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High momentum nucleons in the nucleus

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Abstract. Using Jastrow form for the nuclear wave function, single-particle distributions in the momentum space are extracted for the correlation functions corresponding to the Reid soft core, Hamada-Johnston and Ohmura-Morita-Yamada (OMY) hard core potentials. The correlations functions used for this purpose are the numerical solutions of the Schrödinger type equation for the realistic potentials and analytical form for the OMY potential. It is found that the calculated momentum distributions, with Woods-Saxon basis functions, differ significantly beyond 400 MeV/c. Comparison with the experimental proton momentum distribution from (γ, p) reaction suggests that while the OMY potential results are nearer to the experimental values, the realistic potentials do not introduce the high momentum components to the required extent.

Keywords. Nuclear reactions; single hole states; Jastrow correlations; realistic nucleon-nucleon potentials.

1. Introduction

Since the nucleus consists of neutrons and protons, it is of primary interest to study their momentum distributions in the nucleus. Experimentally the low momentum ($<$ 250 MeV/c) part of this distribution is provided by the knock-out reactions like $(p, 2p)$, $(e, e'p)$, etc (Jacob and Maris 1973). The high momentum components ($>$ 250 MeV/c) seem to be revealed in the high energy (p, d) reactions (Igo 1978), photo- and pion-absorption processes (Brown 1969, Wilkinson 1968), the high energy proton scattering on nuclei at backward angles and the proton and pion inclusive spectra in relativistic heavy ion collisions (Frankel *et al* 1976; Brody *et al* 1977; Nagamiya *et al* 1977, Hatch and Koonin 1979). From the nuclear structure point of view the low momentum components are associated with the independent particle model, while the high momentum components are intimately connected with the short-range behaviour of the nucleon-nucleon interaction in the nucleus. Since the two-nucleon bound and scattering states do not discriminate between the various potentials which differ in their short-range behaviour, it is necessary that the reactions which exclusively look for the high momentum nucleons in nuclei should be used to study the core behaviour of the nucleon-nucleon interaction. In reactions like (p, d), (γ, p) and $(\pi^+, p,)$ which are normally analysed in the distorted wave impulse approximation (DWIA), these studies can be pursued through the study of ' overlap integral'. The overlap integral $\psi_a(1)$ represents the overlap of the initial and final nuclear wave functions, *i.e.*

$$\psi_a(1) = A^{1/2} \langle \Psi_f (A-1) \mid \Psi_i (A) \rangle,$$

where Ψ_i and Ψ_f are the internal wave functions of the initial and final nuclei respectively. The radial coordinate in ψ_a is relative to the centre-of-mass of the residual nucleus. It is the Fourier transform of this function that is extracted from the reactions like (p, d), (γ, p) (π^+, p), etc. Since in DWIA, the final state interaction is implicit, the extracted momentum distribution for closed shell nuclei is customarily interpreted as the momentum distribution of the bound nucleon in the initial nucleus. This interpretation of course ignores the non-orthogonality of the single particle orbitals in Ψ_i and Ψ_f arising due to 're-arrangement effects' (Berggren 1965). Theoretically, the low momentum components of $\psi_a(1)$ are satisfactorily described by the shell model description of Ψ_i and Ψ_f, thus supporting the picture of nucleons moving independently in a common single-particle potential. Once the single-particle potential is known, the high momentum components of the nucleons in the nucleus can be generated, in a consistent fashion, using the Jastrow ansatz (Jastrow 1955), provided the correlation functions are of sufficiently short range. Some efforts have been made in this framework to study the sensitivity of the photo- and pion-absorption to the short range correlations (Dillig and Huber 1974; Ciofi degli Atti 1972). However, since the motivation in these investigations was limited to studying sensitivity, the correlation functions used were generally *ad hoc*. Sometimes, even single particle orbitals were generated in the not so realistic harmonic oscillator potential, which is not consistent with the low momentum components of the wave function. Besides, we realize that it is not enough to detect only the sensitivity of the reaction data to short range correlations, but one needs to use them to measure the correlations. This can be achieved by employing in the reaction analyses the nucleon momentum distribution using correlation functions corresponding to various realistic nucleon-nucleon potentials and realistic single particle potentials consistent with low momentum components.

