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Masses of charmed and *b*-quark hadrons in quasinuclear coloured quark model

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Abstract. Employing non-relativistic quasinuclear coloured quark model, which provides a unified description of mesons and baryons, masses of charmed and *b*-quark hadrons are studied. Various mass splittings are estimated, and mass relations among them are obtained.

Keywords. Hadron masses; charmed hadrons; *b*-quark hadrons; quark model.

1. Introduction

The discovery of resonances ψ and ψ' (Aubert *et al* 1974; Augustin *et al* 1974) has strengthened the idea of a fourth quark-flavour, *viz.* charm (Bjorken and Glashow 1964) in addition to the Gell-Mann trio-*u*, *d* and *s* (Gell-Mann 1964). The subsequent experimental evidence for charmed mesons (Goldhaber *et al* 1976) and baryons (Cazzoli *et al* 1975; Knapp *et al* 1976) has confirmed the theoretical model of charm (Glashow *et al* 1970). Some of the properties of charmed hadrons have also been studied by Rujula *et al* (1975) in a coloured gauge theoretical model. But then, charm is not enough. The strong evidence for a new class of resonance states Υ and Υ' (Herb *et al* 1977; Innes *et al* 1977) which cannot be accommodated within charmed framework, forces the introduction of another quark-*b*, carrying a new flavour. Recently, Thorndike (1980) has reported that Υ (4*s*) is seen at CESR with mass 10·55 GeV and has a large width, suggesting that the mass of the *b*-quark meson may be between $5{\cdot}16 \leqslant D_b \leqslant 5{\cdot}27$ GeV, further confirming the existence of *b*-quark hadrons. Although the experimental data on *b*-quark hadrons are too meagre to reveal a definite pattern and construct a suitable model, it is worthwhile trying earlier known models to see their validity and predictive power in the newly emerging *b*-sector. Many authors (Boal 1978; Ono 1978; Camiz *et al* 1979; Misra and Sastry 1979; Singh *et al* 1981; Singh 1981) have started studying their properties in different frameworks.

In the present work, we study the masses of the charmed and *b*-quark hadrons in the quasinuclear coloured quark model (Lipkin 1978). In this model both the $q\bar{q}$ (mesons) and qqq (baryons) bound systems are assumed to be described by the same Schrödinger equation with two-body flavour-independent logarithmic potential (Quigg and Rosner 1977). The remarkable agreement at the SU(3) level, obtained by Lipkin (1978) shows that mesons and baryons do have a similar structure and should, therefore, lend themselves to a unified description. In § 2, the general expression for the mass operator of a hadron is discussed, and the subsequent sections deal with various mass splittings and their relations.

2. Mass operator

Assuming that hadrons containing heavier strange quark acquire an additional mass contribution proportional to the number of such quarks, Lipkin (1978), obtained an interesting unified mass operator which nicely reproduces their masses. We expect similar agreement for new hadrons, where the non-relativistic models should work more effectively because of the heavier quark-masses. The generalization of the Lipkin's formula to the heavier quark sectors is straightforward and can be written as

$$M = A_{(n)} + B_{n(h)} + \sum_{i>j} K_{ij} \Big\{ [\langle U_{ij}\rangle/(n-1)] + \left[n\, K_{ij} \frac{\sigma_i \cdot \sigma_j}{2\,(m_i + m_j)} \langle V_{ij}\rangle \right] \Big\}, \tag{1}$$

where the colour coupling factor

$$K_{ij} = (3/32) \sum_a \lambda_i^a \lambda_j^a,$$

n and $n\,(h)$ are the total number of all and heavier (s, c and b) quarks antiquarks respectively. A, B are parameters and λ^a are the generators of the SU $(3)_{\text{colour}}$ group. $\langle U_{ij}\rangle$ and $\langle V_{ij}\rangle$ are reduced matrix elements for the two-body spin-independent and spin-dependent interaction respectively.

The colour factor can explicitly be evaluated for mesons and baryons so as to give the final form for the mass operator.

$$M_{(\text{meson})} = A_{(\text{meson})} + B\, n(h) + \langle U_{ij}\rangle + \frac{\sigma_i \cdot \sigma_j}{m_i + m_j} \langle V_{ij}\rangle \tag{2}$$

$$M_{(\text{baryon})} = A_{(\text{baryon})} + B\, n(h) + \sum_{i>j} \left[\frac{1}{4} \langle U_{ij}\rangle + \frac{3}{8} \frac{\sigma_i \cdot \sigma_j}{m_i + m_j} \langle V_{ij}\rangle \right]. \tag{3}$$

Here, 1/4 and 3/8 are the scaling factors, obtained from the log-model which removes ambiguities in the wave functions and matrix elements.

