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A phenomenological bag model with variable bag pressure

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Abstract. We examine the consequences of a variable (density-dependent) bag pressure term and a fixed hadronic size in the phenomenological MIT bag model for hadron spectroscopy. Mass spectrum of the low-lying baryons and mesons, baryon magnetic moments and the hadron mass splittings are estimated. These are found to be in closer agreement with experiment than the MIT results.

Keywords. MIT bag model; hadron spectroscopy; baryon magnetic moments; hadron mass splittings.

1. Introduction

The MIT bag model (Chodos *et al* 1974a, b; De Grand *et al* 1975) in its spherical cavity approximation has come to be established as one of the strikingly successful theories of hadron spectroscopy in the low mass regime. The introduction of the bag pressure term $g_{\mu\nu} B$ in the Lagrangian density to represent a volume tension to balance the outward thrust of the quark gas inside the bag is hailed as an original invention of the MIT team. However, the origin of this term has remained more or less obscure, and in phenomenological applications of the bag model it is treated as a universal parameter, whose value is determined by fitting known masses. We wish to point out that there are indications to the effect that B is not a universal constant. The motivation of the MIT theorists for introducing the pressure term in the bag Lagrangian is apparently the familiar solution of the Einstein general relativity equation with a 'cosmological constant' to generate a closed universe. But todate there is no evidence to give hope that an extension of the cosmological theories to the realm of the microcosm can generate the bag with a pressure term in any order of magnitude agreement with its phenomenologically determined value. It seems that the only hope at present for a field-theoretic derivation of the colour-quark bag rests on quantum chromodynamics (QCD). Johnson (1979) made a proposal for the form of the ground state wavefunction of QCD, where his suggestion for handling QCD quantitatively, leads to the phenomenology of the MIT bag model. The bag pressure term in this case is found to be related to the fundamental scale parameter Λ of QCD.

In the theory of hadronic structure recently proposed by Callan *et al* (1979), qualitative aspects of a bag-like picture that emerges from the properties of QCD vacuum are discussed. According to them, the normal QCD vacuum is populated with instantons and merons forming a dense phase and is colour-repellant. Above a critical field strength the colour fields (due to quarks) find themselves in finite regions (bags) from where instantons are expelled leaving a dilute phase. The bag pressure B is then computed as the zero-field difference in free energy density between the dense and the dilute phases. In a semiquantitative analysis it is shown that B is inversely proportional to the permeability of the dilute phase, which is approximately a constant, although strictly speaking, it is essentially a density-dependent factor. The theory, however, has not been developed into a calculational device.

As a plausible means to relate the bag pressure to the energy density in a hadron, we will seek an equation of state for hadronic matter. Relativistic hydrodynamics when applied to the bag model (Chapline and Nauenberg 1977; Peressutti and Skagerstam 1978) leads to an equation of state

$$P = \tfrac{1}{3}\epsilon - \tfrac{4}{3}B, \tag{1}$$

where ϵ is the total energy density. The condition for the stability of the system, namely, $P = 0$ determines B in terms of the total energy density

$$B = \tfrac{1}{4}\epsilon. \tag{2}$$

This equation relates, of course, to the extreme relativistic case where the quarks have negligible mass. In general, one has

$$B \leqslant \tfrac{1}{4}\epsilon. \tag{3}$$

Writing $$\epsilon = \rho + B, \tag{4}$$

in which ρ represents the contribution from sources other than the volume tension, equation (2) becomes

$$B = \tfrac{1}{3}\rho. \tag{5}$$

For evaluating the properties of the quark gas, Chapline and Nauenberg (1977) make use of the fact that the quark-gluon coupling constant α_s in the MIT bag theory is small enough to permit an agreeable perturbation approximation. However, following the suggestion of Freedman and McLerran (1976), they use a renormalized quark-gluon coupling constant depending on the Gibb's energy per quark rather than a fixed coupling constant as in the MIT theory. This approach naturally lends support to the choice of a variable bag pressure B.

This paper is organised as follows: In § 2 we give a very brief account of the familiar aspects of hadron spectroscopy resulting from the phenomenological application of the MIT bag model. In § 3 the new bag phenomenology is introduced and the mass spectrum of the light baryons and mesons worked out. In § 4 we apply the new phenomenology to a study of the magnetic moments of baryons. In § 5 an analysis of the hadron mass splittings is presented. § 6 contains our concluding remarks.

