Pramāṇa, Vol. 15, No. 5, November 1980, pp. 463–474, © Printed in India.

High energy proton-proton cross-section and Froissart bound

R C BADATYA[†] and P K PATNAIK,
†Present Address: Bhima Bhoi College, Rairakhol, Sambalour, India
Post Graduate Department of Physics, Sambalpur University, Jyoti Vihar, Burla, Sambalpur 768 017, India

MS received 22 October 1979; revised 1 May 1980

Abstract. A model for high energy proton-proton system is developed which obeys several asymptotic bounds including Froissart bound. The amplitude taken has two parts, a Regge part and a pomeron part. The Regge part is described by the exchange degenerate pair w and P'. To calculate the pomeron part the Freund-Harari duality is invoked which states that the pomeron is dual to the s-channel background. The s-channel background is estimated by parametrising the branch cuts in S through a conformal mapping method. Expressions for total cross-section, ratio of real-to-imaginary parts of forward amplitude and the slope parameter are presented. The corresponding quantities for $p\bar{p}$ are also evaluated. Our analysis favours strong exchange degeneracy.

Keywords. Proton-proton cross-section; Froissart bound; duality; pomeron; asymptotic bounds; scattering amplitude.

1. Introduction

High energy proton-proton scattering is characterised by a rising cross-section, shrinking forward peak, almost imaginary nature of the amplitude and a smooth t-dependence of differential cross-section which develops a dip at relatively higher energies. The dominant high energy behaviour of the total cross-section is very close to $(\ln s/s_0)^2$. Doubt still persists whether the data actually saturate the Froissart bound or violates it. Analysis of CERN, ISR and NAL data by Leith (1975) gives a fit for σ_T (in mb);

$$\sigma_T = 38{\cdot}34 + (0{\cdot}376 \pm 0{\cdot}015)\,(\ln s/93{\cdot}7\ \mathrm{GeV}^2)^{2\cdot10\,\pm\,0\cdot07}. \tag{1}$$

This fit does violate Froissart bound. However Leader and Maor (1973) suggested a similar fit which obeys Froissart bound and still agrees with the experiment. Amaldi *et al* (1977, 1978) also presented two sets of data analysis, one preserving the Froissart bound at lower limit and the other violating it explicitly. The expressions are:

$$\sigma_T^{A1} = (41{\cdot}9 \pm 1{\cdot}1)\,E^{-0\cdot37\,\pm\,0\cdot37} - (24{\cdot}2 \pm 1{\cdot}1)\,E^{-0\cdot55\,\pm\,0\cdot02} + (27{\cdot}0 \pm 1{\cdot}0) + (0{\cdot}17 \pm 0{\cdot}08)\,(\ln s)^{2\cdot10\,\pm\,0\cdot10}, \tag{2}$$

$$\sigma_T^{A2} = 4{\cdot}40\,N_q^H\,P_{\mathrm{lab}}^{0\cdot13} + 4{\cdot}00\,N_q^H \cdot N_{\mathrm{ns}}^H\,P_{\mathrm{lab}}^{-0\cdot20} - 7{\cdot}8\left(N_{\bar{d}}^H + 2\,N_{\bar{u}}^H\right) P_{\mathrm{lab}}^{-0\cdot50}. \tag{3}$$

Here different N's stand for the number of particular quarks and antiquarks present inside the hadron.

The forward slope parameter $B(s, 0)$ has been parametrised as a straight line (Abarbanel 1976);

$$B(s, 0) = b_0 + 2\,\alpha' \ln s \tag{4}$$

with $b_0 = 8{\cdot}3 \text{ GeV}^{-2}$ and $\alpha' = 0{\cdot}28 \text{ GeV}^{-2}$.

