\,\Xi_{ct}'^{++}$$

$$B_{[15]6} = -\sqrt{2/3}\,\Xi_{bt}'^{+}$$

$$B_{[16]6} = \Xi_{tt}^{++}$$

$$B_{[23]6} = -\sqrt{2/3}\,\Xi_t'^{0}$$

$$B_{[24]6} = -\sqrt{2/3}\,\Xi_{ct}'^{+}$$

$$B_{[25]6} = -\sqrt{2/3}\,\Xi_{bt}'^{0}$$

$$B_{[26]4} = -\left(\frac{1}{\sqrt{6}}\,\Xi'^{+}_{ct} - \frac{1}{\sqrt{2}}\,\Xi^{+}_{ct}\right)$$

$$B_{[34]4} = \Omega^{+}_{cc}$$

$$B_{[35]4} = -\left(\frac{1}{\sqrt{6}}\,\Omega'^{0}_{bc} - \frac{1}{\sqrt{2}}\,\Omega^{0}_{cb}\right)$$

$$B_{[36]4} = -\left(\frac{1}{\sqrt{6}}\,\Omega'^{+}_{ct} - \frac{1}{\sqrt{2}}\,\Omega^{+}_{ct}\right)$$

$$B_{[45]4} = \Omega^{+}_{ccb}$$

$$B_{[46]4} = \Omega^{++}_{cct}$$

$$B_{[56]4} = \left(\frac{1}{\sqrt{6}}\,\Omega'^{+}_{cbt} + \frac{1}{\sqrt{2}}\,\Omega^{+}_{cbt}\right)$$

$$B_{[26]5} = -\left(\frac{1}{\sqrt{6}}\,\Xi'^{0}_{bt} - \frac{1}{\sqrt{2}}\,\Xi^{0}_{bt}\right)$$

$$B_{[34]5} = -\sqrt{2/3}\;\Omega'^{0}_{cb}$$

$$B_{[35]5} = \Omega^{-}_{bb}$$

$$B_{[36]5} = -\left(\frac{1}{\sqrt{6}}\,\Omega'^{0}_{bt} - \frac{1}{\sqrt{2}}\,\Omega^{0}_{bt}\right)$$

$$B_{[45]5} = \Omega^{0}_{cbb}$$

$$B_{[46]5} = -\left(\frac{1}{\sqrt{6}}\,\Omega'^{+}_{cbt} - \frac{1}{\sqrt{2}}\,\Omega^{+}_{cbt}\right)$$

$$B_{[56]5} = \Omega^{0}_{bbt}$$

$$B_{[26]6} = \Xi^{+}_{tt}$$

$$B_{[34]6} = -\sqrt{2/3}\;\Omega'^{+}_{ct}$$

$$B_{[35]6} = -\sqrt{2/3}\;\Omega^{0\prime}_{bt}$$

$$B_{[36]6} = \Omega^{+}_{tt}$$

$$B_{[45]6} = -\sqrt{2/3}\;\Omega'^{+}_{cbt}$$

$$B_{[46]6} = \Omega^{++}_{ctt}$$

$$B_{[56]6} = \Omega^{+}_{btt}$$

**Appendix 2. Tensor representation for 3/2⁺ baryons**

$$D_{111} = \Delta^{++} \qquad D_{112} = \frac{1}{\sqrt{3}}\Delta^{+} \qquad D_{122} = \frac{1}{\sqrt{3}}\Delta^{0} \qquad D_{222} = \Delta^{-}$$

$$D_{113} = \frac{1}{\sqrt{3}}\Sigma^{*+} \qquad D_{123} = \frac{1}{\sqrt{6}}\Sigma^{*0} \qquad D_{223} = \frac{1}{\sqrt{3}}\Sigma^{*-} \qquad D_{133} = \frac{1}{\sqrt{3}}\Xi^{*0}$$

$$D_{233} = \frac{1}{\sqrt{3}}\Xi^{*-} \qquad D_{333} = \Omega^{*-} \qquad D_{114} = \frac{1}{\sqrt{3}}\Sigma_c^{*++} \qquad D_{124} = \frac{1}{\sqrt{6}}\Sigma_c^{*+}$$

$$D_{224} = \frac{1}{\sqrt{3}}\Sigma_c^{*0} \qquad D_{134} = \frac{1}{\sqrt{6}}\Xi_c^{*+} \qquad D_{234} = \frac{1}{\sqrt{6}}\Xi_c^{*0} \qquad D_{334} = \frac{1}{\sqrt{3}}\Omega_c^{*0}$$

