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Baryon magnetic moments in a quark model

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Abstract. The magnetic moments of uncharmed and charmed baryons are considered to arise through single-quark and two-quark transitions in a quark model. The magnetic moment operator is taken to transform as:
$T^{\alpha}_{\beta} \sim aT^{1}_{1} + bT^{2}_{2} + cT^{3}_{3} + dT^{4}_{4}$, where T^{α}_{β} are members of SU(4) $\underline{20}'$-plet. The assumption, that the magnetic moment operator obtains contribution from the single and two-quark transitions, yields good results for the magnetic moment values of uncharmed baryons. Magnetic moments of charmed baryons can be expressed in terms of one parameter.

Keywords. Magnetic moments; uncharmed baryons; charmed baryons; quark model.

1. Introduction

Magnetic moments have played an important role in our current understanding of the structure of matter. Several attempts have been made to calculate the magnetic moments of baryons in symmetry schemes (Coleman and Glashow 1961; Okubo 1962, Bég *et al* 1964; Choudhary and Joshi 1976; Verma and Khanna 1977; Bohm 1978; Singh *et al* 1979) and in the quark model (Thirring 1965; Rujüla *et al* 1975; Lichtenberg 1977; Singh 1977). However, our theoretical understanding within the SU(3) scheme has so far been beset with difficulties. The recent precise measurement of the magnetic moment of Ξ° (Bunce *et al* 1979), Σ^{+} (Settles *et al* 1979) and Ξ^{-} (Devlin *et al* 1980), has put a tremendous strain on the various versions of the quark model when combined with the previous measurement of Σ^{-}, Λ (Bricman *et al* 1978); it appears that virtually all hyperon moments measurements contribute to the disagreement with any reasonable quark model. However, the conventional quark model, where the magnetic moment operator is assumed to be proportional to the quark charge to mass ratio (Bég and Pais 1965 and Rujüla *et al* 1975), works to a reasonable accuracy. Possible sources of discrepancies in the magnetic moment calculation are the neglect of exchange current and other relativistic effects (Hendry and Lichtenberg 1978). Recently, it has been proposed by Geffen and Wilson (1980) that the effective magnetic moment of quarks in hadrons should have an anomalous moment contribution because of the magnetic coupling of the photon to three or more gluons, and an acceptable fit is possible if the quark magnetic moments are allowed to be arbitrary ($\mu_s \neq \mu_d \neq -\frac{1}{2}\mu_u$).

In the present work we employ a quark model to study the magnetic moments as arising through single-quark and two-quark transitions. The single-quark transition gives the spectator quark contribution, whereas the two-quark transition, which may arise because of the exchange of coloured gauge gluons between the two quark, gives the non-spectator quark contributions to the magnetic moments. Experiments show that about 50% of nucleon's momentum resides on gluons; therefore the exchange of gluons between the quarks may give an important contribution to the magnetic moments. However, we will not consider explicit gluon corrections to the two-quark transition. We simply assume that corrections are absorbed into the wave-functions and so are included into the effective Hamiltonian. A proper understanding of gluon corrections would require a deeper study of the dynamics of quark pair interaction. In the present paper, we study the magnetic moments of $1/2^+$ baryons as arising through single-quark and two-quark transitions. We make the following assumptions for the calculations:

(i) the baryons are nonrelativistic bound states of three quarks in s-state and described by the spin-unitary spin wave function as belonging to 120 representation of SU(8).

(ii) the emission of virtual photon accompanied by the single-quark and two-quark transitions takes place such that these transitions have no dynamical influence on the spectator quarks. In the two-quark transition, the quark pair may interact through the exchange of gauge gluons and may include in some way, the quantum chromodynamic corrections (QCD). The magnetic moment of a baryon is then written as:

$$\langle B | M | B \rangle = \langle B | \sum_{i=1}^{3} M_1^{(i)} + \sum_{i \neq j = 1}^{3} M_2^{(i,j)} | B \rangle, \tag{1}$$

where $M_1^{(i)}$ and $M_2^{(i,j)}$ are the magnetic moment operators associated with the (i)th single-quark and (i, j)th two-quark transitions, respectively. $|B\rangle$ is the SU(8) hadron-state wave functions (Singh 1977).

