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Variation of average charged particle multiplicity in *p*–nucleus interactions with energy and the two component description of particle production at high energies

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Abstract. Experimental data on average shower particle multiplicity ($\langle N_s \rangle$) accumulated on *p*-nucleus interactions in the wide momentum region of 7·1–8000 GeV/c is investigated. It is observed that $\langle N_s \rangle$ is represented exceedingly well as a function of (νS). There are two physical processes which represent the experimental data reasonably well in the two momentum regions *viz* 7·1–67·9 GeV/c and 67·9–8000 GeV/c. $\langle N_s \rangle = a(\nu S)^{\alpha} + b$ fits the data in the low momentum region, whereas $\langle N_s \rangle = a + b \ln (\nu S)$ fits the experimental data in the high momentum region. The two physical processes are unified and represented by a single equation which is shown to be the consequence of two component theory and collective models.

Keywords. Proton-nucleus collisions; charged particle multiplicity; collective models; two component theory.

1. Introduction

Multiparticle production in hadron-nucleus interactions at high energies has been extensively studied in the recent past both at accelerator energies (upto ~ 400 GeV) and at cosmic ray energies. Following the calculations of Glauber (1967), the hadron-nucleus interactions can be regarded as superposition of successive independent hadron-nucleon interactions. This has been used by various authors to study the space-time development (Gottfried 1973) of particle production processes. It was hoped that hadron-nucleus studies would help in discriminating between various models of hadron-nucleon interactions (Fermi 1950, 1951; Belenjki and Landau 1956; Satz 1965; Berger and Krizwicki 1971; Muller 1970). In addition, various models for hadron-nucleus interactions have also been proposed (Dar and Vary 1972; Berlad *et al* 1976; Gottfried 1973; Anderson and Otterlund 1975; Babecki 1976; Afekh *et al* 1976) which in general are extensions of the models of hadron-hadron interactions. A detailed comparison of the systematics of various multiparticle production parameters between hadron-hadron and hadron-nucleus interactions is, therefore, necessary to understand the physical picture of the interaction.

It is observed that the mean charged particle multiplicity, $\langle N_s \rangle$ or the normalised mean multiplicity R_{em} ($=\langle N_s \rangle / \langle N_{ch} \rangle$, where $\langle N_{ch} \rangle$ is the average charged particle multiplicity in *p*–*p* collisions) is one of the most extensively studied parameter in

p-nucleus interactions. However, a detailed study of the variation of this parameter with energy has not been made. In this paper, we discuss the variation of $\langle N_s \rangle$ in the incident momentum range of 7·1–8000 GeV/c and compare it with the predictions of some prominent models of multiparticle production.

2. Experimental data

The experimental data used in the present study are summarised in table 1 where $\langle N_s \rangle$ is the mean-charged shower particle multiplicity in proton-nucleus interactions which includes the leading hadron. The data reported in different experiments have been corrected for the coherent processes*. If at any energy more than one experiment is reported, the weighted mean has been taken. The final values are shown as $\langle N_s \rangle$ weighted. $\langle N_s \rangle$ for the incident momenta $\gtrsim$ 1000 GeV/c has not been corrected for the coherent processes because the experimental errors are very large as compared to the magnitude of such corrections. It may also be noted that at cosmic ray energies (Gibbs *et al* 1974) there may be enough uncertainty in the energy of the incident particle.

Various authors (Bebecki 1976; Gupta 1979; Kaul 1979) have argued that hadron-nucleus data on $\langle N_s \rangle$ needs correction for slow particles. This correction becomes necessary for two reasons. Firstly, one notes that $\langle N_s \rangle$ includes only shower par-

Table 1. Experimental data on shower particle multiplicity, $\langle N_s \rangle$ in the incident momentum range 7·1 – 8000 GeV/c. S is the square of CM energy. $\langle N_s \rangle^{\text{corr}}$ and ν are respectively the corrected shower multiplicity, and the mean number of interactions suffered by the incident particle in average emulsion nucleus.