$$D_{144} = \frac{1}{\sqrt{3}}\Xi_{cc}^{*++} \qquad D_{244} = \frac{1}{\sqrt{3}}\Xi_{cc}^{*+} \qquad D_{344} = \frac{1}{\sqrt{3}}\Omega_{cc}^{*+} \qquad D_{444} = \Omega_{ccc}^{*++}$$

$$D_{115} = \frac{1}{\sqrt{3}}\Sigma_b^{*+} \qquad D_{125} = \frac{1}{\sqrt{6}}\Sigma_b^{*0} \qquad D_{225} = \frac{1}{\sqrt{3}}\Sigma_b^{*-} \qquad D_{135} = \frac{1}{\sqrt{6}}\Xi_{sb}^{*0}$$

$$D_{235} = \frac{1}{\sqrt{6}}\Xi_{sb}^{*-} \qquad D_{335} = \frac{1}{\sqrt{3}}\Omega_{ssb}^{*-} \qquad D_{145} = \frac{1}{\sqrt{6}}\Xi_{cb}^{*+} \qquad D_{245} = \frac{1}{\sqrt{6}}\Xi_{cb}^{*0}$$

$$D_{345} = \frac{1}{\sqrt{6}}\Omega_{scb}^{*0} \qquad D_{445} = \frac{1}{\sqrt{3}}\Omega_{ccb}^{*+} \qquad D_{155} = \frac{1}{\sqrt{3}}\Xi_{bb}^{*0} \qquad D_{255} = \frac{1}{\sqrt{3}}\Xi_{bb}^{*-}$$

$$D_{355} = \frac{1}{\sqrt{3}}\Omega_{sbb}^{*-} \qquad D_{455} = \frac{1}{\sqrt{3}}\Omega_{cbb}^{*0} \qquad D_{555} = \Omega_{bbb}^{*-} \qquad D_{116} = \frac{1}{\sqrt{3}}\Sigma_t^{*++}$$

$$D_{126} = \frac{1}{\sqrt{6}}\Sigma_t^{*+} \qquad D_{226} = \frac{1}{\sqrt{3}}\Sigma_t^{*0} \qquad D_{136} = \frac{1}{\sqrt{6}}\Xi_{st}^{*+} \qquad D_{236} = \frac{1}{\sqrt{6}}\Xi_{st}^{*0}$$

$$D_{336} = \frac{1}{\sqrt{3}}\Omega_{sst}^{*0} \qquad D_{146} = \frac{1}{\sqrt{6}}\Xi_{ct}^{*++} \qquad D_{246} = \frac{1}{\sqrt{6}}\Xi_{ct}^{*+} \qquad D_{346} = \frac{1}{\sqrt{6}}\Omega_{sct}^{*+}$$

$$D_{446} = \frac{1}{\sqrt{3}}\Omega_{cct}^{*++} \qquad D_{156} = \frac{1}{\sqrt{6}}\Xi_{bt}^{*+} \qquad D_{256} = \frac{1}{\sqrt{6}}\Xi_{bt}^{*0} \qquad D_{356} = \frac{1}{\sqrt{6}}\Omega_{sbt}^{*0}$$

$$D_{456} = \frac{1}{\sqrt{6}}\Omega_{cbt}^{*+} \qquad D_{556} = \frac{1}{\sqrt{3}}\Omega_{bbt}^{*0} \qquad D_{166} = \frac{1}{\sqrt{3}}\Xi_{tt}^{*++} \qquad D_{266} = \frac{1}{\sqrt{3}}\omega_{tt}^{*+}$$

$$D_{366} = \frac{1}{\sqrt{3}}\Omega_{stt}^{*+} \qquad D_{466} = \frac{1}{\sqrt{3}}\Omega_{ctt}^{*++} \qquad D_{566} = \frac{1}{\sqrt{3}}\Omega_{btt}^{*+} \qquad D_{666} = \Omega_{ttt}^{*++}$$

