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Mass extrapolation of quarks and leptons to higher generations

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Abstract. An empirical mass formula is tested for the basic fermion sequences of charged quarks and leptons. This relation is a generalization of Barut's mass formula for the lepton sequence (e, μ, τ......). It is found that successful mass extrapolation to the third and possibly to other higher generations ($N>2$) can be obtained with the first and second generation masses as inputs, which predicts the top quark mass m_t to be around 20 GeV. This also leads to the mass ratios between members of two different sequences (i) and (i') corresponding to the same higher generations ($N>2$).

Keywords. Quarks; leptons; generation; mass spectrum; flavour; top-quark.

1. Introduction

Discoveries of heavy lepton τ (Perl *et al* 1975) and the heavy meson families of J/ψ (Aubert *et al* 1974 and Augustin *et al* 1974) and Υ (Herb *et al* 1977; Kephart *et al* 1977; Innes *et al* 1977) have indicated the existence of an increasing sequence of leptons and quarks considered to be the basic fermions of nature. According to Harari (1978) these lepton and quark sequences can be classified into generations with (a, μ_e) and (u, d) belonging to the first generations. It is believed that the higher generation quarks and leptons are the exact copies of the first generation ones in almost every respect excepting their heavier masses. Thus the second generation fermions (μ, ν_μ) and (c, s) are the exact replications of the first generation fermions (e, ν_e) and (u, d) respectively. The same can be true for the third and subsequent possible generations. Although the third generation lepton (τ) and quark (b) belonging to the e-sequence and d-sequence respectively are already known experimentally, the corresponding top-quark t of charge $+ 2/3$ belonging to the u-sequence is yet to be found. According to cosmological predictions (Yang *et al* 1979) the observed He-abundance of the universe implies not more than three light neutrinos, *i.e.* no more than three generations. Nevertheless the possible existence of higher generations beyond the third is still an open question. In any case the mass spectrum of quarks and leptons may indicate a further internal structure which has fascinated many authors in recent years to decode the observed mass pattern of quarks and leptons either phenomenologically (Bjorken 1978; Barut 1978 and 1979) or on the basis of gauge theories (Weinberg 1977; Glashow 1978; Harari 1978; Buras *et al* 1978 and Nanopoulous and Ros 1979) of weak and electromagnetic interactions. In this paper we follow a purely phenomenological approach to extrapolate the masses of the basic fermions to higher generations with an *ad hoc* generalization of the charged lepton mass formula obtained by Barut (1979).

2. Phenomenological mass formula

The occurrence of the charged lepton sequence (e, μ, τ..) is explained by Barut (1979) as due to the magnetic self-interaction of the electron. According to this view, the radiative effects give an anomalous magnetic moment to the electron which when coupled to the self field of the electron, implies an extra magnetic energy. This magnetic energy due to such a system consisting of a charge and a magnetic dipole, when quantized according to Böhr-Sommer-feld procedure, becomes proportional to n^4, where n is a principal quantum number generating different members of the charged lepton sequence. Now with little emphasis on any particular mechanism responsible for the occurrence of all the charged basic fermion sequences in general, we can make an empirical generalization of the mass formula obtained by Barut to generate the quark and lepton mass spectra simultaneously.

We write the basic fermion sequences including the neutral neutrinos in the following manner

$[f]_{iN}$ = (1)

i ↓ \ N →	1	2	3	4(?)	...
1	ν_e	ν_μ	ν_τ	ν_δ	...
2	e	μ	τ	δ	...
3	u	c	t	h	...
4	d	s	b	g	...