P_{lab} GeV/c	S	Expt $\langle N_s \rangle$	$\langle N_s \rangle$ weighted	$\langle N_s \rangle^{\text{corr}}$	ν	References
7·1	15·04	2·80±0·04 2·62±0·05	2·73±0·03	5·00±0·03	2·51	Winzler (1965) Daniel *et al* (1960)
9·9	18·74	3·20±0·20	3·20±0·20	5·49±0·20	2·54	Barashenkov *et al* (1960)
20·5	40·26	5·29±0·13	5·29±0·13	7·66±0·13	2·62	Meyer *et al* (1963)
23·4	45·62	5·61±0·11	5·61±0·11	7·99±0·11	2·64	Winzler (1965)
27·0	52·44	6·16±0·08 6·23±0·20	6·17±0·07	8·57±0·07	2·66	Mayer *et al* (1963)
27·9	54·13	6·60±0·10	6·60±0·10	9·00±0·10	2·66	Barbaro-Galtieri *et al* (1961)
67·9	129·10	9·73±0·23	9·73±0·23	12·23±0·23	2·77	Babecki *et al* (1973)
200	366·96	13·67±0·13 13·27±0·40 14·30±0·20	13·82±0·11	16·46±0·11	2·92	Babecki *et al* (1976) Gurtu *et al* (1974b) Alma-Ata-Leningrad-Moscow-Tashkent collaboration (1975)
300	564·56	15·40±0·20	15·40±0·20	18·11±0·20	3·00	Hebert *et al* (1977)
400	752·16	17·00±0·21	17·00±0·21	19·72±0·21	3·01	Aggarwal *et al* (1977)
1000	1877·74	19·16±1·85	19·16±1·85	22·00±1·85	3·14	Gierula and Wolter (1971)
3000	5629·77	22·50±1·50	22·50± 1·50	25·48±1·50	3·30	Lohrman and Tencher (1962)
8000	15009·74	23·30±2·00	23·30±2·00	26·43±2·00	3·46	Malhotra (1972)

*The coherent processes are considered to contribute ≈ 3·7% of the total number of inelastic interactions. The percentage for interactions with N_s = 1, 3, 5 and 7 (N_h = 0) are 1·3, 1·7, 0·5 and 0·2 respectively (Alma-Ata Collaboration 1975).

ticles ($\beta \gtrsim 0{\cdot}7$) whereas $\langle N_{\text{ch}} \rangle$ for *p-p* interactions reported in various experiments includes all charged particles. Therefore, contribution of slow particles is to be added to experimental $\langle N_s \rangle$. Secondly, one also observes that all the theoretical models of *p*-nucleus interactions treat *p-A* interactions as aggregate of *p-p* type of interactions whereas in fact *p-A* interactions include both *p-p* and *p-n* interactions. Thus for a meaningful comparison with the models of *p*-nucleus interactions, one would like to correct the $\langle N_s \rangle$, such that *p-n* interactions becomes equivalent to *p-p* interactions.

Considering both these arguments, we get

$$\langle N_s \rangle^{\text{corr}} = \langle N_s \rangle^{\text{expt}} + \nu (n_\pi + l_p n_p) + \nu (1 - l_p) \tag{1}$$

where n_π and n_p represent mean number of slow pions and protons per *p-p* interaction. $\nu n_p l_p$ excludes those protons produced from evaporation process. We further assume that n_π remains the same in each *p-n* interaction. These effects are included in the second term of equation (1). n_π and n_p are taken from the experiment of Calcucci *et al* (1974)**. l_p is the fraction of protons among the nucleons of the nucleus. ν represents the average number of inelastic interactions which the incoming particle suffers in the average emulsion nucleus. This is calculated as per procedure given by us earlier (Gurtu *et al* 1974a). The values of l_p, n_π and n_p used here are respectively 0·455, 0·140 and 0·480. Clearly the last term of equation (1) is the addition to the experimental $\langle n_s \rangle$ when all *p-n* interactions are treated as *p-p* type. Details of the formulation of (1) are outlined in appendix A.

The $\langle N_s \rangle^{\text{corr}}$ and ν are also shown in table 1. $\langle N_{\text{ch}} \rangle$ corresponds to the mean multiplicity in *p-p* collisions. $\langle N_{\text{ch}} \rangle$ for $p_{\text{lab}} \leqslant 27{\cdot}9$ GeV/c have been obtained from the relation $\langle N_{\text{ch}} \rangle = 0{\cdot}348 + 1{\cdot}883\, E_{\text{av}}^{0{\cdot}464}$ where $E_{\text{av}} = \sqrt{S} - 2m$ and m is the mass of nucleon. The values of $\langle N_{\text{ch}} \rangle$ for $p_{\text{lab}} = 1000$ to 8000 GeV/c have been calculated from $\langle N_{\text{ch}} \rangle = -3{\cdot}02 + 1{\cdot}81 \ln S$ (Gurtu *et al* 1974a). The values in the intermediate momentum region have been taken from the experiments (Soviet French collaboration 1972; Charlton *et al* 1972; Wolf *et al* 1974; Bromberg *et al* 1973).