In the present paper, we have calculated the single particle momentum distribution for the Hamada-Johnston hard core and Reid soft core potentials (Reid 1968, Hamada and Johnston 1962), using Elton-Swift single particle basis functions (Elton and Swift 1967). For comparison we have also used the OMY potential (Ohmura *et al* 1956) which is generally used in the nuclear matter test calculations. The calculations carried out here are according to the Jastrow framework and following the detailed procedure evolved by Weise and others (Weise and Huber 1971, Weise 1972; Huber 1971). This procedure has certain uncertainties (Ristig and Clark 1975) due to the non-orthogonality in Ψ introduced by the correlation factor and the termination of the cluster expansion at two-body terms. In the present work the cluster expansion upto two-body is considered adequate as the correlation functions arising out of various potentials used here are fairly short range. As regards the non-orthogonality due to the correlation factor, the complete effect could be obtained only if the non-orthogonality arising due to the re-arrangement is also included. In the absence of any tractable method to treat both the non-orthogonalities together, it is hoped that their net effect is small.

2. Formalism

If Φ is the Slater determinant, a correlated wave function in the Jastrow method is written as

$$\Psi_i(A) = N_i^{-1/2} \prod_{j>k=1}^{A} f(j,k)\,\Phi_i(A), \tag{1}$$

for A particles, and

$$\Psi_f(A-1) = N_f^{-1/2} \prod_{j'>k'=2}^{A} f(j',k')\,\Phi_f(A-1), \tag{2}$$

for $(A-1)$ particle system. In equations (1) and (2) N is the normalization constant and is given by

$$N = \int \prod_{j<k} |f(j,k)|^2\,|\Phi|^2\,d\tau,$$

f is the two-nucleon correlation function and is a function of relative coordinate $r_{jk} = |\mathbf{r}_j - \mathbf{r}_k|$. This function modifies the wave function only at short relative distances and that at larger distances remains left unaffected. This means that f should correspond to that part of the two-body interaction which does not contribute to the average potential. For the same reason the uncorrelated wave function Φ is required to be such that it maximizes systematically the average potential aspect of the nucleus. Phenomenologically, it can be achieved by taking Φ as the solution of that single particle potential which simultaneously accounts well for the electron scattering data at low momentum transfer, and $(p, 2p)$ and $(e, e'p)$ type reactions. f itself should be calculated from the Schrödinger type equation introduced by Pandharipande and Bethe (Pandharipande and Bethe 1973):

$$\left[-\frac{\hbar^2}{m}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + k^2\right) + (V-\lambda)\right] f = 0, \tag{3}$$

where V is the two body interaction and λ is interpreted as that part of V which contributes only to the average field. $(V-\lambda)$, therefore, by definition, does not produce any scattering. This is equivalent to the boundary condition that beyond a certain distance d, $f(r>d) = 1$, which ensures that beyond $r>d$, the correlated function Ψ goes over to the uncorrelated function Φ. The parameter d is determined numerically by minimising the binding energy of the nuclear matter.

In terms of Ψ, the single particle amplitude is written as

$$\psi_\alpha(1) = A^{1/2} \langle \Psi_f(A-1) | \Psi_i(A) \rangle. \tag{4}$$

If we retain only the two-body correlation terms, then,

$$\psi_\alpha(1) = (N_i N_f)^{-1/2} \Big[\phi_\alpha(1) - \sum_\beta \langle \phi_\beta(2) | g(1,2) | \phi_\alpha(1)\,\phi_\beta(2) - \phi_\alpha(2)\,\phi_\beta(1) \rangle\Big] \tag{5}$$

where $g(1, 2)$ is related to $f(1, 2)$ through

$$f(1, 2) = 1 - g(1, 2). \tag{6}$$

ϕs are the single-particle wave functions which make up the uncorrelated wave function Φ. In the same approximation the normalizing constant N is given by

$$N = \int \left[1 + \frac{A(A-1)}{2} (|f(1, 2)|^2 - 1) \right] |\Phi(A)|^2 \, d\mathbf{r}_1 \ldots d\mathbf{r}_A$$

In terms of $\psi_\alpha(1)$ the single-particle momentum density is defined as

$$\Omega_{nlj}(q) = (2\pi)^{-3} N_{lj} (2l+1)^{-1} \sum_m \left| \int \exp(i\mathbf{q}\cdot\mathbf{r}_1) \, \psi_{nljm}(\mathbf{r}_1) \, d\mathbf{r}_1 \right|^2 \tag{7}$$

where, in place of α we have written the single particle quantum numbers $nljm$ explicitly. ψ_{nljm} is the spatial part of $\psi_\alpha(1)$. Following Weise (1972), explicit expression for $\psi_{nljm}(\mathbf{r}_1)$ (from equation (5)) may be written for closed shell nuclei as

$$\psi_{nljm}(\mathbf{r}_1) = [R_{nlj}(r_1) - \delta R_{nlj}(r_1)] \; Y_{lm}(\hat{r}_1), \tag{8}$$