Here the suffix $i = 1, 2, 3, 4$ gives the fermion sequences and $N = 1, 2, 3, 4, \ldots$ refers to the generation number giving a particular number of a sequence. Then for the charged fermion sequences ($i > 1$) the mass of the Nth generation fermion is given by the empirical relation

$$m_{iN} = m_{i1}\,[1 + \lambda_i\, G(N)], \tag{2}$$

where
$$G(N) = \sum_{n=1}^{N} (n-1)^4, \tag{3}$$

and λ_i is a constant somewhat dependent on the radiative anomalous magnetic moment which is presumed to be different for different fermion sequences. Barut (1979) has obtained $\lambda_e = 3/2\alpha$ from a simple classical consideration. However in the absence of a concrete theory giving the exact form of λ_i we attach little emphasis on the exact mechanism responsible for the above mass formula (2). On the other hand we treat this mass formula entirely as an empirical one to be tested for its validity. Therefore our approach will be purely phenomenological where we eliminate λ_i by using the known masses of the first two members of a sequence to predict the masses of the other members of the same sequence. This method amounts to determining the constant λ_i with the knowledge of m_{i2} and m_{i1} from equations (2) and (3) as,

$$\lambda_i = \left(\frac{m_{i2}}{m_{i1}} - 1\right). \tag{4}$$

For quark sequences, Minami and Nakashima (1979) have used $\lambda_u = 9/4\alpha$ and $\lambda_d = 1/4\alpha$ with m_{i2} and m_{i1} as the bare current masses taken in the following manner (Bjorken 1978)

$$m_u \simeq 4 \text{ MeV}, \qquad m_d \simeq 7 \text{ MeV}, \tag{5}$$

$$m_c \simeq 1{\cdot}2 \text{ GeV}, \qquad m_s \simeq 250 \text{ MeV}.$$

It is obvious that λ_i values determined in this manner are subject to certain arbitrariness depending on the input quark masses. However since for higher generations ($N > 2$), $G(N)$ in (3) is a very large number, this arbitrariness in λ_i does not substantially affect the predictions of the mass values m_{i_N} for $N > 2$ as long as $(m_{i2} - m_{i1})$ is not altered significantly. This observation will be transparent from the following mass sum-rule obtained from (2) and (4) in the form,

$$m_{i_N} = m_{i1} + (m_{i2} - m_{i1})\, G(N). \tag{6}$$

Therefore we believe that if we take the constituent quark masses as the inputs instead of the bare current masses, the predictions for higher generations would not be significantly different from those obtained by Minami and Nakashima (1979).

3. Mass extrapolations to higher generations

We can now use the phenomenological mass sum-rule (equation (6)) and the knowledge of the first and second generation lepton and quark masses for the extrapolations to higher generations. We take the input masses as given below

$$m_e = 0{\cdot}511 \text{ MeV} \qquad m_u = 0{\cdot}3 \text{ GeV} \qquad m_d = 0{\cdot}3 \text{ GeV}, \tag{7}$$

$$m_\mu = 105{\cdot}659 \text{ MeV}, \quad m_c = 1{\cdot}5 \text{ GeV}, \quad m_s = 0{\cdot}6 \text{ GeV}.$$

Then we predict the third generation lepton and quark masses as follows:

$$\begin{aligned} m_\tau &= m_e + (m_\mu - m_e)\, G(3) &&= 1{\cdot}788 \text{ GeV}, \\ m_b &= m_d + (m_s - m_d)\, G(3) &&= 5{\cdot}4 \text{ GeV}, \\ m_t &= m_u + (m_c - m_u)\, G(3) &&= 20{\cdot}7 \text{ GeV}. \end{aligned} \tag{8}$$

The heavy lepton mass $m_\tau = 1{\cdot}788$ GeV obtained in this manner here and also by Barut (1979) compares very well with the experimental values

$$(m_{\tau_{\rm exp}} = 1{\cdot}782\,^{+3}_{-4}\ \text{GeV}).$$

The quark mass $m_b = 5{\cdot}4$ GeV calculated above is also well within the range of values required to generate the Υ spectrum in potential models. It should also be pointed out that the prediction for the top-quark mass $m_t = 20{\cdot}7$ GeV is in conformity with the lower bound for this value indicated by the recent experiment at PETRA (Barber *et al* 1980).