2. The bag Hamiltonian

The Hamiltonian of the original MIT bag model was developed and its diagonalization was discussed by Chodos *et al* (1974b) and De Grand *et al* (1975). The total energy, and hence the mass of a hadron, is derived from four different sources. The major contribution E_q arises from the motion of the quarks. The kinetic energy of a quark of mass m_i moving in a spherical cavity of radius R is given by

$$\omega\,(m_i\,R) = (p_i^2 + m_i^2)^{1/2} = [(x/R)^2 + m_i^2]^{1/2}, \tag{6}$$

with $x = x\,(m_i\,R)$ being the solution of

$$x/\tan x = 1 - m_i\,R - (x^2 + m_i^2\,R^2)^{1/2} \tag{7}$$

Thus $E_q = \sum_i N_i\,\omega\,(m_i\,R)$,

where N_i represents the number of quarks and antiquarks of the ith flavour. The other contributions are (1) the bag volume energy $= \frac{4}{3}\pi\,R^3\,B$, (ii) the finite part of the zero-point energy $= -Z_0/R$, Z_0 being a phenomenological dimensionless parameter ~ 1, and (iii) the quark-gluon interaction energy. The interaction energy consists of two parts: the contributions E_M and E_E from colour magnetic and electric interactions, respectively, the latter being negligible in the case of ordinary (non-charm) hadrons. In lowest order

$$E_M = \sum_{i>j} \lambda\,a_{ij}\,M_{ij},$$

where

$$a_{ij} = \sigma_i \cdot \sigma_j,$$

$$M_{ij} = \frac{8\alpha_s}{36\,R}\,\mu'\,(m_i\,R)\,\mu'\,(m_j\,R)\,I\,(m_i\,R, m_j\,R),$$

$$\mu'\,(m\,R) = \frac{4\omega\,R + 2m\,R - 3}{2\omega\,R\,(\omega\,R - 1) + mR}. \tag{8}$$

$I\,(m_i\,R, m_j\,R)$ is a slowly varying function of $m_i\,R$ and $m_j\,R$; $\lambda = 1$ for baryons and 2 for mesons. Hence for the mass of a hadron of radius R we have

$$M = E_q + \tfrac{4}{3}\pi\,R^3\,B + \sum_{i>j} \lambda\,a_{ij}\,M_{ij} - Z_0/R. \tag{9}$$

The indices i and j run over the quark/antiquark flavours present in the hadron. The bag radius is estimated by minimizing M with respect to R, a procedure consistent with the nonlinear bag boundary condition.

The parameters B, Z_0, α_s and the quark masses are determined by a fit to known hadron masses. DeGrand *et al* (1975) consider two such fits, keeping the Δ, N, ω and Ω masses correct, which yield the following parameter values.

(i) $m_u = m_d = 0$, $m_s = 0{\cdot}279$ GeV,

$B^{1/4} = 0{\cdot}145$ GeV, $Z_0 = 1{\cdot}84$, $\alpha_s = 0{\cdot}55$.

(ii) $m_u = m_d = 0{\cdot}108$ GeV, $m_s = 0{\cdot}353$ GeV,

$B^{1/4} = 0{\cdot}125$ GeV, $Z_0 = 1{\cdot}95$, $\alpha_s = 0{\cdot}75$.

The former set gives a somewhat better prediction of the static properties of hadrons and seems to be rather universally accepted for bag model computations. With these parameter values De Grand *et al* (1975) reproduced the mass spectrum of the uncharmed hadrons in fairly good agreement with observations, with a spectacular deviation arising only in the case of the pion mass, whose predicted value turns out to be twice the experimental value. A readjustment of the parameters fitting the correct pion mass badly spoils the other predictions.

