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Symmetry breaking for the *BBP*-couplings

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Abstract. Using the consistency requirements arising from the Coleman-Glashow null result for 'tadpole'-type symmetry-breaking, we obtain a simple method of accounting for the SU(3) symmetry-breaking at the *BBP*-vertex, without introducing any new parameters. The results obtained are in excellent agreement with the available numbers. We extend the analysis to the charmed baryon couplings in order to accommodate SU(4)-breaking.

Keywords. Coleman-Glashow theorem; SU(3); SU(4); 'tadpole'-symmetry breaking; current-algebra.

1. Introduction

Extraction of the strong coupling constants from the experimental data involves large theoretical and systematic uncertainties. However, from the numbers available it can be fairly well established that the SU(3)-invariance does not work for the pseudoscalar Yukawa couplings of the mesons with baryons, (called the *BBP*-couplings). (For a recent SU(3) comparison of these couplings, see Nagels *et al* 1979; for a review of the problem, see Gustafson *et al* 1977). Some form of symmetry-breaking at the *BBP*-vertices must be invoked in order to account for the observed couplings. Pinning down the symmetry-breaking mechanism involved assumes special importance at this stage when the study of the hadronic properties of newly observed charmed baryons (Knapp *et al* 1976; Piccolo *et al* 1977) is becoming a real possibility. Knowledge of the symmetry-breaking at the SU(3)-level will give us an indication of how to treat the breaking of SU(4) and get reasonable estimates for the strong couplings involving charmed baryons. Lacking a usable analysis of the symmetry-breaking, estimates of the charmed baryon coupling are often made by using exact SU(4)-symmetry (*e.g.*, Dover *et al* 1977). This use of SU(4)-symmetry is of doubtful validity, in view of the fact that SU(4)-symmetry is much less reliable than SU(3) and that SU(3) itself is rather bad for the strong *BBP*-vertices.

The simplest way to account for SU(3)-breaking is, of course, to break the symmetry by terms linear in the eighth component of an SU(3)-octet. This is the breaking which works so well for the hadron mass splittings, giving the Gell-Mann-Okubo mass formula. This breaking was analysed long back by Glashow and Muraskin (1963). However, this analysis in itself remains useless because of the number of parameters it introduces. In fact for the seven *BBP*-couplings involving pions and kaons, this analysis gives only one relation. A glance at the experimental situation regarding these couplings (Nagels *et al* 1979) is sufficient to convince oneself that one relation among the seven couplings has no predictive value whatsoever.

In this note we assume that the lineaı symmetry breaking term of Glashow and Muraskin (1963) arises through the ' tadpole ' mechanism (Coleman and Glashow 1964). In addition we also assume the PCAC hypothesis for the pion field. Then applying Coleman-Glashow (1964) theorem to the parity-conserving non-leptonic decays generated by the strangeness-changing scalar tadpole, S_6, we are able to get four more relations among the seven couplings referred to above. Besides the *BBP*-coupling constants, these relations also involve baryon masses which, in any case, are well known. This allows us to determine the seven symmetry-broken *BBP*-couplings in terms of only two parameters, no more than the SU(3)-symmetric case. We show that the symmetry-broken *BBP*-couplings obtained thus are in remarkably good agreement with the available experimental numbers (§ 2).

The five relations among the seven couplings obtained above can be reduced to a simple symmetry ansatz (§ 3) wherein the symmetry-broken coupling $g_{BB'P}$ is expected to be proportional to the product of the sum of the two-baryon masses involved, $(m_B + m_{B'})$, and the SU(3)-symmetric expression for the relevant coupling. Besides the baryon masses, there are then only two parameters for the symmetry broken couplings: the proportionality factor g and the SU(3)-parameter α. Extending the same ansatz to SU(4) we obtain the charmed baryon coupling $g^2_{C_1C_0\pi}/4\pi=45{\cdot}7$, in very good agreement with the recent current algebra calculations of the same coupling (Prasad and Singh 1979 and 1980). The value of $g^2_{C_1C_1\pi}/4\pi$ which also has been calculated in the same references, however, does not agree with our calculation.

The symmetry-ansatz obtained in this paper has earlier been presented in literature as following from the application of SU(3)-symmetry at the pseudo-vector *BBP*-couplings (Pilkuhn *et al* 1973). The analysis of this paper gives this ansatz a surer meaning by showing it to be a consequence of ' tadpole '-type symmetry breaking at the pseudoscalar *BBP*-couplings and PCAC.

