Pramāṇa, Vol. 16, No. 1, January 1981, pp. 81–89. © Printed in India.

Lippmann-Schwinger equation

SUPROKASH MUKHERJEE
Saha Institute of Nuclear Physics, Calcutta 700 032, India

MS received 7 August 1980

Abstract. It is established that the solution of the Lippmann-Schwinger equation for scattering is unique, despensing an earlier proof of non-uniqueness.

Keywords. Lippmann-Schwinger equation; non-uniqueness of solution.

1. Introduction

It is commonly accepted (Sandhas 1972) that the solutions of Lippmann-Schwinger equation for the scattering state is not unique and especially so when rearrangement channels are open. A large volume of work has been carried out in the last two decades (Sandhas 1976) which uses this non-uniqueness. Gerjuoy (1958) is one of the first few to have discussed and established this non-uniqueness. In this paper we reanalyse Gerjuoy's work and find some fatal mistakes in his proof and conclude that, contrary to popular belief, the solution of Lippmann-Schwinger equation for scattering is unique.

2. General formalism

In a formal scattering theory (Goldberger and Watson 1964) the total wavefunction for a scattering state is given by limit $\epsilon \to 0$ of $|\Psi_\alpha(Z_1)\rangle$

$$|\Psi_\alpha(Z_1)\rangle = i\epsilon\, G(Z_1)\,|\varphi_\alpha\rangle, \tag{1}$$

where the total Hamiltonian, kinetic and potential energy operators are H, H_0 and V respectively; $H = H_0 + V = H_\alpha + \bar{V}_\alpha$; $H_\alpha = H_0 + V_\alpha$; $V = V_\alpha + \bar{V}_\alpha$; $G(Z) = (Z-H)^{-1}$, $G_\alpha(Z) = (Z-H_\alpha)^{-1}$; the initial state $|\varphi_\alpha\rangle$ in the α-channel satisfies $H_\alpha|\varphi_\alpha\rangle = E_\alpha|\varphi_\alpha\rangle$; $Z_1 = E_\alpha + i\epsilon$, $(\epsilon > 0)$, Gerjuoy (1958) took $|\Psi_\alpha(Z_1)\rangle$ given by equation (1) to be the unique solution of the following differential equation for $|\Psi_\alpha(Z_1)\rangle$:

$$(Z_1 - \overrightarrow{H})\,|\Psi_\alpha(Z_1)\rangle = i\epsilon\,|\varphi_\alpha\rangle, \tag{2}$$

and he also arrived at the following forms of the solutions of the scattering state at complex energy $Z_1 = E_a + i\epsilon$

$$|\Psi_a(Z_1)\rangle = |\varphi_a\rangle + G_a(Z_1)\,\bar{V}_a|\Psi_a(Z_1)\rangle_\rho + \mathcal{G}(G_a(Z_1), |\chi_a(Z_1)\rangle)_\rho, \quad (3)$$

$$|\Psi_a(Z_1)\rangle = |\varphi_a\rangle + G(Z_1)\,\bar{V}_a|\varphi_a\rangle_\rho + \mathcal{G}(G(Z_1), |(\chi_a(Z_1)\rangle)_\rho, \quad (4)$$

$$|\Psi_a(Z_1)\rangle = |\varphi_a\rangle + |\chi_a(Z_1)\rangle, \quad (5)$$

where the Wronskian-terms in equations (3) and (4) are defined as

$$\mathcal{G}(G(Z), |\eta\rangle)_\rho = G(Z)\,\overrightarrow{H_0}\,|\eta\rangle_\rho - G(Z)\,\overleftarrow{H_0}\,|\eta\rangle_\rho. \quad (6)$$

The direction of arrow on H_0 indicates the function immediately to the right or left of H_0 on which it operates. The subscript ρ in these equations has significance similar to that used by Gerjuoy. In the coordinate space representation $G(Z)|\eta\rangle_\rho$ is equal to

$$\int_\rho G(Z;\, \vec{r}, \vec{r}\,')\, \eta(\vec{r}\,')\, d^3 r',$$

where the integration over the dummy variable $\vec{r}\,'$ is taken within a very large sphere of radius ρ with $\vec{r} = \vec{r}\,'$ as centre and finally $\rho \to \infty$ at some suitable stage of the calculation. Equations (1), (2), (3), (4) above are equations (3.1b), (3.1a), (3.3b) and (3.3a) respectively of Gerjuoy (1958).
Using the equations for Green's function, we get