3. The new bag phenomenology: Hadron mass spectrum

We start by relating the bag pressure B to the energy density ρ arising from all contributions to the hadronic mass except the volume energy, through equation (5):

$$B = \tfrac{1}{3}\,\rho.$$

We wish to point out here that this relation follows from the requirement of hydrodynamical stability, namely $P = 0$, and that it is consistent with the traditional stability condition $\partial M/\partial R = 0$ used in the original MIT model to determine the bag size. Writing

$$A = \sum_i N_i\,\omega\,(m_i R) + \sum_{i>j} \lambda\, a_{ij}\, M_{ij} - Z_0/R = \tfrac{4}{3}\,\pi\, R^3\,\rho. \tag{10}$$

We have

$$M = \tfrac{4}{3}\,\pi\, R^3\,(\rho + B) = \tfrac{4}{3}\,A. \tag{11}$$

An important point over which we differ from the MIT bag model calculations is with regard to the bag radius. In the MIT model the bag radius is different for different hadrons, as typified by a value of 5 GeV^{-1} for the nucleon and 3·26 GeV^{-1} for the kaon. Among the baryons the differences are not much, the variation being from 4·91 GeV^{-1} (Ξ) to 5·48 GeV^{-1} (Δ). As for the mesons except for the kaon and the pion, the bag radius has a value slightly less than that of a typical baryon, but the K and π mesons have comparatively much smaller size, 3·26 and 3·34 GeV^{-1} respectively. It is interesting to note that, to get the right mass for the pion, the bag size has to be further reduced considerably. What is curious is the fact that there is no straightforward relationship between the masses of hadrons and their sizes, like the size varying as some power of the mass. However, the radii are such that one can speak of an average baryon size (R_B) and an average meson size (R_M), with SLAC bag model-type relation (Bardeen *et al* 1975), namely, $R_M \sim R_B \times (2/3)^{1/3}$ being obeyed approximately, where the factor 2/3 represents the ratio of the number of quarks and antiquarks in a meson to that in a baryon. It may further be noted that the bag model results are not very sensitive to small variations in the bag size. We are therefore motivated to introduce the idea of a constant bag radius with one value for baryons and a different value for mesons. In (10) the zero-point energy term now becomes a constant. Denoting this term by ϵ, we have

$$M = \tfrac{4}{3} \left[\sum_j N_j \, \omega \, (m_j \, R) + \sum_{i>j} \lambda \, a_{ij} \, M_{ij} - \epsilon \right], \tag{12}$$

in which ϵ has the same value for all baryons but has another fixed value for the mesons.

The quark masses m_q, bag radius R, the strong interaction coupling constant α_s and the zero-point energy ϵ are the parameters of the theory. We make use of the known values of the axial vector coupling constant g_A and the proton magnetic moment μ_P to make a phenomenological estimate of the non-strange quark mass $m_n (m_n = m_u = m_d)$ and the bag size R. In the bag model

$$g_A = \frac{5}{9} \left[\frac{2 \, \omega^2 \, R^2 + 4 \, mR \cdot \omega R - 3 \, mR}{2 \, \omega R \, (\omega R - 1) + mR} \right]. \tag{13}$$

From β decay of the neutron we find $g_A = 1{\cdot}25$. With this value of g_A as input, (13) is solved numerically so as to satisfy the transcendental equation (7), which may be rewritten as

$$\tan \, (\omega^2 \, R^2 - m^2 \, R^2)^{1/2} = -(\omega^2 \, R^2 - m^2 \, R^2)^{1/2} / (\omega R + mR - 1). \tag{14}$$

Putting the values of ωR and mR thus obtained in the expression for the proton gyromagnetic ratio

$$g_P = 2 \, M_P \, \mu_P = \tfrac{1}{3} \, R \, M_P \left[\frac{4 \, \omega R + 2 \, mR - 3}{2 \, \omega R \, (\omega R - 1) + mR} \right], \tag{15}$$

and using the experimental value of $g_P = 2{\cdot}79$, we obtain $R = 8{\cdot}88 \text{ GeV}^{-1}$, and a quark kinetic energy $\omega_n = 0{\cdot}294$ GeV, that corresponds to a quark mass $m_n = 0{\cdot}114$ GeV. The strange quark mass is then determined from the $\Lambda - N$ mass separation which yields $\omega_s = 0{\cdot}427$ GeV corresponding to a bare quark mass $m_n = 0{\cdot}302$ GeV. Finally, we determine the values of α_s and ϵ by fitting the known masses of Λ and Ξ. The values obtained are $\alpha_s = 0{\cdot}94$ and $\epsilon = 68$ MeV. This value of α_s is greater than the MIT value of 0·55.

Large values of α_s are certainly permitted in bag-type confinement schemes. Renormalization group arguments (Politzer 1973) show that the quark-gluon coupling constant can become very large for small momentum transfers. This large increase in quark-gluon coupling at small momentum transfers is conjectured as resulting in quarks being confined to a finite region of space.