3. CM energy in proton-nucleus interactions

The square of the CM energy (S) has been commonly used to express the energy dependence of mean charged shower particle multiplicity in *p-p* interactions ($\langle N_{\text{ch}} \rangle$). The determination of the corresponding parameter in *p*-nucleus interactions is not straightforward and depends on the picture of the interaction. If the *p*-nucleus interactions follow the simple picture as proposed by Glauber (1967), (S_A) can be expressed as (νS). On the other hand, if the CTM model (Berlad *et al* 1976; Afekh *et al* 1976) is followed; $S_A \simeq n_A S$ in the high energy limit where n_A is the total number of nucleons in the tube. Different values of n_A are reported in the

**The values of n_π and n_p used here are for the FNAL data where the errors are small. At present the energy dependence of n_π and n_p is not known. However, one may note that the energy distribution of created particles does not change appreciably with energy (constant p_t). Hence, the value of n_π may not vary appreciably. The value of n_p may slowly depend on the incident momenta. We observe that if we assume $n_p = 1$ for 7 GeV/c and slowly vary it to 0.48 for 400 GeV/c the values $\langle n_s \rangle^{\text{corr.}}$ changes by $\lesssim 10\%$.

literature (see Berlad *et al* 1976; Afekh *et al* 1976; Takagi 1976) and it is difficult to have a definite choice. Therefore, in the present investigation $n_A=19$ (Afekh *et al* 1976) has been used although results for $n_A = 7{\cdot}4$ have also been tried.

4. Analysis of experimental data

The experimental data presented in § 2 is shown in figure 1 as a function of (ν S) and in figure 2 as a function of n_A S for $n_A = 19$. For comparison, the variation of $\langle N_s \rangle$ and $\langle N_s \rangle^{\rm corr}$ with (n_A S) for $n_A = 7{\cdot}4$ is also shown in figure 2.

Considering proton-nucleus interactions as a superposition of *p-p* collisions we consider the same variation of $\langle N_s \rangle$ or $\langle N_s \rangle^{\rm corr}$ with S_A as used in *p-p* interactions. These are as follows:

$$\langle N_s \rangle = a + b\, S_A^{1/3} \quad \text{(Berger and Krizwicki 1971)}, \tag{2}$$

$$\langle N_s \rangle = a + b\, S_A^{1/4} \quad \text{(Belenjki and Landau 1956)}, \tag{3}$$

$$\langle N_s \rangle = a + b \ln S_A \quad \text{(De Tar 1971; Feynman 1969)}, \tag{4}$$

$$\langle N_s \rangle = a + b \ln S_A + C \ln S_A\, S_A^{-a} \quad \text{(Muller 1970)}, \tag{5}$$

$$\langle N_s \rangle = a + b \ln S_A + C (\ln S_A)^2 \quad \text{(Whitmore 1976)}. \tag{6}$$

The results obtained for these fits are shown in tables 2 and 3. Following inferences can be drawn from these results:

(i) ν S is a better parameter than n_A S (χ^2-test). However, it may also be pointed