where $R_{nlj}(r_1)$ is the radial part of the uncorrelated single particle wave function. The correction term $\delta R_{nlj}(r_1)$ in equation (8) is given by

$$\delta R_{nlj}(r_1) = \sum_{l_\beta \, \tau_{z_\beta}} \int dQ \; W(Q) \, [U_\beta^{\tau_{z_\beta}}(Q, r_1) - \delta_{\tau_z \tau_{z_\beta}} V_\beta(Q, r_1)], \tag{9}$$

with $$U_\beta^{\tau_{z_\beta}}(Q, r_1) = (2l_\beta+1) \, j_0(Q r_1) \, R_{nl_j}(r_1) \int_0^\infty dr_2 \, r_2^2 \, |R_{n_\beta l_\beta}^{\tau_{z_\beta}}(r_2)|^2 \, J_0(Q r_2). \tag{10}$$

and $$V_\beta(Q, r_1) = \sum_\lambda (2\lambda+1) \, (l\lambda 00 | l_\beta 0)^2 \, j_\lambda(Q r_1) \, R_{n_\beta l_\beta}(r_1) \times$$

$$\int dr_2 \, r_2^2 \, R_{n_\beta l_\beta}(r_2) \, j_\lambda(Q r_2) \, R_{nl_j}(r_2). \tag{11}$$

τ_z denotes the charge state of the nucleon. $W(Q)$ is related to $g(r)$ through the Fourier-Bessel transform

$$g(r) = \int_0^\infty dQ \; W(Q) \, j_0(Q r),$$

or $$W(Q) = \frac{2}{\pi} Q^2 \int dr \, r^2 \, j_0(Q r) \, g(r). \tag{12}$$

This means that $W(Q)$ is the measure of the momentum package which can be introduced, through $g(r)$, by the short range part of a nucleon-nucleon interaction into the otherwise independently moving nucleons.

3. Results and discussion

We have calculated the single-proton momentum density for ^{16}O for $1p_{1/2}$ and $1s_{1/2}$ shells. The uncorrelated radial function R_{nlj} is generated in a Woods-Saxon potential, the parameters of which are taken from the work of Elton and Swift (1967). These parameters fit the elastic electron scattering (Elton and Swift 1967), $(p, 2p)$ and $(e, e'p)$ reactions (Shanta and Jain 1971). The correlation functions for the Reid soft core and Hamada-Johnston potentials are taken from the calculations of Pandharipande and Wiringa (1976) who obtain them numerically by solving equation (3). We have used their k and l averaged values corresponding to Fermi momentum, k_F, equal to 1.4 fm^{-1}. This value of k_F is very close to the normal nuclear matter density. The results corresponding to these two potentials are shown in figure 1 along with that for the uncorrelated wave function. It can be seen that the momentum distributions for Reid and Hamada-Johnston potentials do not differ much from the uncorrelated distribution upto about 500 MeV/c, and Beyond this value they differ considerably among themselves as well as with the uncorrelated distribution. The results for $1s$ shell are similar. Figure 2, for example, shows the $1s$ results for the Reid soft core potential along with that for the uncorrelated wave function.

For comparison we have also calculated the OMY potential (Ohmura *et al* 1956) commonly used in the nuclear matter calculations. The correlation function for this potential is

$$f(r) = \begin{cases} 0 & , r \leqslant c \text{ (0·6 fm)}, \\ [1-\exp(-\mu_1 (r-c))]\ [1+\gamma \exp(-\mu_2 (r-c))], & r > c \end{cases}$$

where the parameters μ_1, μ_2 and γ are determined by the energy minimization. For $k_f = 1{\cdot}4$ fm^{-1}; $\mu_1 = 2{\cdot}05$ fm^{-1}, $\mu_2 = 1{\cdot}35$ fm^{-1} and $\gamma = 0{\cdot}484$ (Chakraborty 1978, 1979). The results for the $1p_{1/2}$ and $1s$ protons given in figures 1 and 2 show that for the OMY potential, the momentum distribution starts deviating from the uncorrelated one from 300 MeV/c itself. Also the effect for this potential is much larger than due to either Reid or HJ potentials.

In figure 1 we also show the experimental momentum distribution for $1p_{1/2}$ protons extracted from the (γ, p) reaction (Findlay *et al* 1978). This distribution may be subjected to some uncertainty due to the probable inadequacy of the impulse approximation for the analysis of (γ, p) reaction (Londergan and Nixon 1979). It shows that in the Jastrow framework the correlations corresponding to the realistic potentials like Reid soft core and HJ do not enhance the high momentum components strongly enough to bring them close to the probable experimental values. The OMY potential does much better. The calculated momentum distribution for the OMY potential reproduces the magnitude and shape of the experimental distribution upto about 550 MeV/c and the positions of maxima and minima even beyond it.