2. The ' tadpole ' mechanism, Coleman-Glashow theorem and relations among the *BBP*-couplings

The tadpole mechanism (Coleman and Glashow 1964) is designed to generate all symmetry-breaking effects, *i.e.* the medium strong, the electromagnetic, and the weak effects, through a single symmetry-breaking term, which transforms like an SU(3)-octet. Except for this ' tadpole ' term the hadronic Hamiltonian is supposed to be exactly SU(3)-invariant. This mechanism offers a simple description of the linear Gell-Mann-Okubo mass splitting and can cause the Muraskin-Glashow (1963) breaking in the *BBP*-couplings. The tadpole mechanism succeeds admirably in explaining the medium strong and the electromagnetic effects, but fails to describe the strangeness changing weak effects. This happens because, if all symmetry-breaking effects are generated through a single term transforming as an SU(3)-octet, then one can always find an SU(3)-transformation that diagonalizes the symmetry-breaking term leaving the rest of the Hamiltonian invariant. This implies that the strangeness changing scalar tadpole S_6, transforming as the sixth component of the symmetry-breaking octet, can be rotated away, leaving behind the medum strong and the electromagnetic tadpoles, S_8 and S_3 respectively, transforming as the diagonal eighth and third components of the SU(3)-octet. The strangeness changing effects generated through the S_6-tadpole, therefore, must vanish. This is the Coleman-Glashow (1964) theorem. Below we exploit this null-result to find relations between symmetry-broken couplings.

The S_6-tadpole coupled with the PCAC hypothesis for the pion field and the current-algebra can be used to write expressions for the parity-conserving non-leptonic decays of hyperons in terms of the strong *BBP*-coupling constants and the hadron masses. Such expressions have been written by Marshak *et al* (1969). The Coleman-Glashow null-result, discussed above, requires that these expressions must all equal zero. Marshak *et al* (1969) have explicitly shown that these amplitude expressions indeed go to zero, if all couplings and masses are assumed to be SU(3)-invariant. The null result, however, must hold even when the SU(3)-symmetry for the couplings and the masses is broken via the tadpole mechanism (Bajaj *et al* 1978). The requirement that each of the four independent non-leptonic decay amplitudes generated through the S_6-tadpole must be zero, then yields four relations among the symmetry-broken coupling constants and the hadron masses. These relations are:

$$g_{\Lambda NK}+\frac{m_\Lambda+m_N}{2\,m_N}\left(\sqrt{3}\,g_{NN\pi}-\frac{2\,m_N}{m_\Lambda+m_\Sigma}\,g_{\Lambda\Sigma\pi}\right)=0, \tag{1a}$$

$$g_{\Xi\Lambda K}-\frac{m_\Lambda+m_\Xi}{2\,m_\Xi}\left(\sqrt{3}\,g_{\Xi\Xi\pi}+\frac{2\,m_\Xi}{m_\Lambda+m_\Sigma}\,g_{\Lambda\Sigma\pi}\right)=0, \tag{1b*}$$

$$g_{\Sigma NK}-\frac{m_\Sigma+m_N}{2\,m_N}\left(g_{NN\pi}-\frac{m_N}{m_\Sigma}\,g_{\Sigma\Sigma\pi}\right)=0, \tag{1c}$$

$$2\,g_{NN\pi}-\frac{m_N}{m_\Sigma}\,g_{\Sigma\Sigma\pi}-\frac{2\,m_N}{m_\Lambda+m_\Sigma}\,\sqrt{3}\,g_{\Lambda\Sigma\pi}=0. \tag{1d}$$

In writing relations (1) we have used the relation

$$-\sqrt{3}\,\frac{\delta M}{2}=f_\pi, \tag{2}$$

*The corresponding expression for the amplitude $B(\Xi_-^-)$ in Marshak *et al* (1969) is wrong. The correct expression is:

$$B(\Xi_-^-)=\frac{G_p}{\sqrt{2}\,f_\pi}\left[g_{\Xi\Lambda K}+\frac{2\,f_\pi}{\sqrt{3}\,\delta M}\,\frac{m_\Lambda+m_\Xi}{2m_\Xi}\left(\sqrt{3}\,g_{\Xi\Xi\pi}+\frac{2m_\Xi}{m_\Lambda+m_\Sigma}\,g_{\Lambda\Sigma\pi}\right)\right].$$

Equating $B(\Xi_-^-)$ above to zero gives the relation (1b).

where δM is given by the expression for the mass matrix:

$$M = M_0 + \delta M \cdot S_8. \tag{3}$$

The relation (2) can be verified by a uniform treatment of the Gell-Menn-Okubo mass splitting (Marshak *et al* 1969). The relations (1) represent a consistency requirement imposed by the Coleman-Glashow null result. The relations must be valid if the tadpole mechanism is responsible for the breaking of the SU(3)-symmetry and if pion PCAC and current-algebra commutation relations hold.