**Appendix 3. Tensor representations for $J^p = 0^-$ mesons ($P^a_b$)**

$$P^a_b = \begin{vmatrix} P^1_1 & \pi^+ & K^+ & D^0_c & D^+_b & D^0_t \\ \pi^- & P^2_2 & K^0 & D^-_c & D^0_b & D^-_t \\ K^- & \overline{K^0} & P^3_3 & F^-_c & F^0_b & F^-_t \\ \overline{D}^0_c & D^+_c & F^+_c & P^4_4 & G^+_b & G^0_t \\ D^-_b & \overline{D}^0_b & \overline{F}^0_b & G^-_b & P^5_5 & H^-_t \\ \overline{D}^0_t & D^+_t & F^+_t & \overline{G}^0_t & H^+_t & P^6_6 \end{vmatrix}$$

Where

$$P^1_1 = P_3/\sqrt{2} + P_8/\sqrt{6} + P_{15}/\sqrt{12} + P_{24}/\sqrt{20} + P_{35}/\sqrt{30} + P_0/\sqrt{6}$$

$$P^2_2 = -P_3/\sqrt{2} + P_8/\sqrt{6} + P_{15}/\sqrt{12} + P_{24}/\sqrt{20} + P_{35}/\sqrt{30} + P_0/\sqrt{6}$$

$$P^3_3 = -2P_8/\sqrt{6} + P_{15}/\sqrt{12} + P_{24}/\sqrt{20} + P_{35}/\sqrt{30} + P_0/\sqrt{6}$$

$$P^4_4 = -\sqrt{3}/2\, P_{15} + P_{24}/\sqrt{20} + P_{35}/\sqrt{30} + P_0/\sqrt{6}$$

$$P^5_5 = -4/\sqrt{20}\, P_{24} + P_{35}/\sqrt{30} + P_0/\sqrt{6}$$

$$P^6_6 = -5/\sqrt{30}\, P_{35} + P_0/\sqrt{6}$$

For vector mesons ($1^-$) similar representations can be obtained.

## References


Aubert J J *et al* 1974 *Phys. Rev. Lett.* **33** 1404

Aubrecht II G J and Scott D M 1979 SU(6) as an internal symmetry: Meson mass spectra and decay widths, Ohio State Univ. Preprint No. COO-1545-240.

Aubrecht II G J and Scott D M 1979 *Nuovo Cimento* **A50** 241

Augustin J *et al* 1974 *Phys. Rev. Lett.* **33** 1406

Berger Ch *et al* 1978 *Phys. Lett* **B76** 243

Boal D H 1978 *Phys. Rev.* **D18** 3446

Camiz P, Dattoli G, and Mignani R 1979 *Nuovo Cimento* **26** 15

Cazzoli E G *et al* 1975 *Phys. Rev. Lett.* **34** 1125

Eliezer S and Singer P 1973 *Phys. Rev.* **D8** 2235

Gell-Mann M 1962 *Phys. Rev.* **125** 1067

Glashow S L, Illiopoulos J and Maiani L 1970 *Phys. Rev.* **D2** 1285

Goldhaber G *et al* 1976 *Phys. Rev. Lett.* **37** 255

Harari H 1975 *Proc. Int. Symp. on Lepton and Photon interactions at higher energies*, Stanford.

Herb S W *et al* 1977 *Phys. Rev. Lett.* **39** 252
Khanna M P 1980 *Lett. Nuovo Cimento* **28** 201
Knapp B *et al* 1976 *Phys. Rev. Lett.* **37** 882
Kobayashi M and Maskawa K 1973 *Prog. Theor. Phys.* **49** 652
Machacek M and Tomozawa Y 1976 *J. Math. Phys.* **17** 458 and ref. therein
Mathur V S, Okubo S and Borchardt S 1975 *Phys. Rev.* **D11** 2572
Misra D and Sastry C V 1979 *Pramana* **13** 163
Okubo S 1962 *Prog. Theor. Phys.* **27** 949
Perl M L *et al* 1975 *Phys. Rev. Lett.* **35** 1489
Perl M L *et al* 1977 *Phys. Lett.* **B70** 487
Singh C P, Verma R C and Khanna M P 1980 *Phys. Rev.* **D21** 1388
Singh C P, Kanwar S and Khanna M P 1980 *Pramana* **14** 433
Singh C P 1981 *Phys. Rev.* **D23** (to appear)
Thorndike E H 1980 *Proc. Int. Conf. on HEP*, Madison, Wisconsin
Verma R C and Khanna M P 1978 *Phys. Rev.* **D18** 828
Wilk B H 1980 DESY-80/129