The ratio ρ of real-to-imaginary parts of forward amplitude has been experimentally obtained. This quantity is positive at low energy, decreases rapidly to change sign and attain maximum negative value at about 10–15 GeV^2. Then this increases monotonically and becomes positive at about 500 GeV^2. The differential cross-section is almost a smooth function of t, but develops a dip at relatively higher energies. The dip position shifts to lower $|t|$ as energy increases. After the dip the cross-section decreases monotonically at a much slower rate.

There have been several attempts to explain the different aspects of these phenomena (Kasman 1976; Mishra and Maharana 1976; Parida 1979; Sidhu and Wang 1975; Collins *et al* 1974 a,b). Kasman (1976) studied the high energy pp scattering within the context of geometric scaling model and obtained good fits to differential cross-sections at different energies. Kasman (1976), Torgerson and Kamal (1975), Chou and Yang (1979) predicted the existence of additional dips at higher t-values, but the experiment at $s=53$ GeV^2 does not show any such indication. Mishra and Maharana (1976) used the powerful method of group contraction to extract the forward slope parameter for the different hadronic processes. Parida (1979), using the optimal convergence method of Cutkosky and Deo (1970) and Ciulli (1969) obtained expressions for slope parameters for the different processes. He has also proposed a certain scaling variable which has good agreement with the experiment. On the other hand Collins *et al* (1974 a,b) obtained, within the framework of the Regge pole model, expressions for σ_T, ρ and $d\sigma/dt$ at different s and t values. Their scattering amplitude has two parts, a Regge pole part and a pomeron part. For elastic hadronic processes like pp, πp and kp they build up exchange degenerate Regge partners. Specifically for pp they take p' and w contributions only. For pomeron part they take the usual pomeron term, but take intercept α_p (0) greater than one. This term explicitly violates the Froissart bound. The total cross-section is

$$\sigma_T = 26{\cdot}3\, s^{0.06} + 39{\cdot}5\, s^{-1/2}. \tag{5}$$

For large t they add one extra term namely ‘ core term ’ to get satisfactory results. They have fairly good agreement with experiment. However, their theory has some undesirable features:

(i) It explicitly violates the Froissart bound.
(ii) For the forward slope parameter $B(s, 0)$, the agreement with experiment is rather poor.
(iii) pp being an exotic channel, duality demands that Regge pole contribution should be zero or at least very small. On the contrary the contribution is significant.

We feel that it is still possible to get agreement with experiment within the Harari-Freund duality (Harari 1968; Freund 1968) and without violating the Froissart bound (Froissart 1961). In this paper we take up a part of this programme. We restrict to forward directions only and obtain expressions for σ_T, ρ and $B(s, 0)$. Here the formulation is valid for small t values ($-t < 4\mu^2$) only. In future we plan to extend this to all t values.

In a formal Regge pole theory the scattering amplitude is written as a sum of two parts, a Regge pole part and a pomeron part. According to Harari-Freund duality the t-channel Regge poles are dual to the resonances in s-channel and the pomeron is dual to the non-resonant s-channel background. Thus a calculation with Regge poles and a pomeron is equivalent, in principle, to any other calculations which take into account effectively the resonances and the background in s-channel. There has also been calculations taking contributions from pomeron and a set of resonances (Ross *et al* 1973) only. Here we do just the opposite. We take Regge pole term as given by Collins *et al* (1974) and add a background term corresponding to the pomeron.

The non-resonant background part of the scattering amplitude has no poles, but has branch cuts corresponding to the multiparticle exchanges in s-channel. For pp this amplitude $f_p(s, t)$ has cuts from $s = 4m^2$ to ∞ and from $s = 4(m^2 - \mu^2) - t$ to $-\infty$. So a fixed t dispersion relation can be written down as

$$f_p(s, t) = \int_{-\infty}^{4(m^2-\mu^2)-t} ds' \frac{\operatorname{Im} f_p(s', t)}{(s' - s)} + \int_{4m^2}^{\infty} ds' \frac{\operatorname{Im} f_p(s', t)}{s' - s} \tag{6}$$