The above parameter values are used to work out the mass spectrum of the low-lying baryons. Results of our calculation, along with the experimental values as well as the MIT results for two sets of parameters, are listed in table 1.

The parameters appropriate to the baryon spectrum do not fit with the meson spectrum. Hence we seek another set of parameters for the low-lying mesons. There is no reason to change the quark masses. But it is reasonable to assume that the ratio of the meson to baryon bag radius is dependent on the ratio of the number of quarks and antiquarks in mesons and baryons. Thus following the SLAC approach (Bardeen *et al* 1975), we assume $R_M = R_B\,(2/3)^{1/3}$. With $R_B = 8{\cdot}88$ GeV^{-1}, $R_M = 7{\cdot}75$ GeV^{-1}. This decrease in bag size causes an increase in quark kinetic energies. Thus, for mesons we obtain $\omega_n = 0{\cdot}321$ GeV, and $\omega_s = 0{\cdot}448$ GeV. Using the same value of the colour coupling constant α_s as for baryons, and fitting the experimental kaon mass we find $\epsilon = 0{\cdot}178$ GeV. These parameters generate quite an agreeable meson spectrum. The pion mass has not improved considerably over the MIT value. However, a new value of α_s determined from the chromomagnetic splitting of ω and π, namely $\alpha_s = 1{\cdot}25$ and the corresponding value of $\epsilon = 0{\cdot}175$ GeV needed to fit the mass of ω bring about substantial improvement over the MIT result (table 2). Our values of the meson masses for the two sets of parameters along with the MIT results are presented in table 2. The K mass has come out somewhat poorer.

Table 1. Masses (in GeV) of the low-lying baryons for the parameters: $m_n = 0{\cdot}114$ GeV, $m_s = 0{\cdot}302$ GeV, $R = 8.88$ GeV^{-1}, $\alpha_s = 0{\cdot}94$, $\epsilon = 0{\cdot}068$ GeV. The MIT results are for the sets of parameters (i) and (ii) mentioned in the text.

Particles	Expt.	MIT (i)	MIT (ii)	Present calculation
N	0·938	0·938	0·938	0·937
Λ	1·116	1·105	1·103	1·116
Σ	1·189	1·144	1·145	1·163
Ξ	1·321	1·289	1·286	1·321
Δ	1·236	1·236	1·233	1·233
Σ^*	1·385	1·382	1·381	1·386
Ξ^*	1·533	1·529	1·528	1·542
Ω	1·672	1·672	1·672	1·702

Table 2. Masses (in GeV) of the low-lying mesons for the sets of parameters: (a) $m_n = 0{\cdot}114$ GeV, $m_s = 0{\cdot}302$ GeV, $R = 7{\cdot}75$ GeV^{-1}, $\alpha_s = 0{\cdot}94$, $\epsilon = 0{\cdot}178$ GeV (b) $m_n = 0{\cdot}114$ GeV, $m_s = 0{\cdot}302$ GeV, $R = 7{\cdot}75$ GeV^{-1}, $\alpha_s = 1{\cdot}25$, $\epsilon = 0{\cdot}175$ GeV. The MIT results are for the sets of parameters (i) and (ii) mentioned in the text.

Particles	Expt.	MIT(i)	MIT(ii)	Present calculation (a)	Present calculation (b)
π	0·139	0·280	0·175	0·256	0·139
K	0·495	0·497	0·371	0·495	0·432
ω	0·783	0·783	0·783	0·740	0·783
ρ	0·77±0.01	0·783	0·783	0·740	0·783
K^*	0·892	0·928	0·925	0·885	0·921
φ	1·020	1·068	1·063	1·034	1·062

But, significantly, there is overall improvement which includes an exact fit with the pion mass.