The condition that the symmetry is broken by terms linear in the eighth component of an SU(3)-octet gives one more relation among the seven couplings appearing in relations (1). This relation is (Glashow and Muraskin 1963):

$$\frac{2}{\sqrt{3}} g_{NN\pi} + \frac{1}{\sqrt{3}} g_{\Sigma\Sigma\pi} - \frac{4}{\sqrt{3}} g_{\Xi\Xi\pi} - 3\, g_{\Lambda\Sigma\pi}$$

$$+ g_{\Lambda NK} + 2\, g_{\Xi\Lambda K} + \sqrt{3}\, g_{\Sigma NK} = 0. \tag{4}$$

Thus we have five relations ((1a) to (1d) and (4)) among the seven *BBP*-couplings being considered. To check the experimental validity of these relations we use the experimental values of $g^2_{NN\pi}/4\pi$ and $g^2_{\Lambda\Sigma\pi}/4\pi$, and estimate the remaining five. The results are shown in table 1 where we have also listed the SU(3)-symmetric values obtained from the same two inputs. The calculated couplings are all in good agreement with the experimental numbers, and are definitely superior to the SU(3)-symmetric values. A note on the experimental numbers used in table 1: The input values of $g^2_{NN\pi}/4\pi$ and $g^2_{\Lambda\Sigma\pi}/4\pi$

Table 1. Estimated and the SU(3)-symmetric values of the *BBP*-couplings.

Coupling	Estimated value	Symmetric value	Experimental value
$g^2_{NN\pi}/4\pi$	14·64*	14·64*	14·64 $^{+0.54}_{-0.72}$
$g^2_{\Lambda\Sigma\pi}/4\pi$	11·1*	11·1*	11·1 ± 1·2
$g^2_{\Lambda NK}/4\pi$	18·26	10·9	20·4 ± 3·7
$g^2_{\Sigma NK}/4\pi$	0·99	3·8	1·9 ± 3·2
$g^2_{\Sigma\Sigma\pi}/4\pi$	13·95	3·5	12·5 ± 2·0 13·4 ± 2·1
$g^2_{\Xi\Xi\pi}/4\pi$	1·53	3·8	—
$g^2_{\Xi\Lambda K}/4\pi$	2·39	0·0	—

*Input values

are taken from Pilkuhn *et al* (1973). The latter number has been obtained by using Goldberger-Treiman relation on the decay $\Sigma \to \Lambda e \bar{\nu}$. This value should be relevant for an analysis involving current-algebra that we have done in this paper. Numbers for the $g_{\Lambda\Sigma\pi}$ coupling obtained from other analyses are not much different (see, Nagels *et al* 1979). For $g_{\Sigma\Sigma\pi}$ we quote both values listed in the latest compilation (Nagels *et al* 1979). The calculated value is in agreement with both while the SU(3)-symmetric value is way-off. For $g_{\Lambda NK}$ and $g_{\Sigma NK}$ we have quoted the numbers from Baillon *et al* (1976). This analysis includes the new data on the real part of KN forward scattering. The numbers listed in Nagels *et al* (1979) compilation do not include the new data. Only other analysis that includes the new data is that of Martin (1976). Martin's number for $g^2_{\Lambda NK}/4\pi$ is, however, off by two standard deviations from our calculated number.

3. A symmetry-breaking ansatz and extension to SU(4)

The analysis of § 2 led to five relations amongst the seven couplings considered. Therefore, it should be possible to write the seven couplings in terms of only two parameters. It can be easily checked that the five relations above imply

$$g_{BB'P} = \frac{(m_B + m_{B'})}{m} g^{\text{symm}}_{BB'P}, \tag{5}$$

where m_B, $m_{B'}$ are the masses of the two baryons; m is a constant with the dimensions of mass (say, the mean mass of the baryon multiplet) introduced to get the correct dimensions; and $g^{\text{symm}}_{BB'P}$ is the SU(3)-invariant expression for the relevant $BB'P$-coupling written in terms of the two SU(3) parameters g and α (see Dover *et al* (1977), for definitions). Thus relation (5) expresses the symmetry-broken couplings in terms of only two parameters (g/m) and α. Incidentally, for the fit in table 1, $\alpha=0{\cdot}61$, very close to the SU(6) value of 0·6.