Given $\operatorname{Im} f_p(s, t)$ over the branch cuts $f_p(s, t)$ can be known. However this is not known and we have to use approximate methods. One such method is to map with a suitable mapping, the whole of analyticity region into a regular domain and approximate the amplitude by a series of suitable polynomials of the mapped variable (Erdelyi *et al* 1953). A particular mapping is chosen which confirms to some well-defined behaviour of the amplitude. For this calculation we choose a mapping which retains the high energy behaviour of the amplitude. The mapping we choose in this case is

$$Z = -i \sin^{-1}\left(\frac{2(s - 4m^2) + 4\mu^2 + t}{4\mu^2 + t}\right). \tag{7}$$

This maps the whole of analyticity plane into a strip along real axis with boundaries at $\pm i\pi/2$. Now the amplitude $f_p(s, t)$ is analytic in this strip as a function of Z, and can be expanded as

$$f_p(s, t) = \sum_{n=0}^{\infty} a_n H_n(Z), \tag{8}$$

provided it satisfies certain growth conditions. $H_n(Z)$ are Hermite polynomials. The a_n's are independent of Z, but can be functions of t. For large s ($s \gg 4m^2$) and small t ($|t| < 4\mu^2$)

$$Z = \left(\ln \frac{s}{\mu^2} - i\frac{\pi}{2}\right) = \ln\left(\frac{s}{\mu^2} \exp(-i\pi/2)\right). \tag{9}$$

With this mapping we have achieved two objectives. The amplitude has explicit s-channel analyticity and it is very closely related to the Regge variable $(s/s_0 \exp(-i\pi/2))$. Further it goes to infinity with s as $\ln s$.

Regarding the growth condition we impose that the amplitude not only grows infinitely with Z but also saturate the Froissart bound. This restriction will help in terminating the series in (8) after a small number of terms. For example with $t=0$, only terms upto $n=2$ will contribute. This is supposed to be a good approximation at high energy behaviour in the form of Froissart bound. Actually this gives an extremely good fit upto quite low energies ($s \sim 15$ GeV^2) when an appropriate Regge pole contribution is added to it. The Regge contribution is (Collins *et al* 1974a, b) given by

$$f_R(s, t) = -G_R\,[S \exp(-i\pi/2)]^{\alpha_R(t)}$$
$$\exp(a_R t)\,[1 - i\beta \exp(a_3 t)\,(1 + t/t_0)]. \tag{10}$$

Here β is the exchange degeneracy parameter signifying the amount of exchange degeneracy present. Our best fit gives $\beta = 1$ suggesting strong exchange degeneracy and agreement with duality.

In § 2 we derive expressions for σ_T, ρ and $B(s, 0)$. Expressions for the first two are immediately obtained. However to obtain an expression for $B(s, 0)$ we take the help of some more asymptotic theorems on the behaviour of the scattering amplitude at $t = 0$. In § 3 numerical calculations are presented and the results are compared with the experiment.

2. The scattering amplitude

We assume that spin and isospin effects are small at high energy and normalise the scattering amplitude as

$$d\sigma/dt = \frac{1}{16\pi}|f(s, t)|^2, \tag{11}$$

$$\sigma_T = \operatorname{Im} f(s, 0). \tag{12}$$

We also have

$$\rho = \operatorname{Re} f(s, 0)/\operatorname{Im} f(s, 0), \tag{13}$$

and the forward slope parameter

$$B(s, 0) = \frac{d}{dt} \ln \frac{d\sigma}{dt}\bigg|_{t = 0}.$$

In our model the scattering amplitude has two parts, the Regge part $f_R(s, t)$ and the pomeron part $f_p\,(s, t)$. We adopt the form of $f_R\,(s, t)$, suggested by Collins *et al* (1974 a, b),

$$f_R\,(s, t) = -\,G_R\,[s \exp\,(-\,i\pi/2)]^{a_R\,(t)}$$
$$\exp\,(a_R\,t)\left[1 - i\beta \exp\,(a_3\,t)\left(1 + \frac{t}{t_0}\right)\right]. \tag{10}$$