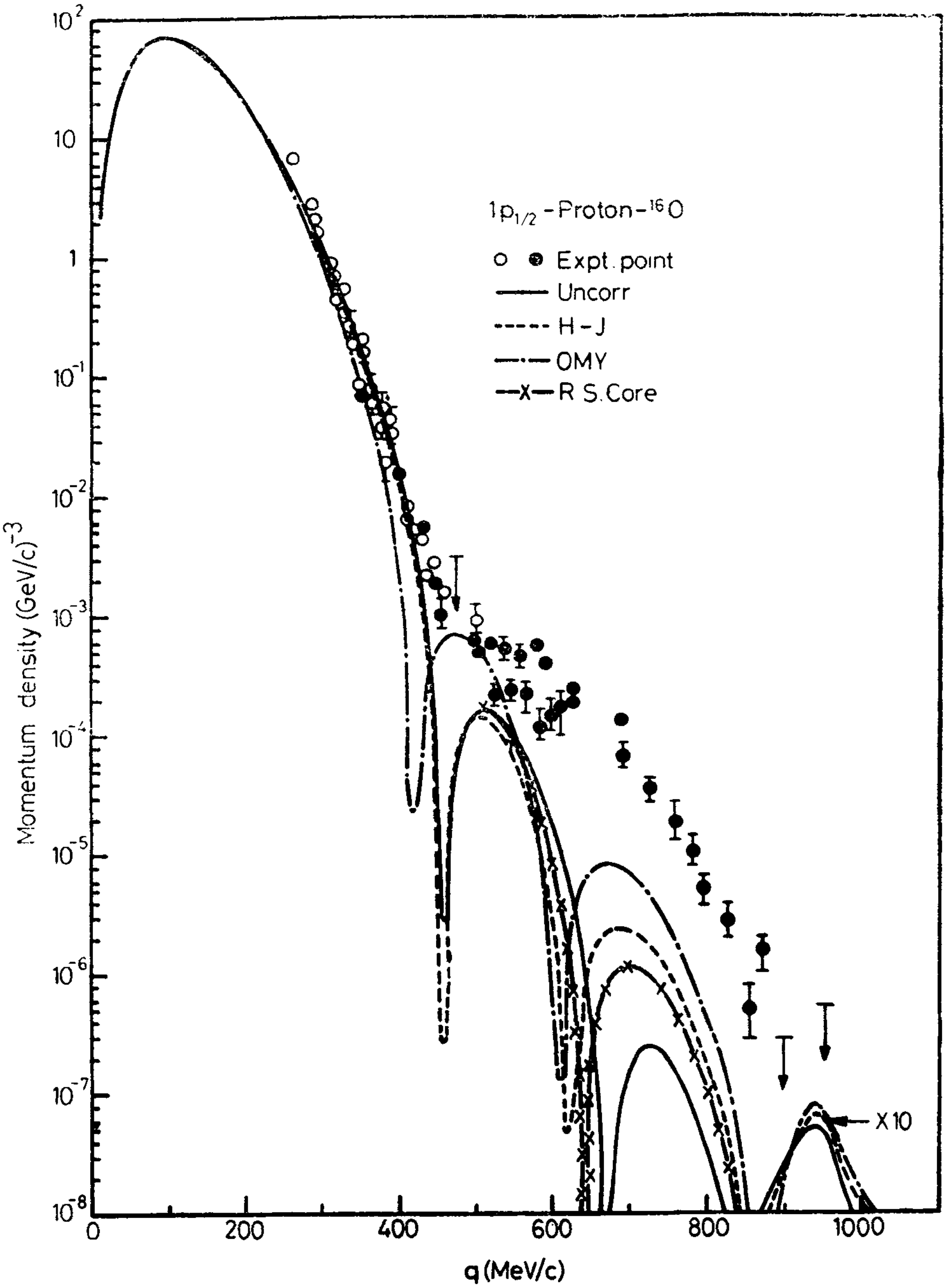


Figure 1. Single particle momentum distribution for $1p_{1/2}$ protons. Experimental points are from Findlay *et al* (1978). Uncorrelated distribution is for Elton-Swift (1967).

This big difference in the momentum distributions of OMY and the realistic potentials and the similarity between the Reid and HJ potentials may be understood from a study of the momentum package the various correlation functions contain.