4. Magnetic moments of baryons

The magnetic moment of a single quark of mass m confined to a bag of radius R is given by the integral

$$\mu_q = \int_0^R (d^3 x \, \tfrac{1}{2} \vec{r} \, X : q^+ (x) \, \vec{a} \, e_q \, q (x):)_z, \tag{16}$$

in which $q(x)$ is the quark wavefunction and e_q is the electric charge of quark q. On evaluation, the integral yields

$$\mu_q = \frac{R}{6}\left[\frac{4\,\omega R + 2\,mR - 3}{2\,\omega R\,(\omega R - 1) + mR}\right] e_q \tag{17}$$

Remembering that $e_u = \frac{2}{3} e$, $e_d = e_s = -\frac{1}{3} e$, we write

$$\mu_u = \frac{\mathrm{Re}}{9}\left[\frac{4\,\omega_u R + 2\,m_u R - 3}{2\,\omega_u R\,(\omega_u R - 1) + m_u R}\right], \tag{18a}$$

$$\mu_d = -\frac{\mathrm{Re}}{18}\left[\frac{4\,\omega_d R + 2\,m_d R - 3}{2\,\omega_d R\,(\omega_d R - 1) + m_d R}\right], \tag{18b}$$

$$\mu_s = -\frac{\mathrm{Re}}{18}\left[\frac{4\,\omega_s R + 2\,m_s R - 3}{2\,\omega_s R\,(\omega_s R - 1) + m_s R}\right]. \tag{18c}$$

Since we have assumed that $m_u = m_d \,(= m_n)$, we have

$$\mu_d = -\tfrac{1}{2}\,\mu_u. \tag{19}$$

In the present calculations the bag radius is the same for all the baryons. Hence μ_u, μ_d, μ_s do not vary from baryon to baryon as in the original MIT bag calculations (De Grand *et al* 1975). We have

$$m_n R = 1{\cdot}01,\ m_s R = 2{\cdot}68,$$

$$\omega_n R = 2{\cdot}61 \text{ and } \omega_s R = 3{\cdot}79,$$

so that

$$\mu_u = 0{\cdot}9915\, e = 0{\cdot}9915\, e \times 2\, M_p/e = 1{\cdot}86 \text{ nm.},$$

$$\mu_d = -\tfrac{1}{2}\,\mu_u = -0{\cdot}93 \text{ nm.},$$

$$\mu_s = -0{\cdot}68 \text{ nm.}$$

(nm means nuclear magneton).

The baryon magnetic moments are computed by making use of the well-known additivity assumption of the quark model. Thus ignoring the possibility of quark anomalous magnetic moments, we express the baryon magnetic moment as the vector sum of the quark moments, assuming further that the orbital angular momenta of the quarks are zero. Hence we write

$$\vec{\mu}_B = \sum_i \vec{\mu_q}\,(m_i\, R), \tag{20}$$

i being flavour index. The value of μ_B is estimated in terms of μ_q from a knowledge of the flavour and spin wavefunctions of the baryon. Denoting the combined spin and flavour wavefunctions of the quarks by u, d, s we have for the magnetic moment of the proton (Franklin 1968)

$$\begin{aligned}\mu_P &= \left(u\,u\,d,\ \left(\sum_i \vec{\mu_q}\,(m_i\, R)\right)_z u\,u\,d\right),\\ &= \tfrac{1}{3}\,(4\,\mu_u - \mu_d),\\ &= 2{\cdot}79 \text{ nm.}\end{aligned} \tag{21}$$

The neutron magnetic moment

$$\mu_N = \tfrac{1}{3}\,(4\,\mu_d - \mu_u) = -\,1{\cdot}86 \text{ nm.} \tag{22}$$

For the ratio of these moments we get

$$\mu_N/\mu_P = -2/3 \tag{23}$$

in agreement with the well-known quark model prediction. Our values for the proton and neutron magnetic moments are in excellent agreement with experiment. The MIT group, on the other hand, gets too small a value for the proton magnetic moment, namely, $\mu_P = 1{\cdot}9$ nm., a fact that is considered as a serious discrepancy among their predictions. In general, the magnetic moments of baryons resulting from the present calculation are in better agreement with experiment than the corresponding MIT results (see table 3).

Table 3. Magnetic moments of baryons in nuclear magnetons

Particle	Expt.	MIT result	Present result
P	2·79	1·90	2·79
N	−1·91	−1·27	−1·86
Λ	−0·6138 ± 0·0047	−0·484	−0·68
Σ^+	2·62 ± 0·41	1·843	2·707
Σ°	...	0·589	0·847
Σ^-	−1·48 ± 0·37	−0·684	−1·013
Ξ°	...	−1·064	−1·527
Ξ^-	−1·85 ± 0.75	−0·437	−0·597
Ω	...	−1·452	−1·84

We would like to mention here a recent observation made by Fritzsch (1979). Based on the basic assumptions of the nonrelativistic SU(6) quark model he makes the following precise predictions of the hyperon magnetic moments: $\mu\,(\Sigma^{+}) = 2{\cdot}69$ nm., $\mu\,(\Sigma^{-}) = -1{\cdot}09$ nm., $\mu\,(\Xi^{\circ}) = -1{\cdot}44$ nm., $\mu\,(\Xi^{-}) = -0{\cdot}495$ nm.