$$G_a(Z)(Z - \overleftarrow{H_a}) = 1 = (Z - \overrightarrow{H_a})\, G_a(Z), \quad (7)$$

$$G(Z_1)|\varphi_a\rangle = G_a(Z_1)(Z_1 - \overleftarrow{H_a}) \cdot G(Z_1)|\varphi_a\rangle, \quad (8)$$

$$G_a(Z_1)|\varphi_a\rangle = G_a(Z_1) \cdot (Z_1 - \overrightarrow{H})\, G(Z_1)|\varphi_a\rangle, \quad (9)$$

so that from equations (8), (9) and (1) we have

$$|\Psi_a(Z_1)\rangle = |\varphi_a\rangle + G_a(Z_1)\,\bar{V}_a|\Psi_a(Z_1)\rangle_\rho + \mathcal{G}(G_a(Z_1), |\Psi_a(Z_1)\rangle)_\rho, \quad (10)$$

$$|\Psi_a(Z_1)\rangle = |\varphi_a\rangle + G(Z_1)\,\bar{V}_a|\varphi_a\rangle_\rho - \mathcal{G}(G(Z_1), |\varphi_a\rangle)_\rho, \quad (11)$$

where equation (11) uses (8) and (9) with $G_a(Z_1)$, $G(Z)$ interchanged, along with equation (1). Equation (10) is just the result of application of equation (1) on the resolvent identity

$$G(Z) = G_a(Z) + G_a(Z)\,\bar{V}_a\, G(Z) + \mathcal{G}(G_a(Z), G(Z)), \quad (12)$$

which can be derived from (8) and (9) as well. Gerjuoy (1958) also obtained this identity, but he threw away the last term, i.e. the Wronskian-term, on the right side of (12), which he evaluated to be zero, after converting it into surface integral at infinity. Usually, the resolvent identity has only the first two terms on the right in (12), as used in literature. But in the present context, since we are dealing with surface integrals at infinity, we cannot drop this last term in (12) as Gerjuoy did, particularly when we notice that terms similar to this are retained elsewhere, even by Gerjuoy, which can be seen in equations (3) and (4) above. Finally, to see that the Wronskian term of (12) is important, we can directly verify the correctness of (10) by evaluating the Wronskian-term of (10) which came from the Wronskian-term of (12). Thus using (6) and (2), we get in a straightforward way,

$$\mathcal{J}\,(G_\alpha(Z_1), \,|\,\Psi_\alpha(Z_1)\rangle)$$

$$= G_\alpha(Z_1)\cdot\overrightarrow{H}_\alpha\,|\,\Psi_\alpha(Z_1)\rangle - G_\alpha(Z_1)\,\overleftarrow{H}_\alpha\cdot|\,\Psi_\alpha(Z_1)\rangle, \tag{13a}$$

$$= G_\alpha(Z_1)\cdot[-i\,\epsilon\,|\,\varphi_\alpha\rangle + (Z_1-\overline{V}_\alpha)\,|\,\Psi_\alpha(Z_1)\rangle]$$

$$+ [1-Z_1(G_\alpha(Z_1)]\cdot|\,\Psi_\alpha(Z_1)\rangle, \tag{13b}$$

$$= -\,|\,\Psi_\alpha\rangle - G_\alpha(Z_1)\,\overline{V}_\alpha\,|\,\Psi_\alpha(Z_1)\rangle + |\,\Psi_\alpha(Z_1)\rangle, \tag{14}$$

which is the same as equation (10). This shows that if equation (1) or (2) is valid then (10) follows merely from the definition, equation (6) of Wronskian. We, therefore, treat (10) (and similarly (11)) as correct and comparing with Gerjuoy's (1958) equations for $|\,\Psi_\alpha(Z_1)\rangle$ at complex energy Z_1 (*i.e.* (3) and (4) above) we get

$$\mathcal{J}\,(G_\alpha(Z_1), \,|\,\varphi_\alpha\rangle)_\rho = 0, \tag{15}$$

$$\mathcal{J}\,(G(Z_1), |\,\Psi_\alpha(Z_1)\rangle)_\rho = 0. \tag{16}$$

These may be compared with similar relations obtained by Gerjuoy (his equation (2.14b), (2.13a) for 'real energy E_α'

$$\mathcal{J}\,(G_\alpha(E_\alpha), \,|\,\varphi_\alpha\rangle) = |\,\varphi_\alpha\rangle, \tag{17}$$