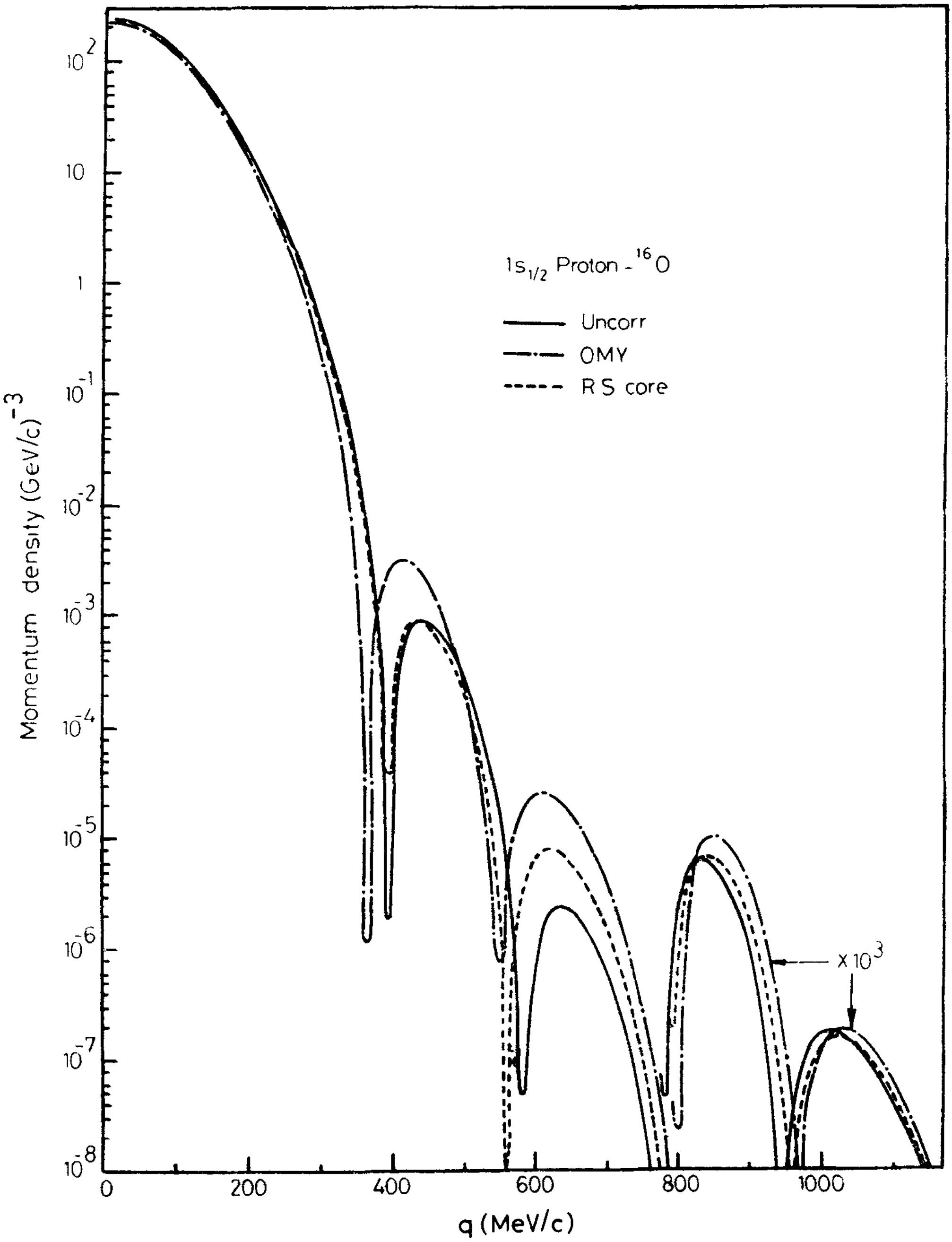


Figure 2. Single particle momentum distribution for $1s_{1/2}$ protons.

In figure 3 we plot the correlation function $f(r)$ and the corresponding momentum package $W(Q)$ for the three potentials used here. It shows that for the Reid and HJ potentials $W(Q)$ peak around the same value of $Q(\sim 800$ MeV/c). The magnitudes at the peak of course differ by a factor 2. In contrast $W(Q)$ for the OMY potential