There is very good agreement between these and the results of our calculation (see table 3). Also, it is instructive to compare these predictions with those of De Rújula, *et al* (1975):

$$\mu(\Sigma^{+}) = 2{\cdot}67 \text{ nm.}, \quad \mu(\Sigma^{-}) = -1{\cdot}05 \text{ nm.},$$

$$\mu(\Xi^{\circ}) = -1{\cdot}39 \text{ nm.}, \quad \mu(\Xi^{-}) = -0{\cdot}46 \text{ nm.}$$

Large discrepancy between theoretical predictions and the experimental value is oted in the case of the strange hyperon Ξ^{-}

$$\mu(\Xi^{-})_{\text{exp.}} = -1{\cdot}85 \pm 0{\cdot}75 \tag{24}$$

$$\begin{aligned} \mu(\Xi^{-})_{\text{theor.}} &= -0{\cdot}495 \text{ (Fritszch 1979)}, \\ &\quad -0{\cdot}46 \text{ (De Rújula, } et\ al \text{ 1975)}, \\ &\quad -0{\cdot}597 \text{ (present calculation)}. \end{aligned} \tag{25}$$

If the basic additivity assumption of the quark model be true and the very good agreement between theory and experiment in the case of μ_N/μ_P (equation (23)) not accidental, then as Fritzsch (1979) has pointed out,

$$|\,\mu(\Xi^{-})\,| < |\,\mu\,(\Lambda)\,|. \tag{26}$$

This inequality sets a bound on $\mu(\Xi^{-})$. A recent high precision measurement (Schachinger *et al* 1978) has yielded

$$\mu\,(\Lambda) = (-0{\cdot}6138 \pm 0{\cdot}0047) \text{ nm.} \tag{27}$$

Hence (26) implies

$$|\,\mu(\Xi^{-})\,| < 0{\cdot}614. \tag{28}$$

All theoretical predictions (equation (25)) are compatible with (28), but the present experimental value badly violates this bound. We should, therefore, agree with the observation made by Fritzsch (1979) that the experimental value of $\mu(\Xi^{-})$ is incorrect and should emphasize the need for a new precise measurement of this quantity.

5. Hadron mass splittings

Mass splittings among the light hadrons arise mainly from SU(3) breaking effects introduced through quark mass difference between nonstrange and strange flavours.

Also there are the short range chromomagnetic forces depending on quark spins and masses that produce hyperfine mass splittings. These effects have already been taken into account in the bag formula for hadronic mass, where the chromomagnetic spin-spin interaction is assumed to arise from exchange of colour octet vector gluons between the quarks. Thus in the mass spectrum presented above degeneracy between baryon decuplet and octet, as well as vector and pseudoscalar mesons and that among the various isospin multiplets are lifted. The mass splitting among the members of an isomultiplet is due to the mass difference between u and d quarks and the electromagnetic interaction between the quarks. These effects are not incorporated in the present calculations; the u and d quarks are assumed to have the same mass. The absolute value of the chromomagnetic interaction energy can be computed in the bag model in terms of the colour coupling constant α_s unlike in other models which invariably employ, in addition, a phenomenologically determined universal mass parameter depending on unknown details of the wavefunction.

From the energy (mass) equation (12) the various SU(3) and SU(6) mass formulae can be deduced. The equal spacing rule for the baryon decuplet:

$$\Omega - \Xi^* = \Xi^* - \Sigma^* = \Sigma^* - \Delta, \tag{29}$$

(where particle symbols stand for the masses of the corresponding particles) that follows from the linear Gell-Mann-Okubo mass formula (Gell-Mann 1962, Okubo 1962) implies that

$$2(M_{ns} - M_{ss}) = M_{nn} - M_{ss} = 2(M_{nn} - M_{ns})$$

or

$$M_{ns} = \tfrac{1}{2}(M_{nn} + M_{ss}), \tag{30}$$

where M_{ij} is given by equation (8). The Gell-Mann-Okubo formula for the baryon octet:

$$2N + 2\Xi = 3\Lambda + \Sigma, \tag{31}$$

also implies the relation (30) among the colour magnetic interaction terms. This relation is very well satisfied by the new bag phenomenology. Also the SU(6) relation (Paos 1964):

$$\Sigma^* - \Sigma = \Xi^* - \Xi, \tag{32}$$

is satisfied exactly, while the relation (Federman *et al* 1966):

$$2\Delta - 2N = 3\Sigma^* + \Sigma - 3\Lambda, \tag{33}$$

is satisfied within 2·5%.