The expression (5) for $BB'P$-couplings is not new. It has been used earlier in the symmetry analysis of the $BB'P$-couplings (see, *e.g.*, Pilkuhn *et al* 1973). This expression arises if SU(3) is applied to the pseudo-vector couplings of the pseudo-scalar mesons to baryons. Gustafson *et al* (1977) have arrived at a very similar expression* through an analysis of the baryon decays in a dynamical model. The special merit of the present analysis is that it gives expression (5) a simple meaning as a consequence of 'tadpole'-type symmetry-breaking, PCAC for the pion field and current algebra, at least for the cases when $P \equiv \pi$ or is related to π through S_6, *i.e.* $P \equiv K$.

*Gustafson *et al* expression is

$$\frac{g^2_{BB'P}}{4\pi} \equiv \frac{[(m_B+m_{B'})^2-m_P{}^2]}{m^2_\pi} \frac{\left(g^{\text{symm}}_{BB'P}\right)^2}{4\pi}$$

which is same as relation (5) except for the m_{P^2} term, which in any case is negligible compared to $(m_B+m_{B'})^2$.

The symmetry-broken couplings involving η or η' require one more parameter which is not determined by the analysis of § 2 (see, Glashow and Muraskin 1963). To determine this parameter we must know at least one coupling involving η or η'. Lacking this input, we can perhaps assume the expression (5) to be valid for $P \equiv \eta$, η' also. But it should be kept in mind that from the assumptions of this paper, *i.e.*, S_8 breaking of SU(3), and the validity of PCAC for the pion field and SU(3) $\times$ SU(3) current algebra, one gets expression (5) only for $P \equiv \pi$ and $P \equiv K$.

Expression (5) will be valid for B, B' being charmed baryons also provided we make the same assumptions about SU(4): SU(4) is a good symmetry for the hadrons, broken only by terms transforming as components of a 15-plet of SU(4), PCAC for the pion field holds and SU(4)$\times$SU(4) current algebra relations are valid. In extending expression (5) in this way to cover the charmed baryon couplings we do not introduce any new parameters. $g^{\text{symm}}_{BB'P}$, of course, now means the SU(4)-invariant expression for the $BB'P$-couplings which also involve only two parameters g and α (see, Dover *et al* 1977). Since expression (5) works so well for the uncharmed case, we expect it to be good for the charmed-baryon case also. In any case, this expression offers a simple way of accounting for the SU(4) symmetry-breaking without any cost in terms of new parameters.

Using g/m and α obtained from the inputs of table 1 we obtain

$$g^2_{C_1C_0\pi}/4\pi = 45{\cdot}7 \text{ and } g^2_{C_1C_1\pi}/4\pi = 57{\cdot}5.$$

There are no experimental numbers available with which to compare these numbers. However, it is interesting to note that the former number agrees very well with calculations using Melosh transformations and PCAC (Lee *et al* 1977), and others using current algebra techniques (Prasad and Singh 1979, 1980). The latter number however is larger than the number calculated by Prasad and Singh (1979, 1980). At this stage of experimental knowledge about charmed resonances there is hardly any way of explaining this discrepancy. The situation is however reminiscent of the situation in the uncharmed case where the experimental number for $g^2_{\Sigma\Sigma\pi}/4\pi$ turns out to be way above the SU(3)-symmetry value.

4. Conclusion

By exploiting the Coleman-Glashow (1964) null result for 'tadpole'-type symmetry-breaking we arrive at a simple method of accounting for the SU(3) symmetry-breaking at the pseudoscalar BBP-vertex, without introducing any new parameters. We show that the same method will work for charmed baryon couplings also provided in analogy with the SU(3) case one assumes that: SU(4) is a good symmetry except for linear-breaking terms that transform as components of a 15-plet of SU(4) and PCAC for the pion field and SU(4) $\times$ SU(4) current-algebra remain valid. This analysis may not be of much use at this stage in view of the rather scarce data on charmed baryon decays. However, the fact that this symmetry-breaking analysis does not require any new parameters and that it works very well at the SU(3) level should recommend it for use in calculations, *e.g.*, of the charmed baryon-antibaryon bound states (Dover *et al* 1977, Bhamathi and Prema 1979), where SU(4)—invariant couplings have been used so far. We also expect that the consistency requirements of the type used in this paper should be of use in extracting information at other places where some null result can be proved.

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