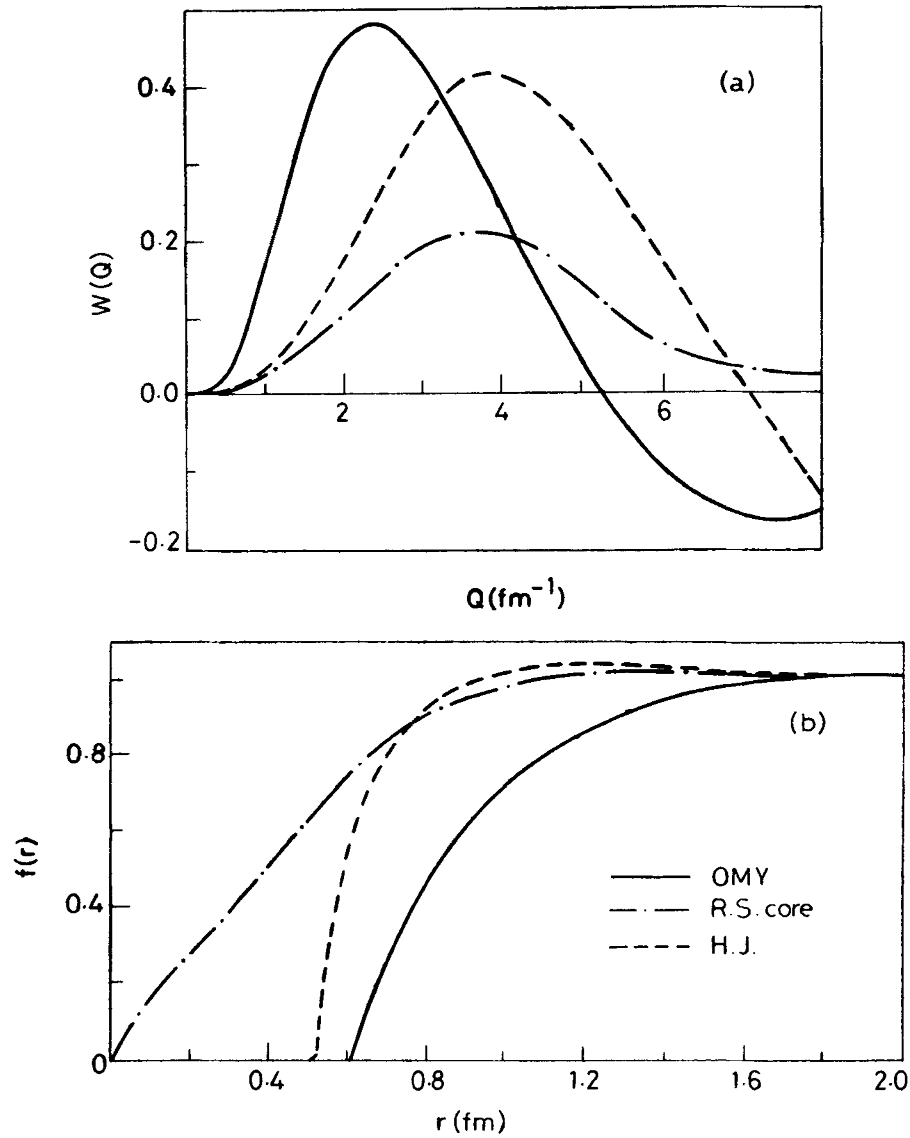


Figure 3. **a.** Momentum package $W(Q)$ corresponding to three correlation functions. **b.** Correlation function $f(r)$ for three potentials.

differs considerably from the other two. It peaks around 400 MeV/c. This arises due to the larger hard core radius for the OMY potential. As we see from equation (12), $W(Q)/Q$ is the sine transform of r g(r), where r g(r) peaks at the hard core radii for the HJ and the OMY potentials and around 0·45 fm (which is close to the HJ hard core radius) for the Reid soft core potential.

In order to demonstrate the importance of the use of the proper basis function Φ in equations (1) and (2) to determine $f(r)$ correctly, we have compared in figure 4, the $1p_{1/2}$ distribution for ^{16}O using Elton-Swift and the harmonic oscillator potentials with the oscillator parameter b equal to 1·76 fm^{-1}. This oscillator parameter fits the elastic electron scattering upto the momentum transfer which is less than that fitted by the Elton-Swift wave function. As could be seen, the difference between the two distributions is considerable. Accordingly the inference about the correlation functions could be considerably different depending upon which basis functions are used.