(iii) the magnetic moment operator $M_1^{(i)}$ and $M_2^{(i,j)}$ of baryons are described as sum of the contributions from the constituent quarks.

2. Magnetic moment operator

We assume that the electromagnetic current transforms as a $T^{\alpha}_{\beta} \sim aT^1_1 + bT^2_2 + cT^3_3 + dT^4_4$ component of SU(4) and the effects of symmetry-breaking interaction arising due to the strong interaction dynamics are expressed in terms of four independent parameters **a**, **b**, **c** and **d**. The first three parameters can be evaluated using the experimental values of $\mu(p)$, $\mu(n)$ and $\mu(\Lambda^\circ)$ as input. But it is hard to calculate the value of **d**, until the magnetic moment of any one of charmed baryons is experimentally known.

3. Magnetic moment of baryons

In the following, we derive various magnetic moments sum rules for $1/2^+$ baryons. The notations used to describe the particles is given in Verma and Khanna (1977).

3.1 *Single quark transition*

In the single-quark transition, we do not add the corrections to the magnetic moments due to the exchange of coloured gauge gluons between the quarks. The magnetic moment operator M_1 for the single-quark transition can be written as

$$M_1 = k\,[(q^{-m}\,\vec{\sigma}\,q_n)\,T^n_m], \tag{2}$$

where T^n_m is the electromagnetic current transforming as above. The choice of $\vec{\sigma}$ is to ensure the vector nature of the interaction. The magnetic moment of a baryon is then given by,

$$\mu(B) = k\,\langle B\,|\,(q^{-m}\,\vec{\sigma}\,q_n)\,T^n_m\,|\,B\rangle, \tag{3}$$

The evaluation of magnetic moment is straightforward (Nakagawa and Trofimenkoff 1968; Kanwar *et al* 1980) and the calculated values are displayed in table 1. We obtain the following sum rules:

Table 1. The baryon magnetic moments in nuclear magneton.

μ (in units of nm)	Single-quark transition	(Single quark + two quark transitions)	Experimental
B(8) multiplet:			
p	2·79 (input)	2·79 (input)	2·793
n	−1·91 (input)	−1·91 (input)	−1·913
Λ	−0·61 (input)	−0·61 (input)	−0·606 ± 0·034
Σ^+	2·67	2·72	2·33 ± 0·13 (Settles *et al* 1979)
Σ^0	0·79	0·50	—
Σ^-	−1·09	−1·487	−1·48 ± 0·37
Ξ^0	−1·42	−1·29	−1·20 ± 0·06 (Bunce *et al* 1979)
Ξ^-	−0·49	−0·80	−0·75 ± 0·07 (Devlin *et al* 1980)
$\Lambda\Sigma^0$	1·68	1·72	$1{\cdot}82^{+0{\cdot}25}_{-0{\cdot}18}$
B(6) multiplet:			
Σ_1^{++}	2·47−0·33x	2·81−0·67x	
Σ_1^+	1·88−0·33x	1·39−0·67x	—
Σ_1^0	−1·29−0·33x	−1·60−0·67x	—
Ξ_1^+	0·83−0·33x	0·60−0·67x	—
Ξ_1^0	−1·05−0·33x	−1·30−0·67x	—
Ω_1^0	−0·81−0·33x	0·99−0·67x	—
B(3*) multiplet:			
$\Lambda_1'^+$	x	0·11+3x	—
$\Xi_1'^+$	x	0·16+3x	—
$\Xi_1'^0$	x	2·5x	—
B(3) multiplet:			
Ξ_2^+	−0·62+1·33x	−0·33x	—
Ξ_2^{++}	0·522+1·33x	0·19x	—
Ω_2^+	0·202+1·33x	0·12+5·67x	—