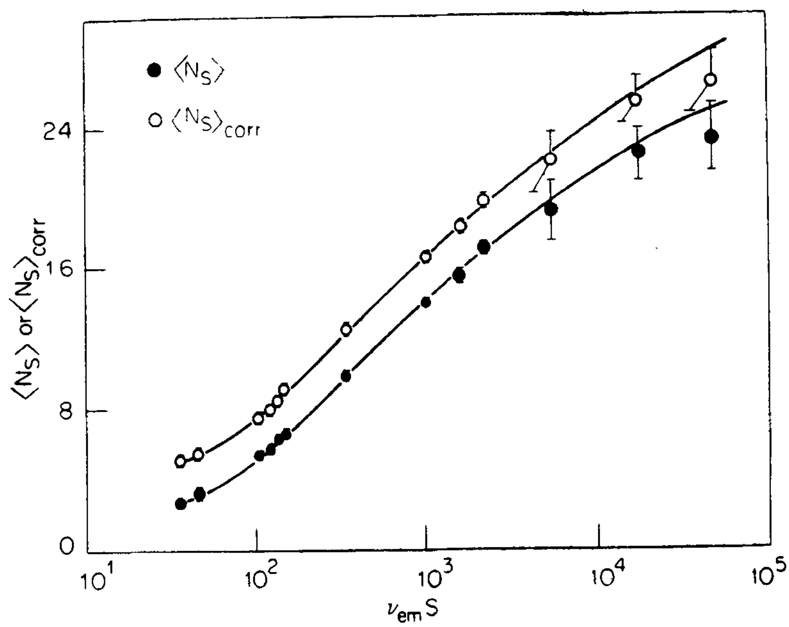

Figure 1. $\langle N_s \rangle$ or $\langle N_s \rangle^{\rm corr}$ versus ($\nu_{\rm em}$ S). The fits carried out are also shown. The curves correspond to the fit to equation (5) in the momentum range of 7·1–8000 GeV/c.

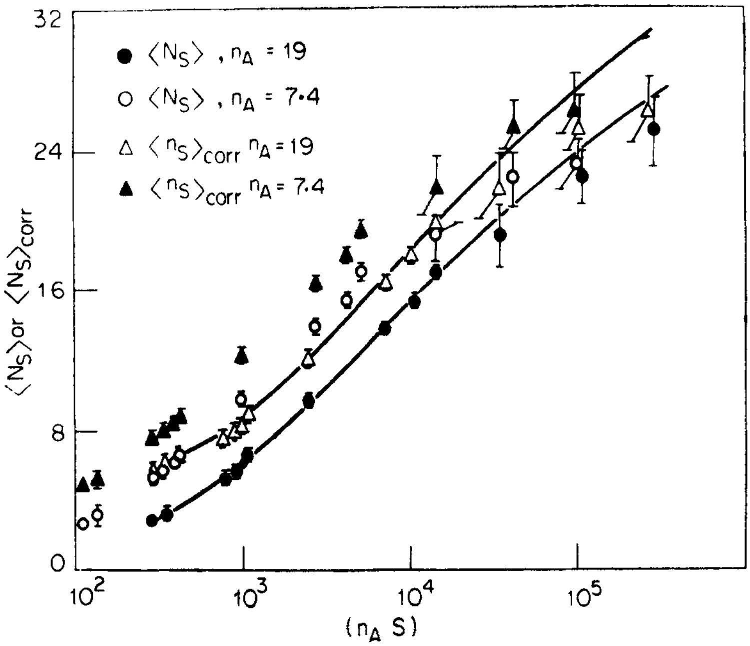

Figure 2. $\langle N_s \rangle$ or $\langle N_s \rangle^{corr}$ versus $(n_A S)$. The fits carried out are also shown. The curves correspond to the fit to equation (5) in the momentum range of 7·1–8000 GeV/c for $n_A = 19$.

Table 2. Results of various fits in the momentum region 7·1–27·9, 7·1–67·9, 7·1–8000 GeV/c for different empirical forms for *p*-nucleus interactions.

Function form for $\langle N_s \rangle$	Momentum range (GeV/c)	Constants *a*	*b*	*c*	α	χ^2/DOF
$a+b(\nu S)^{1/3}$	7·1– 67·9	−3·80±0·12	1·9 ±0·02	—	—	2·30
	7·1–8000·0	−1·10±0·03	1·4 ±0·06	—	—	36·40
$a+b(\nu S)^{1/4}$	7·1– 67·9	−6·80±0·02	3·8 ±0·04	—	—	2·50
	7·1–8000·0	−4·70±0·04	3·2 ±0·01	—	—	17·70
$a+b\ln(\nu S)$	7·1– 27·9	−7·60±0·02	2·8 ±0·01	—	—	4·30
	67·9–8000·0	−10·60±0·40	3·5 ±0·08	—	—	1·30
$a+b\ln(\nu S)+c\ln(\nu S)(\nu S)^{-\alpha}$	7·1–8000·0	+36·70±1·53	−0·20±0·12	−29·90±0·98	0·33±0·01	1·00
$a+b\ln(\nu S)+c(\ln \nu S)^2$	7·1–8000·0	−4·90±0·30	1·30±0·09	0·19±0·01	—	4·70

out that the values of n_A lying between 7 to 19 do not alter the situation in any way.