Finally, in view of the results of this paper we may also make certain comments

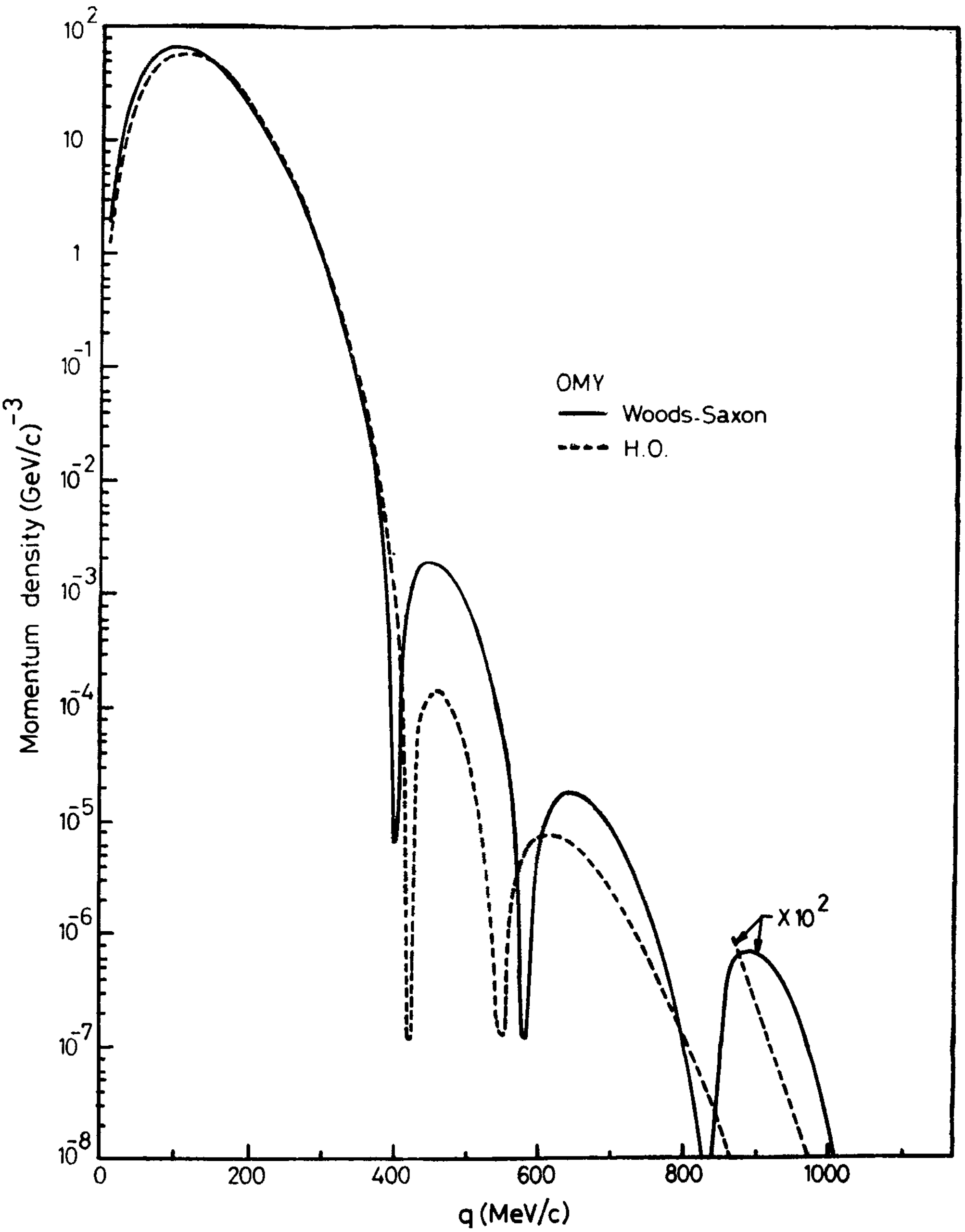
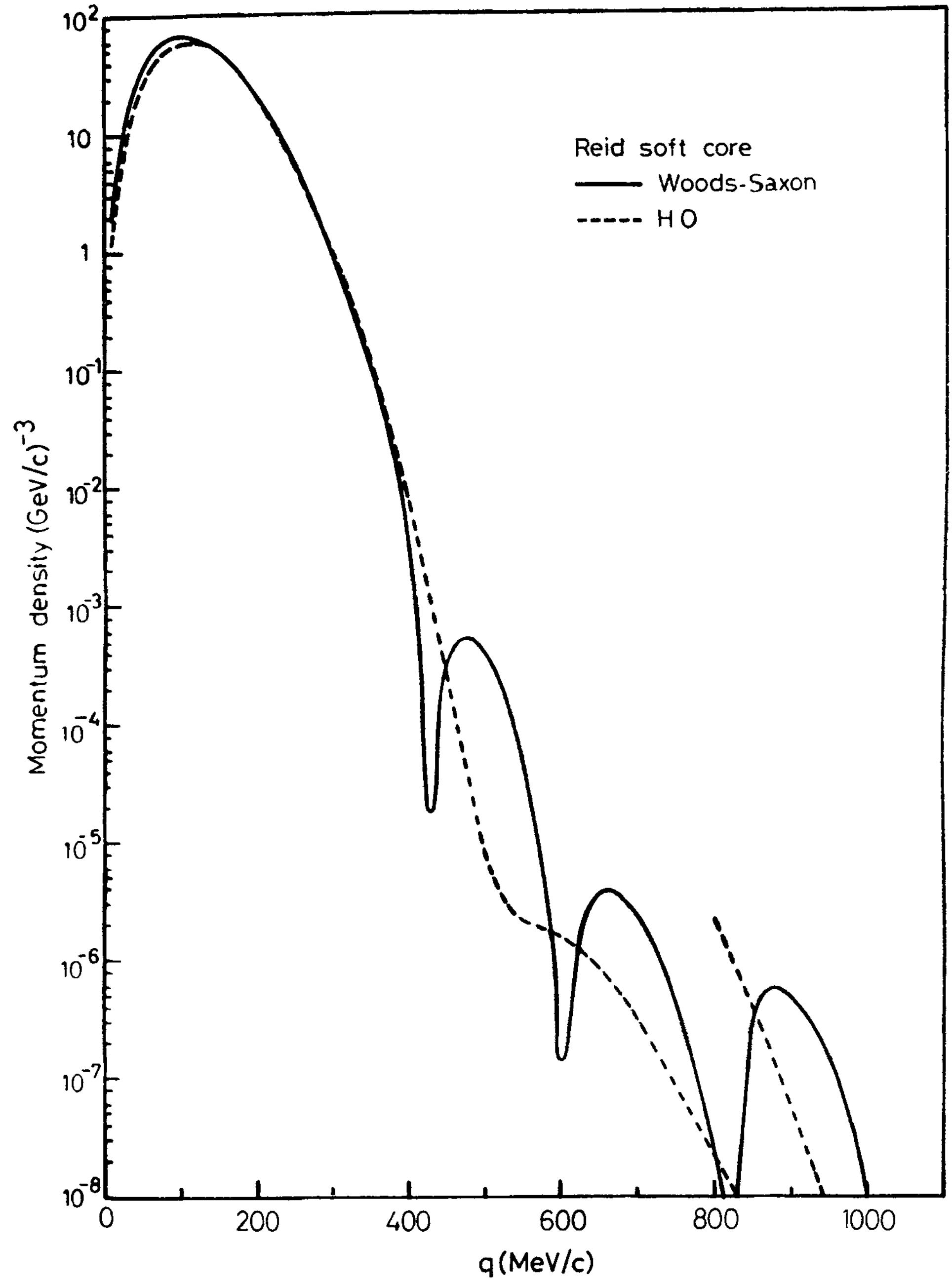