B (8) *multiplet:*

$$2\mu(\Sigma^0) = \mu(\Sigma^+) + \mu(\Sigma^-)\ (0{\cdot}85 \pm 0{\cdot}50)$$
$$= \tfrac{4}{3}\,[\mu(n) + \mu(p)] - \tfrac{2}{3}\,\mu(\Lambda^0) \tag{4}$$
$$(1{\cdot}574 \pm 0{\cdot}06)$$

$$5[\mu(\Sigma^+) - \mu(\Sigma^-)] = 4[\mu(p) - \mu(n)]$$
$$(19{\cdot}05 \pm 2{\cdot}5) \qquad (18{\cdot}80 \pm 0{\cdot}00), \tag{5}$$

$$\mu(\Xi^0) - \mu(\Xi^-) \quad = \quad -\tfrac{1}{4}\,[\mu(\Sigma^+) - \mu(\Sigma^-)]$$
$$(-0{\cdot}55 \pm 0{\cdot}13) \qquad (-0{\cdot}95 \pm 0{\cdot}16), \tag{6}$$

$$\mu(\Lambda\Sigma^0) = \frac{-1}{\sqrt{3}}\,[\tfrac{3}{5}\,\mu(p) - \mu(n)] + \mu(\Lambda^0)$$
$$(-1{\cdot}27 \pm 0{\cdot}02). \tag{7}$$

B(6) *multiplet:*

$$\mu(\Sigma_1^{++}) - \mu(\Sigma_1^0) = 2\,[\mu(\Xi_1^+) - \mu(\Xi_1^0)]$$
$$= \mu(\Sigma^+) - \mu(\Sigma^-). \tag{8}$$

$$\mu(\Omega_1^0) - \mu(\Xi_1^0) = \tfrac{1}{2}\,[\mu(\Xi^0) - \mu(n)]. \tag{9}$$

$$\mu(\Omega_1^0) - \mu(\Sigma_1^0) = 4\,[\mu(\Sigma^+) - \mu(p)]. \tag{10}$$

B(3*) *multiplet:*

$$\mu(\Lambda_1'^+) = \mu(\Xi_1'^+) = \mu(\Xi_1'^0). \tag{11}$$

B(3) *multiplet:*

$$\mu(\Omega_2^+) - \mu(\Xi_2^+) = \mu(\Sigma^+) - \mu(p). \tag{12}$$

$$\mu(\Xi_2^{++}) - \mu(\Xi_2^+) = \tfrac{1}{5}\,[\mu(n) - \mu(p)]. \tag{13}$$

The sum rules are essentially the same as obtained by Rujüla *et al* (1975) in quark model and in broken SU(4) symmetry (Verma and Khanna 1977) and Singh *et al* (1979).

3.2 *Two-quark transition*

In the two-quark transition, we assume that the quark pair interaction may include the QCD correction to the magnetic moments due to the exchange of coloured gauge gluons. In the gauge theory model (Rujüla *et al* 1975), quarks have colour, and the baryon wave function is antisymmetric in the colour indices. This puts restriction on gluons exchange between quark pairs. We, however, treat the two-quark transition in the general symmetry arguments and do not go into the detailed dynamical nature of the quark pair-interaction. As the quarks have been assumed to be in S-state, the magnetic moment operator M_2 causing the two-quark transition is written as:

$$M_2 = k' \left[\{(q^{-m} \vec{\sigma}\, q_e)(q^{-e}\, q_n) + (q^{-m}\, q_e)(q^{-e} \vec{\sigma}\, q_n) + (q^{-m} \vec{\sigma}\, q_e) \times (q^{-e} \vec{\sigma}\, q_n)\} \, T^n_m \right]. \tag{14}$$

We now assume that the magnetic moment arises through the single-quark and two-quark transitions, where two-quark transition may give QCD corrections to the single-quark transition. Then the magnetic moment operator becomes

$$M = k'' \left[\{(q^{-m} \vec{\sigma}\, q_n)\} + \{(q^{-m} \vec{\sigma}\, q_e)(q^{-e}\, q_n) + (q^{-m} \vec{\sigma}\, q_e) \times (q^{-e} \vec{\sigma}\, q_n) + (q^{-m} \vec{\sigma}\, q_e) \times (q^{-e} \vec{\sigma}\, q_n)\} \, T^n_m \right]. \tag{15}$$