3. Masses of the hadrons

Using the wavefunctions (Singh *et al* 1981) and (2) and (3), masses of the hadrons can be written in terms of the parameters A and B. As the n dependence of A is not known, the absolute masses cannot be calculated. We discuss only the different mass splittings. The numerical value of B in different sectors governs the mass splittings and dominates over other model-dependent terms. Taking the meson mass splittings in various sectors as inputs, we predict the baryon mass splittings (table 1). The available experimental data favourably match with our predictions. The notation used to describe the particles is given in Singh *et al* (1980a) and (1981). In table 1 and in the relations given below the particle symbol stands for its mass.

4. Mass relations

Apart from the relations obtained by Lipkin (1978), we obtain the following mass relations among the various mass splittings.

Table 1. Hadron mass splittings in MeV

Mass difference	Present analyses	Experimental Values	Mass difference	Present analyses	Experimental Values
Spin splittings			*Charm splittings*		
$(D_c^* - D_c)$		140†	$(D_c^* - \rho)$	—	1240†
$(F_c^* - F_c)$	126	110	$(D_c - \pi)$	1710	1730
$(D_b^* - D_b)$	40		$(F_c^* - K^*)$	1280	1248
$(F_b^* - F_b)$	38·5		$(F_c - K)$	1550	1536
$(G_b^* - G_b)$	32		$(G_b^* - D_b^*)$	1348	
$(\Sigma_c^* - \Sigma_c)$	68	60	$(G_b - D_b)$	1356	
$(\Sigma_c - \Lambda_c)$	183	190	$(\Lambda_c - N)$	1350	1325
$(\Xi_c^* - \Xi_c)$	69		$(\Sigma_c - N)$	1534	1520
$\Omega_c^* - \Omega_c)$	71		$(\Sigma_c^* - \Delta)$	1258	1248
$(\Sigma_b^* - \Sigma_b)$	23		$(\Xi_{cc}^* - \Sigma_c^*)$	1299	
$(\Sigma_b - \Lambda_b)$	221		$(\Omega_{ccc}^* - \Xi_{cc}^*)$	1340	
$(\Omega_b^* - \Omega_b)$	21·7				
$(\Omega_{ccb}^* - \Omega_{ccb})$	18				
Strangeness splittings			*Beauty splittings*		
$(F_c^* - D_c^*)$	176	130	$(D_b^* - \rho)$	—	4460†
$(F_c - D_c)$	189	160	$(D_b - \pi)$	5090	
$(F_b^* - D_b^*)$	179		$(F_b^* - K^*)$	4547	
$(F_b - D_b)$	182		$(F_b - K)$	4910	
$(\Omega_{cc} - \Xi_{cc})$	185		$(G_b^* - D_c^*)$	4615	
$(\Omega_{cc}^* - \Xi_{cc}^*)$	177		$(G_b - D_c)$	4730	
$(\Omega_{bb}^* - \Xi_{bb}^*)$	180		$(\Lambda_b - N)$	4635	
$(\Omega_{bb} - \Xi_{bb})$	181		$(\Sigma_b^* - \Delta)$	4520	
			$(\Sigma_b - N)$	4805	

† input,

4.1 *Spin splittings*

The following mass relations are obtained among the splittings between hadron pairs having same quark contents but differing in spin.

$$(\Sigma_c^* - \Sigma_c) = (\Xi_{cc}^* - \Xi_{cc}) \tag{4}$$

$$\underset{(225\ \text{MeV})}{3/2(\Sigma_c - \Lambda_c)} = \underset{(235\ \text{MeV})}{(\Delta - N) - (\Sigma_c^* - \Sigma_c)}, \tag{5}$$

$$3(\Xi_c^* - \Xi_c) = \tfrac{1}{4}(\Sigma^* - \Sigma) + (\Sigma_c^* - \Sigma_c) + (\Omega_c^* - \Omega_c), \tag{6}$$

$$(\Sigma_b^* - \Sigma_b) = (\Xi_{bb}^* - \Xi_{bb}) \tag{7}$$

$$3/2\,(\Sigma_b - \Lambda_b) = (\Delta - N) - (\Sigma_b^* - \Sigma_b), \tag{8}$$

$$3(\Xi_b^* - \Xi_b) = \tfrac{1}{4}(\Sigma^* - \Sigma) + (\Sigma_b^* - \Sigma_b) + (\Omega_b^* - \Omega_b), \tag{9}$$

$$3(\Xi_{cb}^* - \Xi_{cb}) = \tfrac{1}{4}(\Sigma_c^* - \Sigma_c) + (\Sigma_b^* - \Sigma_b) + (\Omega_{ccb}^* - \Omega_{ccb}). \tag{10}$$

The relation (5) agrees reasonably well with experimental data. Other relations can be tested when more information is available.