Figure 4 a. Single particle momentum distribution for $1p_{1/2}$ protons for OMY hard core potential using Elton-Swift (——) and harmonic oscillator (- - - -) potentials.



b. Single particle momentum distribution for $1p_{1/2}$ protons for Reid soft core potential using Elton-Swift (——) and harmonic oscillator (- - - -) potentials.

on the empirical determination of the correlation functions from the data on pion absorption and other reactions. Empirically the information on the correlation function is essentially determined by finding the appropriate momentum package $W(Q)$ required to fit the experimental data. In most of the investigations (Dillig and

Huber 1974; Ciofi degli Atti 1972; Huber 1971; Weise and Huber 1971) $W(Q)$ is found to peak around $Q \sim 300$–350 MeV/c, which, according to equation (1) may be interpreted as the most probable momentum exchanged between the shell model nucleons. From the results in figure 3 and equation (12), these momentum packages also imply the correlation functions which have large hard core radius (~ 1 fm) and heal slowly (range around 1·7 fm or so, see figure 3). Considering that the inter-nucleon spacing in the nucleus is of the order of 2 fm, the slowly healing correlation functions, which is not in accord with the assumptions of the Jastrow ansatz, would modify the nuclear wave functions at longer distances also. Therefore, it appears that for those nuclear processes which require the enrichment of the shell model wave function by about 300–350 MeV/c, the Jastrow ansatz is not the correct prescription. These range of momenta may be introduced by modifying the shell model wave function by configuration mixing through extra-core residual interaction.

In conclusion we may summarize that

(i) the realistic correlation functions corresponding to the Reid and Hamada Johnston potentials modify the single particle momentum distribution beyond about 550 MeV/c only;

(ii) the momentum components in the region of 300–500 MeV/c can be enhanced over the shell model values only by using the correlation functions which have larger hard core radius and heal very slowly. These correlation functions are therefore not in accord with the basic assumption of the Jastrow ansatz that the correlation function should leave the long range part of the nucleon wave function unaffected.

(iii) The use of the proper basis function is very important for correctly determining the effect of the correlation functions.

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