(ii) The linear function of the type $\langle N_s \rangle = a + b\, S_A^{\alpha}$ fits the data in the lower momentum ($P_{lab} \lesssim 67$ GeV/c) region only.

(iii) The linear variation of the type $\langle N_s \rangle = a + b \ln S_A$ represents the experimental data in the higher momentum ($P_{lab} \gtrsim 67$ GeV/c) region.

(iv) The function $\langle N_s \rangle = a + b \ln \sqrt{S}$ also fits the data in the high momentum ($P_{lab} \geqslant 20$ GeV/c) region as observed by Aggarwal *et al* (1977). This result implied a linear variation of $\langle N_s \rangle$ with $\langle N_{ch} \rangle$ which has been further discussed by Otterlund *et al* (1979) and Kaul *et al* (1980).

(v) None of these linear expressions are valid to represent the data over the entire range.

(vi) The expression (5) is obviously the best to represent the variation of $\langle N_s \rangle$ with (νS) over the entire range. The quadratic expression (6) can in no way be preferred over (5).

Similar results are obtained for $\langle N_s \rangle^{corr}$ as shown in tables 4 and 5. For further discussions we confine only to the variation of $\langle N_s \rangle$ with S_A $(= \nu S)$.

Table 3. Computed parameters for the variation of mean n_s with n_A S for various functional forms in the nomentum range of 7·1–8000 GeV/c for $n_A = 19$.

Functional form for $\langle N_s \rangle$	Momentum range GeV/c	Constants			α	χ^2/DOF
		a	b	c		
$a+b\,(n_A\ S)^{1/3}$	7·1– 67·9	– 4·9±0·05	1·11±0·01	—	—	2·5
	7·1–8000·0	– 2·9±0·02	0·85±0·002	—	—	54·9
$a+b\,(n_A\ S)^{1/4}$	7·1– 67·9	– 6·9±0·07	2·33±0·02	—	—	2·5
	7·1– 8000	– 5·9±0·02	2·12±0·01	—	—	18·3
$a+b\ln(n_A\ S)$	7·1– 67·9	–13·5±0·11	2·87±0·02	—	—	6·0
	67·9– 8000	–20·4±0·53	3·87±0·06			2·5
	7·1– 8000	–16·4±0·05	3·35±0·01			20·2
$a+b\ln(n_A\ S)+$ $C\ln(n_A\ S)(n_A\ S)^{-\alpha}$	7·1– 8000	–20·4±0·09	3·85±0·01	(3·4 ±0·06)10^5	2·5±0·10	4·5
$a+b\ln(n_A\ S)+$ $c\ln(n_A\ S)^2$	7·1– 8000	– 7·7±0·20	0·8 ±0·06	0·17±0·01	—	12·6

Table 4. Results of various fits of $\langle N_s \rangle^{corr}$ in the momentum region 7·1–27·9, 7·1–67·9 and 7·1–8000 GeV/c for different functional forms.

Function form for $\langle N_s \rangle^{corr}$	Momentum range (GeV/c)	Constants			α	χ^2/DOF
		a	b	c		
$a+b\,(\nu\ S)^{1/3}$	7·1– 67·9	– 1·65±0·01	+1·98±0·01	—	—	2·40
	7·1–8000					55·90
$a+b\,(\nu\ S)^{1/4}$	7·1– 67·9	– 4·46±0·07	3·81±0·02	—	—	2·70
	7·1–8000	– 3·32±0·02	3·40±0·006	—	—	22·90
$a+b\ln(\nu\ S)$	7·1– 67·9	– 5·38±0·07	2·85±0·02	—	—	7·00
	67·9–8000	–10·14±0·04	3·81±0·05			2·50
	7·1–8000	– 7·26±0·03	3·33±0·006			21·44
$a+b\ln(\nu\ S)+$ $c\ln(\nu\ S)(\nu\ S)^{-\alpha}$	7·1–8000	+ 39·8±0·90	–0·17±0·07	–30·7±0·06	0·325± 0·005	1·00
$a+b\ln(\nu\ S)+$ $c\,(\ln\nu\ S)^2$	7·1–8000	– 0·75±0·15	0·64±0·06	0·26±0·01	—	5·70

Table 5. Results of various fits of $\langle N_s \rangle^{corr}$ in the momentum region 7·1–67·9, 7·1–8000 GeV/c for different functional form of S-variable using $n_A = 19$.