4.2 *Flavour splitting*

This pertains to the mass splitting between hadron pairs differing by unit strangeness, charm or beauty only. Here, the following sum rules are obtained

$$3(\Lambda - N) = 2(\Omega_{cc}^* - \Xi_{cc}^*) + (\Omega_{cc} - \Xi_{cc})$$

$$= 2(\Omega_{bb}^* - \Xi_{bb}^*) + (\Omega_{bb} - \Xi_{bb}) \tag{11}$$

$$(\Xi_{cc}^* - \Sigma_c^*) = (\Xi_{cc} - \Sigma_c), \tag{12}$$

$$2(\Omega_{ccc}^* - \Xi_{cc}^*) = (\Lambda_c - N) + (\Sigma_c - N), \tag{13}$$

$$3(\Lambda_c - N) = 2(\Omega_{cbb}^* - \Xi_{bb}^*) + (\Omega_{cbb} - \Xi_{bb}), \tag{14}$$

$$(\Xi_{bb}^* - \Sigma_b^*) = (\Xi_{bb} - \Sigma_b), \tag{15}$$

$$2(\Omega_{bbb}^* - \Xi_{bb}^*) = (\Lambda_b - N) + (\Sigma_b - N). \tag{16}$$

4.3 *Hybrid relations*

We do not get any relation among the meson splittings alone. However, meson mass splittings can be related to the baryon mass splittings, through the following relations

$$(\Lambda_c - N) = [\tfrac{3}{4}(D_c^* - \rho) + \tfrac{1}{4}(D_c - \pi)]$$
(1325 MeV) (1360 MeV) (17)

$$(\Lambda_b - N) = [\tfrac{3}{4}(D_b^* - \rho) + \tfrac{1}{4}(D_b - \pi)] \qquad (18)$$

$$(\Sigma_c - \Lambda_c) = \tfrac{3}{8}\,[(\rho - \pi) - (D_c^* - D_c)] \qquad (19)$$

(190 Mev) (184 MeV)

$$(\Sigma_b - \Lambda_b) = \tfrac{3}{8}\,[(\rho - \pi) - (D_b^* - D_b)] \qquad (20)$$

$$\frac{\Sigma_b^* - \Sigma_b}{D_b^* - D_b} = \frac{\Sigma_c^* - \Sigma_c}{D_c^* - D_c} = \frac{\Sigma^* - \Sigma}{K^* - K}$$
(0·42) (0·48)

$$= \frac{\Delta - N}{\rho - \pi} = 9/16 \qquad (21)$$
(0·46)

We see that relations (17), (19) and (21) are reasonably satisfied and we expect that the other relations, which cannot be tested presently, may also give a satisfactory fit subject to the availability of the data in future.

5. Conclusions

The masses of the mesons ($q\bar{q}$) and baryons (qqq), which are made up of similar type of quarks have been studied in different techniques *viz* additive quark model (Lichtenberg 1975 and 1976); symmetry scheme (Okubo *et al* and 1975 Singh *et al* 1980b) and current algebra framework (Simard and Suzuki 1975), but there they are treated separately. In the present analysis they are described by the same Schrödinger equation with the two-body flavour-independent Quigg-Rosner (1977) logarithmic potential. We take meson mass splittings as the input and predict the various baryons mass splittings. Some relations among the mass splittings are also obtained. In the charm sector the available data compare favourably with our predictions, *viz.* $(\Sigma_c - \Lambda_c) = 183$ MeV [190 MeV], $(\Sigma_c^* - \Sigma_c)$ 68 MeV [60 MeV]; $(F_c^* - F_c) = 126$ MeV [110 MeV], $(\Lambda_c - N) = 1350$ MeV [1325 MeV] and $(\Sigma_c^* - \Delta) = 1258$ MeV [1248 MeV]. In the absence of experimental data, the other predictions in the charm and b-sector are found to comparable with the theoretical predictions of Ono (1978) Misra and Sastry (1979) and Singh (1981).

As the above analysis gives a reasonable agreement, it can be inferred that mesons and baryons, which have similar constituent quarks, can be understood by a unified prescription and their mass spectra can be related. This view has an added relevance within the context of QCD (Close 1979) wherein $q\bar{q}$ and qq interactions, having assumed to be arising from coloured gluon exchange get related. The validity of the model and assumptions involved for heavier hadrons can be tested when more experimental information from CERN, PETRA, etc. emerges.

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