For the vector mesons, the linear Gell-Mann-Okubo mass formula, taking into account the octet singlet mixing, reads

$$2\varphi + \omega + \rho = 4K^*. \tag{34}$$

This relation is well satisfied by the present model. The equation follows exactly from the already mentioned relation (equation (30)) among the magnetic interaction terms. However the empirical relation

$$(K^* - K)_{us}/(\rho - \pi)_{uu} = 0{\cdot}667, \tag{35}$$

is only poorly satisfied. In the present model, the ratio of the vector-pseudoscalar splittings for the differing flavour combinations is obtained as

$$M_{ns}/M_{nn} \simeq 0{\cdot}8. \tag{36}$$

It is interesting to compare these results with those of the original MIT calculations (De Grand *et al* 1975). In the MIT model the linear Gell-Mann-Okubo mass formula for baryons implies, as in our case $\frac{1}{2}(M_{nn}+M_{ss}) = M_{ns}$ and is satisfied well, if it be assumed that R does not vary from state to state which, of course, is not the case. The SU(6) relation (32) requires

$$M_{ns} = M_{ss} \tag{37}$$

and the equal spacing rule for the decuplet demands

$$M_{ns} = M_{ss} = M_{nn}. \tag{38}$$

Neither of these formulae is satisfied.

An important aspect of the present model is the simple qualitative explanation it provides for the mass splittings among baryons. The decuplets are heavier than the corresponding octets for the fact that, even though the quark rest and kinetic energy contributions are identical, the colour magnetic interaction contribution is negative in the case of the octets, while it is positive for the decuplets. Besides, the contributions are different in magnitude as a result of SU(3) symmetry-breaking effects and the different spin orientations of the constituent quarks. Thus the octet-decuplet splitting turns out to be a pure hyperfine splitting. The situation is not so straightforward in the MIT model as the zero-point energy contributions are different for different hadrons.

Note that the Σ-Λ mass difference also arises from colour magnetic interaction. Here the energy contributions are different as a result of the mass difference between the non-strange and strange quarks and the difference in relative spin orientations of the quarks. In Λ, the spin-zero combination of the u and d quarks leads to the result

$$\sum_{i>j} a_{ij} M_{ij} = -3 M_{nn}. \tag{39}$$

In Σ, u and d quarks form a spin-1 state with the consequence that

$$\sum_{i>j} a_{ij} M_{ij} = M_{nn} - 4 M_{ns}. \tag{40}$$

In the exact SU(3) limit, $M_{nn} = M_{ns}$ and the Σ-Λ degeneracy is restored.

6. Concluding remarks

We have introduced two major modifications in the phenomenological MIT bag model by choosing a variable bag pressure term B and a fixed bag size R. The bag size is fixed so as to reproduce the proton magnetic moment, and hence it differs considerably from the average radius in the original bag model. In our theory B is not a universal constant parameter as in the MIT model; it is determined by the density of hadronic matter constituting each particle. Its value thus varies from hadron to hadron. We have been able to reproduce the mass spectrum of the light hadrons, with an exact fit with the pion mass and have computed the magnetic moments of the baryons in far closer agreement with measured values than the MIT model. Where there is a large deviation from the experimental number, it is pointed out that it is in violation of basic quark model assumptions and is suggestive of the need for a fresh experimental determination of the quantity. Our analysis of the hadron mass splittings, taking into account the chromomagnetic hyperfine interactions of quarks, leads to well-established mass relations among hadrons. The comparatively better results obtained in the present approach are a clear manifestation of the fact that the universal character of the bag pressure term is not an essential requirement at least as far as the bag phenomenology is concerned. We wish to emphasize here that whatever success the bag model has achieved is essentially due to its being a quark model, and that the model permits a lot of flexibility in the choice of its parameters.

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