Function form for $\langle N_s \rangle^{corr}$	Momentum range GeV/c	Constants a	b	c	α	χ^2/DOF
$a+b(n_A S)^{1/3}$	7·1– 67·9	– 2·87±0·05	1·15±0·01	—	—	2·6
	7·1–8000	– 0·78±0·02	0·88±0·003	—	—	58·7
$a+b(n_A S)^{1/4}$	7·1– 67·9	– 4·93±0·07	2·42±0·02	—	—	2·5
	7·1–8000	– 3·93±0·02	2·19±0·01	—	—	19·5
$a+b\ln(n_A S)$	7·1– 67·9	–11·84±0·11	2·97±0·02	—	—	6·0
	67·9–8000	–18·94±0·51	4·00±0·06			2·4
	7·1–8000	–14·80±0·04	3·47±0·01			20·9
$a+b\ln(n_A S)+$ $c\ln(n_A S)(n_A S)^{-\alpha}$	7·1–8000	–19·60±0·12	4·1 ±0·01	(2·21±0·04)10⁵	2·40±0·10	1·7
$a+b\ln(n_A S)+$ $c(\ln(n_A S)^2$	7·1–8000	– 8·15±0·52	0·57±0·09	2·28±0·01	—	5·7

5. Comparison with p-p interactions

Many authors have studied the variation of $\langle N_{ch} \rangle$ with S. Ganguli and Malhotra (1972) considered that *p-p* data also fit Ragge-Muller theory (Muller 1970) in the entire available momentum region 4–10^4 GeV/c, while Whitmore (1976), shows that a quadratic fit with ln S may be the best representation of the variation of $\langle N_{ch} \rangle$. The *p-p* data also exhibit a linear variation with S in the low momentum region ($\lesssim$ 67 GeV/c). The results of the present study for hadron-nucleus interactions show an agreement with the observation of Ganguli and Malhotra (1972) rather than that of Whitmore (1976).

The striking similarity between *p-p* and *p*-nucleus data lends further support to the assumption that the physical processes involved in the two cases are not very much different (Glauber 1967).

6. Discussion

The empirical fit exhibited by $\langle N_s \rangle$ with (νS) as discussed in § 3, indicates that $C(\nu S)^{-\alpha}$ is the dominant term in low energy region and $\ln(\nu S)$ is the controlling term when (νS) is large. The change in mathematical fits takes place at $E \sim 60$ GeV. These variations with incident energy can be associated with the arguments that two different physical processes may be involved in the two energy regions both in hadron-hadron and hadron-nucleus collisions. The physical process in the low energy region is well explained by statistical (or hydrodynamical) models and in the high energy region by multiperipheral models. It is difficult to argue as to why a clear-cut separation in the two physical processes occurs around 60 GeV. Therefore, we first prefer to understand the variation of $\langle N_s \rangle$ with (νS) as a single physical process rather than two separate processes. This means that we prefer the variation $\langle N_s \rangle =$

$a+b \ln (\nu S)+C \ln (\nu S)(\nu S)^{-\alpha}$ over $\langle N_s\rangle = a + b\,(\nu S)^{1/3}$ or $\langle N_s\rangle = a + b \ln (\nu S)$. It has been indicated earlier that the above variation is consistent with the multi-peripheral model of Muller and Regge (Muller 1970). However, it may be noted that the values of the constants a, b, c and α obtained in the present study do not show any systematic relation with the corresponding constants for *p-p* interactions. It is difficult to express the fitted constants of *p*-nucleus data as functions of ν and constants of *p-p* data. Thus a naive extension of *p-p* models to explain the *p*-nucleus interactions may not be possible.

The expression (5) can also be shown to be a consequence of multiperipheral theory (Fubini 1964). It is shown by Horn (1972) that if one follows Fubini's approach and assumes that the total cross-section for nucleon-nucleon interactions has a leading power behaviour, then the total cross-section for nucleon-nucleon interaction can be expressed as:

$$\sigma_T(\lambda) \simeq \beta(\lambda)\, S^{\alpha(\lambda)-1}, \tag{7}$$

where β and α are constants and λ is a continuous parameter associated with coupling constants, such that in the asymptotic region

$$\alpha(\lambda)\,|_{\lambda=1} = 1$$

i.e. σ_T saturates at very high energies. Taking the conventional definition of mean-charged particle multiplicity as

$$\langle N_{ch}\rangle = \sum_n n\,\sigma_n \Big/ \sum_n \sigma_n \tag{8}$$

with σ_n as partial cross-section, Horn (1972) further shows that

$$\langle N_{ch}\rangle = \frac{\lambda}{\sigma_T(\lambda)}\, \frac{\partial}{\partial(\lambda)}\, \sigma_T(\lambda)\Big|_{\lambda=1}. \tag{9}$$