$$\mathcal{J}\,(G(E_\alpha), \,|\,\Psi_\alpha(E_\alpha + i\,0)) = |\,\Psi_\alpha(E_\alpha + i\,0)\rangle, \tag{18}$$

where $G_\alpha(E_\alpha)$ is the outgoing Green's function which is the limit $\epsilon \to 0$ performed on $G_\alpha(E_\alpha + i\epsilon)$. Under very general conditions of convergence as discussed by Gerjuoy, one would expect the limits $\rho \to \infty$, $\epsilon \to 0$ to exist for the left sides of (15) and (16), and they then contradict (17) and (18), the results obtained by Gerjuoy. It may be instructive to evaluate the left side of (15) for a simple system. We take a 3-body system of equal mass particles with zero spin, $\chi_0(r_{12})$ being the bound state of particles 1 and 2 with third particle free so that $\varphi_\alpha = \chi_0(r_{12}) \exp(i\vec{k}_0\cdot\vec{R})$ and since only the bound state component survive we get, directly using (6), in coordinate representation,

$$\mathcal{I}\,(G_\alpha\,(E_\alpha+i\,\epsilon),\,|\,\varphi_\alpha\rangle)_\rho$$

$$=\frac{\chi_0(r_{12})}{(2\pi)^3}\int\frac{\exp\,(i\vec{k}'\cdot\vec{R})\,d^3k'}{k_0^2+i\,\epsilon-k'^2}\left[(k_0^2-k'^2)\int_0^\rho\exp\,[i(\vec{k}_0-\vec{k}')\cdot\vec{R}_1]\,d^3R_1\right] \quad (19)$$

$$=\frac{\chi_0(r_{12})}{(2\pi)^3}\int\frac{\exp\,(i\,\vec{k}'\cdot\vec{R})\,d^3k'}{k_0^2+i\,\epsilon-k'^2}\left[(k'^2-k_0^2)\int_\rho^\infty\exp\,[i(\vec{k}_0-\vec{k}')\cdot\vec{R}_1]\,d^3R_1\right], \quad (20)$$

for $$(k_0^2-k'^2)\,\delta\,(\vec{k}_0-\vec{k}')=0. \quad (21)$$

The integration from 0 to ρ within the square bracket term in (19) refers to integration over the dummy variable $\vec{R}_1$ being done within a sphere of radius ρ and that in (20) being external to this sphere. Equation (20) follows from (19) if (20) is used. A number of different results follow if we pass to the limits $\epsilon\to 0$, $\rho\to\infty$ in different order if we use (19), but not so if we use (20). For example, if we use (19) and keep ρ fixed and pass to $\epsilon\to 0$ first and then pass to the limit $\rho\to\infty$, then we get $|\,\varphi_\alpha\rangle$ just as (17) suggests. But if we keep ϵ fixed and pass to $\rho\to\infty$ first and then pass to $\epsilon\to 0$, we get zero by (21), as in (15). Thus (19) gives (17) or (15) by passing to $\epsilon\to 0$, $\rho\to\infty$ in different order and since the results of these limits are different we may conclude that either the double limit $\epsilon\to 0$, $\rho\to\infty$ does not exist or (19) is not the correct form to pass to the double limit. Equation (19) can, however, be rearranged slightly as in (20), using (21) and now the double limit is seen to exist uniquely, for $\epsilon\to 0$ and $\rho\to\infty$ taken in any order gives zero *i.e.* equation (15). We, therefore, conclude that the double limit $\epsilon\to 0$, $\rho\to\infty$ of the left sides of (19) or (15) exists and the result is zero

$$\mathscr{L}t_{\substack{\epsilon\to 0\\ \rho\to\infty}}\;\mathcal{I}\,(G_\alpha\,(E_\alpha+i\,\epsilon),\,|\,\varphi_\alpha\rangle)=0, \quad (22)$$

and this is consistent with (15) and not with (17). Alternatively, from (13), we have for $V_\alpha=0=\overline{V}_\alpha$, (dropping the subscript ρ)

$$F_\alpha\,(Z)=\mathcal{I}\,(G_\alpha\,(Z),\,|\,\varphi_\alpha\rangle), \quad (23)$$

$$=G_\alpha\,(Z)\cdot E_\alpha\,|\,\varphi_\alpha\rangle+[1-Z\,G_\alpha\,(Z)]\,|\,\varphi_\alpha\rangle, \quad (24)$$

$$=|\,\varphi_\alpha\rangle-(Z-E_\alpha)\cdot G_\alpha\,(Z)\,|\,\varphi_\alpha\rangle, \quad (25)$$

$$=|\,\varphi_\alpha\rangle-(Z-E_\alpha)\cdot\frac{1}{Z-E_\alpha}|\,\varphi_\alpha\rangle=0. \quad (26)$$