Strong interaction dynamics is implicit in the parameter k''. Here, we assume the same breaking parameter for the single-quark and two-quark transitions. In view of the nebulous state of knowledge about the dynamics of quark pair interaction, it is difficult to comment on the above assumption. However, it may get support from the fact that QCD is flavour-independent. The calculated magnetic moment of uncharmed and charmed baryons are displayed in table 1. The magnetic moment of uncharmed baryons are in excellent agreement with the recent experimental values (Bingham *et al* 1970, Cool *et al* 1974; Bricman *et al* 1978) except $\mu(\Sigma^+)$, which shows a large deviation from the predicted value. The value of $\mu(\Xi^-)$ is in excellent agreement with the recent accurate measurement by Michigan, Minnesota, Rutgers and Wisconsin collaboration (Devlin *et al* 1980). The magnetic moment of charmed baryons is expressed in terms of one parameter. By knowing the magnetic of one charmed baryon, the others can be estimated. We obtain the following magnetic moment sum rules for $1/2^+$ baryons;

B(8) *multiplet*

$$-15[2\mu(\Sigma^0) - \{\mu(\Sigma^+) + \mu(\Sigma^-)\}] = 4[\mu(n) + \mu(p)] \tag{16}$$

$$\begin{aligned} 2[\mu(\Sigma^+) - \mu(\Sigma^-)] &= 17[\mu(\Xi^-) - \mu(\Xi^0)] \\ (7{\cdot}62 \pm 1{\cdot}00) &\qquad (9{\cdot}33 \pm 2{\cdot}1) \\ &= \tfrac{34}{19}[\mu(p) - \mu(n)] \\ &\qquad (8{\cdot}41 \pm 0{\cdot}00), \end{aligned} \tag{17}$$

$$\begin{aligned} 8\sqrt{3}\,\mu(\Lambda\Sigma^0) &= 42[\mu(\Xi^0) - \mu(\Xi^-)] - [3\{\mu(\Sigma^+) + \mu(\Sigma^-)\} - \tfrac{17}{5}\{\mu(p) + \mu(n)\}] \\ &= 42[\mu(\Xi^0) - \mu(\Xi^-)] + [\mu(\Lambda^0) - \tfrac{1}{10}\{\mu(p) + \mu(n)\}], \end{aligned} \tag{18}$$

B(6) *Multiplet*

$$\mu(\Sigma_1^{++}) + 2\,\mu(\Omega_1^0) + \mu(\Sigma_1^0) = 2\,[\mu(\Xi_1^+) + \mu(\Xi_1^0)] \tag{19}$$

$$\frac{17}{13}\,[\mu(\Sigma_1^{++}) - \mu(\Sigma_1^0)] = \frac{17}{19}\,[\mu(p) - \mu(n)] = [\mu(\Sigma^+) - \mu(\Sigma^-)]. \tag{20}$$

B(3*) *multiplet*

$$4\,[\mu(\Xi'^+_1) - \mu(\Lambda'^+_1)] = -3\,[\mu(\Sigma^+) - \mu(\Sigma^-)]. \tag{21}$$

4. Conclusion

The electromagnetic phenomena involving strongly interacting particles, like the radiative decays of hadrons, magnetic moments, etc are not well explained in the conventional models of electromagnetic interaction. However, most of the data can be understood if we assume that magnetic moment of baryons are considered to arise through single-quark and two-quark transitions. The two-quark transition, where the quark pair may interact through the exchange of coloured bosons, may give in some way the QCD correction to the single-quark transition.

In this paper, the magnetic moments of charmed as well as uncharmed baryons are calculated with quark transitions in a quark model. Several relations among the magnetic moments of $1/2^+$ baryons have been obtained. The magnetic moment values for uncharmed sector are in excellent agreement with the recent experimental values. Magnetic moments of charmed baryons are expressed in terms of one parameter.

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