This leads to

$$\langle N_{ch}\rangle = a + b \ln S. \tag{10}$$

If we now extend these arguments and incorporate an additional term in the leading power behaviour of the total cross-section such that

$$\sigma_T(\lambda) \simeq \beta(\lambda)\, S^{\alpha(\lambda)-1} + \gamma(\lambda)\, S^{-[m(\lambda)-1]} \tag{11}$$

Keeping the asymptotic conditions the same as in (7), using (9), we can easily write

$$\frac{\partial}{\partial\lambda}\sigma_T(\lambda) = [\beta'(\lambda) + \alpha'(\lambda)\beta(\lambda)\ln S]\, S^{\alpha(\lambda)-1} + [\gamma'(\lambda) - m'(\lambda)\,\gamma(\lambda)\ln S]\, S^{-m(\lambda)+1} \tag{12}$$

From (11) it can be physically interpreted that two separate classes of inelastic hadron collisions contribute to the total cross-section. Such two-component models have been considered by many authors. Fialkowski (1972) showed that the experimental multiplicity distributions justify the two-component theory. Van Hove (1973) interpreted the linear relation $D = A\langle N_{ch}\rangle - B$ on this idea, where $D^2 = \langle N_{ch}^2\rangle - \langle N_{ch}\rangle^2$. The second term in the cross-section and its contribution estimated at FNAL energies is of $\sim 7{\cdot}7$ mb. Assuming that the component $\gamma(\lambda)\, S^{-m(\lambda)+1}$ is similar to the diffractive dissociation component considered by Van Hove (1973) and knowing that physically its contribution is rather small as compared to the non-diffractive component (represented by the first term of (11), and following the same procedure as given by Horn (1972), we get (using (8), (11) and (12))

$$\langle N_{ch}\rangle = \frac{\lambda\beta'(\lambda)}{\beta(\lambda)} + \lambda\alpha'(\lambda)\ln S + \left(\frac{\lambda\gamma'(\lambda)}{\beta(\lambda)} - \frac{\lambda\, m'(\lambda)\,\gamma(\lambda)}{\beta(\lambda)}\ln S\right) S^{-m(\lambda)-\alpha(\lambda)+2}\Bigg|_{\lambda=1} \tag{13}$$

For $\gamma(\lambda) \ll e^{m(\lambda)\ln S}$, equation (13) can be written as

$$\langle N_{ch}\rangle = \frac{\lambda\beta'(\lambda)}{\beta(\lambda)} + \lambda\alpha'(\lambda)\ln S - \frac{\lambda\, m'(\lambda)\,\gamma(\lambda)}{\beta(\lambda)}\ln S\,.\, S^{-m(\lambda)-\alpha(\lambda)+2}\Bigg|_{\lambda=1}. \tag{14}$$

This assumption can be treated valid, as the choice $\lambda = 1$ leads to the expected asymptotic result $m=1$. Equation (14) can be written as

$$\langle N_{ch}\rangle = a + b\ln S + C\ln S\, S^{-\alpha}, \tag{15}$$

where
$$\frac{\lambda\beta'(\lambda)}{\beta(\lambda)}\Bigg|_{\lambda=1} = a$$

$$\lambda\,\alpha'(\lambda)\Big|_{\lambda=1} = b$$

$$-\frac{\lambda\, m'(\lambda)\,\gamma(\lambda)}{\beta(\lambda)}\Bigg|_{\lambda=1} = c \tag{16}$$

and
$$-m(\lambda) - \alpha(\lambda) + 2\big|_{\lambda=1} = -\alpha.$$

In order to obtain the numerical values of $\beta(\lambda)$, $\alpha(\lambda)$, $m(\lambda)$ and $\gamma(\lambda)$ at $\lambda=1$ from (16) it becomes necessary to know the actual forms for the above parameters as a function of λ. In its absence it is difficult to calculate the numerical values by using (16).