If we further consider that higher generations beyond the third occur, then the lepton and quark masses in these generations can be obtained in a trivial manner from (6). As an example we give the predictions of the fourth generation masses as,

$$\begin{aligned} m_\delta &= m_e + (m_\mu - m_e)\, G(4) = 10{\cdot}3 \text{ GeV},\\ m_g &= m_d + (m_s - m_d)\, G(4) = 29{\cdot}7 \text{ GeV},\\ m_h &= m_u + (m_c - m_u)\, G(4) = 117{\cdot}9 \text{ GeV}. \end{aligned} \tag{9}$$

From (6) it is obvious that for a particular sequence i the members of the higher generations ($N > 2$) would lie on a straight line with $(m_{i2} - m_{i1})$ as the slope in a plot of $(m_{iN} - m_{i1})$ vs $G(N)$. Therefore different fermion sequences would correspond to different straight lines in such a plot.

Finally we note that if we neglect the small contribution of the first generation fermion mass as compared to the large second term in (6), we would obtain the following mass relations for higher generation ($N > 2$) fermions belonging to different sequences,

$$\frac{m_{iN}}{m_{i'N}} = \frac{(m_{i2} - m_{i1})}{(m_{i'2} - m_{i'1})}. \tag{10}$$

This would imply the following mass ratios between quarks and leptons.

$$\begin{aligned} \frac{m_b}{m_t} &= \frac{m_g}{m_h} = \ldots \simeq \frac{(m_s - m_d)}{(m_c - m_u)} = 0{\cdot}25,\\ \frac{m_t}{m_\tau} &= \frac{m_h}{m_\delta} = \ldots \simeq \frac{(m_c - m_u)}{(m_\mu - m_e)} = 11{\cdot}4,\\ \frac{m_b}{m_\tau} &= \frac{m_g}{m_\delta} = \ldots \simeq \frac{(m_s - m_d)}{(m_\mu - m_e)} = 2{\cdot}85. \end{aligned} \tag{11}$$

It will be interesting to compare these quark-lepton mass ratios with those obtained from some grand unified models (Barbieri *et al* 1980 and Buras *et al* 1978). Buras *et al* (1978) obtain

$$\frac{m_b}{m_\tau} \simeq \left[\frac{\alpha_s(\Upsilon)}{\alpha_s(M_1)}\right]^{12/(33-2F)} \tag{12}$$

where $\alpha_s(\Upsilon)$ is the strong coupling constant evaluated at the mass of Υ and $M_1 = 10^{15}$ GeV and F is the number of quark flavours. Using experimental data to determine α_s and taking $F = 6$, they get $(m_b/m_\tau) \simeq (2\cdot7 - 3)$ which is the observed value that compares well with $(m_b/m_T) = 2\cdot85$ from (11). Barbieri *et al* (1980) also find the mass ratio

$$\frac{m_t}{m_\tau} = \frac{m_c}{m_\mu}\left[\frac{\alpha_s(\Upsilon)}{\alpha_s(\psi)}\right]^{4/7}, \tag{13}$$

which with appropriate values of $\alpha_s(\Upsilon)$ and $\alpha_s(\psi)$ leads to $(m_t/m_\tau) \simeq 11\cdot2$ that agrees well with the predictions of (11).

4. Conclusions

Following Barut (1979) if we treat higher generation quarks and leptons as the excitation of their first generations counterparts, then we can explain empirically the observed mass spectra of the charged quarks and leptons in a general manner. Although such a hypothesis is heuristic in nature, its phenomenological implications in reproducing the masses of μ, τ and b successfully cannot be completely ignored. Therefore this approach may provide a guiding link towards the development of a more complete theory to derive the mass formula from first principles to explain the occurrence of the fermion sequences. Of couıse within the scope of the above hypothesis we have no answer at the moment to the existence of the neutral lepton sequence (ν_e, ν_μ, ν_τ, ..). Although neutrino masses are believed to be consistent with zero, recent evidences of neutrino oscillations indicate the possibility of nonzero masses. If it is found to be true then it may provide a testing ground for the empirical formula given in (6) to have a further understanding of the exact mechanism responsible for generating different fermion sequences. Therefore the neutrino mass spectrum may play a key role to decide which picture makes more sense.

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