We thus note that $F_a(Z)$ as a function of complex variable Z is identically zero everywhere in the complex plane, excepting probably at one point $Z=E_a$, as seen from (26), where it is undefined and even there

$$\mathop{\mathscr{L}t}_{Z \to E_a} F_a(Z) = 0$$

exists and the result is zero as in (22). In the presence of the square-root cut on the positive real energy axis, the value of $F_a(Z)$ as positive real energy axis is approached from above or below is of concern only, (and not that at an isolated point $Z=E_a$). We therefore regard eqs (22) and (15) as valid and discard Gerjuoy's result, equation (17), as wrong. In the same way we accept (16) as true and discard (18), as follows from steps of (23) to (26) repeated with $G_a(Z)$ and $|\varphi_a\rangle$ replaced by $G(Z)$ and $|\Psi_a(Z)\rangle$, respectively. A possible source of mistake leading to (17) and (18) may be located from a discussion in the Appendix.

Finally, contrary to the claim of Gerjuoy, we now prove that (11) provides a unique solution to (10) and that the Lippmann-Schwinger equation for scattering state does have an unique solution which is also given by (11). We write, (without explicitly writing the subscript ρ)

$$|\Psi(Z_1)\rangle = |\varphi_a\rangle + G(Z_1)\bar{V}_a|\varphi_a\rangle - \mathscr{I}(G(Z_1), |\varphi_a\rangle) \tag{27}$$

and substitute for $|\varphi_a\rangle$ in it from (10) and get

$$\begin{aligned} |\Psi(Z_1)\rangle = {} & |\Psi_a\rangle + [G(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle - G_a(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle \\ & - G(Z_1)\bar{V}_a\cdot\{G_a(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle\}] \\ & + \mathscr{I}(G(Z_1), G_a(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle) \\ & + \mathscr{I}(G(Z_1), \mathscr{I}(G_a(Z_1), |\Psi_a(Z_1)\rangle)) - G(Z_1)\bar{V}_a\cdot\mathscr{I}(G_a(Z_1), |\Psi_a(Z_1)\rangle) \\ & - \mathscr{I}(G_a(Z_1), |\Psi_a(Z_1)\rangle) - \mathscr{I}(G(Z_1), |\Psi_a(Z_1)\rangle). \end{aligned} \tag{28}$$

It can be shown directly from the definition, equation (6), of Wronskian and equation (2) that,

$$\begin{aligned} & \mathscr{I}(G(Z_1), G_a(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle) \\ & = \{G_a(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle - G(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle \\ & + G(Z_1)\bar{V}_a\cdot\{G_a(Z_1)\bar{V}_a|\Psi_a(Z_1)\rangle\}. \end{aligned} \tag{29}$$

$$\begin{aligned} & \mathscr{I}(G(Z_1), \mathscr{I}(G_a(Z_1), |\Psi_a(Z_1)\rangle)) \\ & = \mathscr{I}(G_a(Z_1), |\Psi_a(Z_1)\rangle) + G(Z_1)\bar{V}_a\cdot\mathscr{I}(G_a(Z_1), |\Psi_a(Z_1)\rangle) \end{aligned} \tag{30}$$

Equations (29) and (30) put in (28) gives

$$|\Psi(Z_1)\rangle = |\Psi_a(Z_1)\rangle - \mathcal{J}(G(Z_1), |\Psi_a(Z_1)\rangle) \tag{31}$$