Following Glauber (1967), Berlad *et al* (1976) and Afekh *et al* (1976) *i.e.* replacing S by $S_A = \nu S$ or $n_A S$ one can easily write

$$\langle N_s \rangle = a + b \ln S_A + C \ln S_A S_A^{-a}. \tag{17}$$

7. Conclusions

(i) It is observed that two different linear relations are required to cover the data from 7–8000 GeV/c *viz.* $\langle N_s \rangle = a(\nu S)^a + b$ and $\langle N_s \rangle = a + b \ln (\nu S)$. The line of demarcation between the two linear fits is observed around 60 GeV/c. In terms of the existing models, it means that statistical models are valid in the lower incident momentum region and multi-peripheral models are valid in the high energy region.

(ii) The variation of $\langle N_s \rangle$ with (νS) over the entire energy region is satisfactorily represented by the expression

$$\langle N_s \rangle = a + b \ln (\nu S) + C \ln (\nu S) (\nu S)^{-a}.$$

The quadratic expression $(\langle N_s \rangle) = a + b \ln (\nu S) + C (\ln \nu S)^2$ can in no way be preferred over the above expression.

(iii) The above mathematical expression is consistent with the multi-peripheral model (Regge Muller theory) which is again a two-step mechanism. In an effort to obtain the above expression from the simple version of Muller Regge theory, one has to introduce an additional term in the leading particle behaviour. However, we are not sure if this is the only method and whether it satisfies the deep theoretical requirements.

(iv) There is no systematic relation between the coefficients obtained in expression (5) with those of the coefficients obtained in *p-p* collisions (Ganguli and Malhotra 1972). This can be understood in the *p*-nucleus collisions when we replace S by S_A $(=\nu S)$ where ν corresponds to the average number of nucleons or collisions which the incoming particle encounters in the emulsion. In other words ν has a distribution whereas this is not the case for pure *p-p* collisions. Therefore $\nu = 1$ may not reproduce the *p-p* data or as a result there will not be any systematic relation between the coefficients of *p-p* and *p*-nucleus data.

Appendix A

The experimental multiplicity in emulsion data contains only shower particles $(\beta \geqslant 0{\cdot}7)$ which consists of fast protons and created particles (mostly pions). This experimental multiplicity (which comes in *p*-nucleus interactions both from the basic *p-p* type and *p-n* type) need correction so as to include all 'non-evaporation' particles and as superposition of only *p-p* type of interactions.

A simple representation of the above picture will be

$$pp \rightarrow \underset{\text{(i)}}{\text{shower particles}} + \underset{\text{(ii)}}{\text{slow protons}} + \underset{\text{(iii)}}{\text{slow pions}} \tag{1}$$

(i) Shower particles—These are created particles+fast protons as explained above.

(ii) Slow protons—These are protons of $\beta<0{\cdot}7$ (non-evaporation).

(iii) Slow pions—These are all slow created particles.

Clearly the number of slow protons and fast protons should be equal to the number of baryons on the L.H.S.–I. In general projectile would give fast protons and the target, the slow ones. However, we do not know their exact fractions. In emulsion one considers only charged particles which register visible tracks.

In the case of *p-n* interactions following the same logic one can write

$$pn \to \underset{\text{(i)}}{\text{shower particles}} + \underset{\text{(ii)}}{\text{slow protons}} + \underset{\text{(iii)}}{\text{slow pions}} \tag{2}$$

Here (i), (ii), (iii) have the same meaning as for equation (1) above. If we believe that contribution to (ii) mainly comes from the target then in *pn* case it will be zero (because we consider only charged particles). The equivalence of parts (i) and (ii) of equations (1) and (2) assumes that all strong interaction process in *p-p* and *p-n* interactions are identical. The charge conservation alone demands that the total RHS of equation (1) should differ by equation (2) by unity irrespective of values of parts (i), (ii), and (iii) in these equations.

The experimental multiplicity is given by (i) of equations (1) and (2) respectively for *p-p* and *p-n* interactions. These do not differ by unity. Their difference may be less than one depending on what fraction of target proton is fast or slow in interactions and remembering that such a contribution is absent in *p-n* interactions.

As an illustration, if target nucleons are always slow the difference would be zero (i.e. experimental multiplicity in *p-p* and *p-n* interactions would be identical); if 50% of target nucleons are fast the difference would be 0·5.

The fraction of target protons as slow (or fast) may depend on the incident momenta. Then the difference of experimental multiplicity in *p-p* and *p-n* interactions would show energy dependence. However, both these arguments would not effect the formulation of equation (1) of the text.

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