which, by (16), reduces to

$$|\Psi(Z_1)\rangle = |\Psi_a(Z_1)\rangle. \tag{32}$$

We thus prove that any solution $|\Psi_a(Z_1)\rangle$ of (10) is necessarily identical to $|\Psi(Z_1)\rangle$ of (27). In other words, (11) provides the unique solution to (10). It may be pointed out that Gerjuoy (1958) obtained only the first two terms on the right of (28), and he showed that the square-bracket term in (28) does not vanish or equivalently uniqueness (32) above does not hold unless an extra condition, $\mathcal{J}(G(Z_1), |\chi_a\rangle) = 0$, is assumed. From our analysis it is obvious that no such condition is required for the said uniqueness, for the square bracketed term in (28) is removed exactly by virtue of (29). Again, if the Wronskian term of (10) is absent, it reduces to the usual Lippmann-Schwinger equation for scattering states and the right side of (28) will only miss the terms that appear in (30). But since (29) is an identity, it will again remove all the undesirable terms in (28) and we again get back (31) and (32). This has a very important implication. It simply means that the Lippmann-Schwinger equation for scattering states has an *unique* solution and that solution is given by (11). According to our present analysis, the Wronskian-term of (11) remains there and its presence actually helps remove the undesirable terms in (28) proving the uniqueness of (32). But the situation can be improved further and it has been shown elsewhere (Mukherjee 1980) that the Wronskian term of (11) is actually zero. Here we indicate only how one can go about proving it. This follows from a simple overstation that $\mathcal{J}(G(Z), |\varphi_a\rangle)$ for any complex Z is an eigenfunction of H with a complex eigenvalue Z

$$(Z - \overrightarrow{H})\, \mathcal{J}(G(Z), |\varphi_a\rangle) = 0. \tag{33}$$

Equation (33) can be verified directly from the definition given by (6). Since the spectrum of H considered in a conventional scattering theory consists of points on the real axis, (33) can hold for any complex Z if and only if

$$\mathcal{J}(G(Z), |\varphi_a\rangle) = 0, \tag{34}$$

identically, whenever Z is not within the spectrum of H. Similar agreements also show why $F_a(Z)$ of (23) should be zero, as proved by alternative method in (23) to (26). This result adds further that the old Chew-Goldberger solution (equation (11) without the Wronskian term) is *the unique solution* of the Lippmann-Schwinger equation for scattering states (equation (10) without the Wronskian term).

We thus conclude that the Lippmann-Schwinger equation for scattering states in a given channel has a unique solution and this observation does not depend whether or not other channels, particularly rearrangement channels are open. This observation directly contradicts the results of Gerjuoy (1958) and others (see Sandhas 1972) which states that

$$\text{(i)} \quad |\Psi_\alpha^\dagger\rangle = \mathscr{L}\underset{\epsilon\to 0}{t}\, |\Psi_\alpha(Z_1)\rangle$$

is non-unique anyhow in the channel α-itself and further (ii) when the rearrangement channels are open, homogenous integral equations results for wavefunctions in other channels and that gives further non-uniqueness of $|\Psi_\alpha^\dagger\rangle$. In this paper, the first category of non-uniqueness is shown to be non-existent, and this eliminates the second category too, for as noted above, our discussions do not depend on the closed or openness of rearrangement channel. The said homogenous equation (Sandhas 1972) are ($\beta \neq \alpha$)

$$|\Psi_\beta^\dagger\rangle = G_\alpha (E_\alpha + io)\, \bar{V}_\alpha |\Psi_\beta^\dagger\rangle, \tag{35}$$

$$|\Psi_\alpha^\dagger\rangle = G_\beta (E_\alpha + io)\, \bar{V}_\beta |\Psi_\alpha^\dagger\rangle. \tag{36}$$

Recently Adhikari and Glöckle (1980) claimed that the homogenous equations are quite consistent with inhomogenous equations of scattering. They, in fact, obtained the following inhomogenous equations

$$|\Psi_\alpha^\dagger\rangle = \mathscr{I}(G_\alpha (E_\alpha + io), |\varphi_\alpha\rangle) + G_\alpha (E_\alpha + io)\, V_\alpha |\varphi_\alpha^\dagger\rangle \tag{37}$$

starting from the homogenous equation (36), and then using (17), which they established at length, they obtained the usual Lippmann-Schwinger equation. That this claim is wrong is easily seen from our discussion above where we show that (17) is wrong and hence the homogenous equation (36), which give rise to (37) does not reproduce the inhomogenous Lippmann-Schwinger equation. Since a lot of work is done using homogenous equations of scattering, the question of their existence, in view of the above, will be discussed at length in a future publication.

Appendix 1

For any Z in the complex plane the Green's function satisfy

$$G_\alpha(Z)\,(Z-\overleftarrow{H}_\alpha) = (Z-\overrightarrow{H}_\alpha)\, G_\alpha (Z)=1. \tag{A1}$$

It is tempting to write, as is often done, the equivalent of (A1) on real axis as

$$G_\alpha^\dagger(E_\alpha)\,(E_\alpha-\overleftarrow{H}_\alpha) = (E_\alpha-\overrightarrow{H}_\alpha)\, G_\alpha^\dagger (E_\alpha)=1, \tag{A2}$$

where

$$G_\alpha^\dagger(E_\alpha) = \mathscr{L}\underset{\epsilon\to 0}{t}\; G_\alpha(E_\alpha+i\epsilon)$$

is the Green's function with outgoing wave boundary condition, for real energy E_a. If (A2) is regarded as valid, then corresponding to (24) we have

$$\mathcal{I}(G_a^\dagger(E_a), |\varphi_a\rangle) = G_a^\dagger(E_a) \,.\, E_a |\varphi_a\rangle + [1 - E_a G_a^\dagger(E_a)] |\varphi_a\rangle, \tag{A3}$$

$$= |\varphi_a\rangle, \tag{A4}$$

which agrees with (17), the result of Gerjuoy. The source of the error in (A3) is easy to locate when it is compared with (24), and that is, when $Z = E_a + i\epsilon \to E_a + i0$, $ZG_a(Z)$ does not tend to $E_a G_a^\dagger(E_a)$ but becomes

$$\underset{Z \to Ea+i0}{\mathscr{L}\mathrm{t}} \; ZG_a(Z) = E_a G_a^\dagger(E_a) + \underset{\epsilon \to 0}{\mathscr{L}\mathrm{t}} \; [i\epsilon G_a(E_a + i\epsilon)], \tag{A5}$$

and the limit of the operator $i\epsilon G_a(E_a + i\epsilon)$ as $\epsilon \to 0$ does not go to zero, in strong or weak sense or any other acceptable topology used in scattering theory (that is, the product of the limits is not the limit of the product of operators). In other words, the claim is that, if

$$\underset{\epsilon \to 0}{\mathscr{L}\mathrm{t}} \; i\epsilon G_a(E_a + i\epsilon) |\chi\rangle = P |\chi\rangle, \tag{A6}$$

then $P|\chi\rangle \neq 0$ for all $|\chi\rangle$. This is obvious for,

$$i\epsilon G_a(E_a + i\epsilon) |\varphi_a\rangle = |\varphi_a\rangle$$

for every ϵ and hence so for $\epsilon \to 0$, and this is also the result for the so-called Lippman's identity (Sandhas 1976). Thus we cannot accept the second term on the right of (A5) as zero, and for the same reason we note that (A1) does not tend to (A2) for, we can rewrite (A1) as

$$G_a(Z)(E_a - \overleftarrow{H}_a) = (E_a - \overrightarrow{H}_a) G_a(Z) = 1 - (Z - E_a) G_a(Z), \tag{A7}$$

and because of (A5) and (A6) and the discussion above, the last term in (A7) does not go to zero as $Z \to E_a + i0$, and hence (A4) is never really true. We, therefore, again have, from (6) and (A7),

$$\mathcal{I}(G_a^\dagger(E_a), |\varphi_a\rangle) = G_a^\dagger(E_a)\,(E_a - \overleftarrow{H}_a) \,.\, |\varphi_a\rangle = 0, \tag{A8}$$

and this is consistent with our equation (15) and contradicting equation (17).

It thus appears that a possible explanation of the wrong equations, (17) and (18), could be the use of equation (A2) which is proved to be wrong above. The afilure of (A2) for Green's function does not come in the way of solving the equation like $(E_a - \vec{H}_a)\,|\varphi_a\rangle = |\xi\rangle$ with E_a real by the method of Green's function as $|\varphi\rangle = G_a^\dagger(E_a)\,|\,\xi\rangle$. It is straightforward to show that it is sufficient for $G_a^\dagger(E_a)$ to be defined as

$$G_a^\dagger(E_a) = \mathscr{L}\underset{\epsilon \to 0}{t}\; G_a(E_a + i\epsilon) = \frac{P}{E_a - H_a} - i\pi\delta(E_a - H_a)$$

which may not satisfy (A2), but will always give

$$(E_a - \vec{H}_a)\,[G_a^\dagger(E_a)\,|\,\xi\rangle] = |\,\xi\rangle,$$

and that is enough for solving the above inhomogenous equation by the method of Green's function.

References

Adhikari S K and Glöckle W 1980 *Phys. Rev.* **C21** 54
Gerjuoy E 1958 *Phys. Rev.* **109** 1806
Goldberger M L and Watson K M 1964 Collision theory (New York: John Wiley)
Mukherjee S 1980 To be published
Sandhas W 1972 *Acta Phys. Austr. Supl.* **10** 57
Sandhas W 1976 *Few body problems* ed. A N Mitra (Amsterdam: North Holland)