As we are only interested in the forward direction this can equivalently be written as

$$f_R\,(s, t) = -\,G_R\,[s \exp\,(-i\pi/2)]^{a_R\,(t)}$$
$$\exp\,(a_R\,t)\;[1 - i\beta \exp\,(a_4\,t)] \tag{14}$$

where $\quad a_4 = a_3 + \dfrac{1}{t_0}$

As suggested in § 1 the pomeron part $f_p\,(s, t)$ is obtained by parametrising the s-channel branch cuts. For $f_p\,(s, t)$ we have branch cuts from $s=4m^2$ to ∞ and from $s=4m^2-4\mu^2-t$ to $-\infty$. Here m and μ are the masses of the proton and pion respectively (see figure 1). For $-t < 4\mu^2$ the two cuts are non-overlapping. As this is very essential for our analysis we restrict to the region $-t < 4\mu^2$ only. This will present no problem for the calculation of σ_T, ρ and $B(s, 0)$. In future we would like to extend our analysis to higher $|\,t\,|$ values.

Now our aim is to bring the whole of analytic domain inside a regular region by a suitable conformal mapping and then expand the amplitude in a suitable series. This method was used by Frazer (1961) to parametrise the differential cross-section. With a suitable mapping he transformed the whole analytic domain inside a circle and then expanded the amplitude as a power series in the mapped variable. The same techniques were also used by Levinger and Peierls (1964) and Greenberger and Margolis (1961) to analyse form factors. In place of power series expansion, polynomial expansion can also be used. In this connection a general result can be stated like this (Erdelyi *et al* 1953); ' A series of Jacobi polynomials converge in an ellipse whose foci are at ± 1 and every function which is analytic in such an ellipse may be expanded in a series of Jacobi polynomials. In case of Lagurre polynomials the region of convergence is a parabola around the positive real axis, with its focus at the origin. In the case of Hermite polynomials the region of convergence is a strip

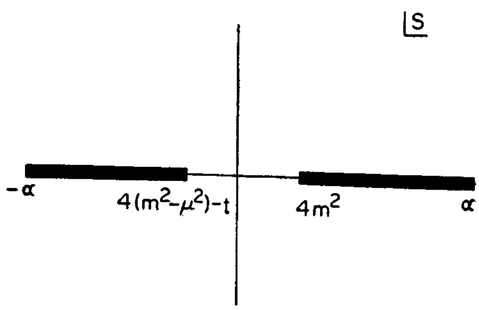

Figure 1. s-channel cut structure.

whose central line is the real axis. In both cases the region of convergence is unbounded and an analytic function which is to be expanded in a series of Lagurre and Hermite polynomials must satisfy certain growth conditions in addition to being analytic in an appropriate region.'

For our case we know that the total cross-section is increasing and we assume that it will continue to increase saturating the Froissart bound. Such an asymptotic behaviour can be obtained only by taking an expansion in Lagurre or Hermite polynomials. For the present case expansion in Hermite polynomials is more suitable and the corresponding mapping which brings the whole of analytic domain inside a strip along the real axis is (see figure 2)

$$Z = -i \sin^{-1}\left[\frac{2(s-4m^2)+t+4\mu^2}{t+4\mu^2}\right]. \tag{7}$$

For large s and small t

$$Z = \ln\frac{s}{\mu^2} - i\frac{\pi}{2} = \ln\left(\frac{s}{\mu^2}\exp(-i\pi/2)\right). \tag{9}$$

This shows that the new variable is very closely related to the usual Regge variable

$$[s/s_0 \exp(-i\pi/2)].$$

With this mapping we write

$$f_p(s,t) = \sum_n a_n H_n(Z), \tag{15}$$

In this infinite series a_n's are independent of Z but can be functions of t. To bring out the details of the structure of $a_n(t)$ we impose the following high energy restrictions:

(i) Froissart bound (Froissart 1961)

$$\operatorname{Im} f(s,0) \leqslant \text{const}\left(\ln\frac{s}{s_0}\right)^2 \tag{16}$$

(ii) Martin bound (Martin 1963)

$$\frac{d}{dt}\ln \operatorname{Im} f(s,0) \geqslant \frac{\sigma_T}{32\pi}. \tag{17}$$

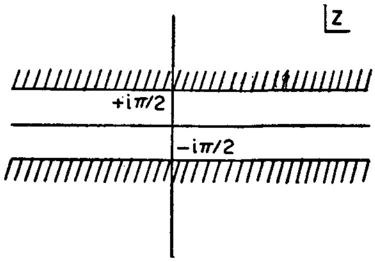

Figure 2. Cut structure in mapped variable.

(iii) Popov-Mur and Singh-Roy bound (Popov and Mur 1965, Singh and Roy 1970)

$$\frac{1}{\operatorname{Im} f(s, 0)} \frac{d^n}{dt^n} \operatorname{Im} f(s, 0) \geqslant \frac{1}{n!\,(2n+1)} \left(\frac{2n+2}{2n+1} \frac{\sigma_T^2}{16\sigma_{el}}\right)^n \tag{18}$$

As $\sigma_T/\sigma_{el} \geqslant 1$ we rewrite this in a less restrictive form

$$\frac{1}{\operatorname{Im} f(s, 0)} \frac{d^n}{dt^n} \operatorname{Im} f(s, 0) \geqslant \text{const}\ \sigma_T^n. \tag{19}$$

(iv) Kinoshita bound (Kinoshita 1966)

$$\frac{d}{dt} \ln \operatorname{Im} f(s, 0) \leqslant \text{const}\ [\ln (s/s_0^2\ \sigma_T)]^2. \tag{20}$$

Again we rewrite this in a less restrictive form as

$$\frac{d}{dt} \ln \operatorname{Im} f(s, 0) \leqslant \text{const}\left(\ln \frac{s}{s_0}\right)^2. \tag{21}$$

The first restriction imposes that

$$a_n(0) = 0 \text{ for } n > 2.$$

The other restrictions suggest that $a_3^{(t)}$ and $a_4^{(t)}$ must vanish as t at $t = 0$. The third restriction requires that the series must extend upto infinity and at $t = 0$ $a_{2+n+m}^{(t)}$ must vanish as t^{n+1} where n is even and m takes values of 1 or 2. If we intend to have a finite number of terms and consistency with this restriction we have to accept s-dependence into a_n. A finite number of terms with exponential dependence on $t\,F(Z)$ where Re $F(Z) = \sigma_T$ can satisfy all the bounds. Thus we construct our model as

$$f_p(s, t) = i\,\{b_0 \exp\,[\alpha_0 F(Z)\,t] + b_1 Z \exp\,(\alpha_1 F(Z)\,t) + b_2 Z^2 \exp\,(\alpha_2 F(Z)\,t)\}, \tag{22}$$

where α_i's and b_i's are real constants,

$$\text{and } F(Z) = b_0 + b_1 Z + b_2 Z^2. \tag{23}$$

The corresponding equations for σ_T, and $B(s, 0)$ are given by

$$\sigma_T = \frac{G_R(1-\beta)}{(2s)^{1/2}} + f_1(s) \tag{24}$$

$$\rho = \left(\frac{-G_R(1+\beta)}{(2s)^{1/2}} - f_2\right) \Big/ \sigma_T \tag{25}$$

$$B(s,0)=2\Bigg[\frac{G_R}{(2s)^{1/2}}A(\beta)+b_0\,a_0 f_1+b_1\,a_1\left(y f_1+\frac{\pi}{2}f_2\right)$$

$$+b_2\,a_2\left(y^2 f_1-\frac{\pi^2}{4}f_1+\frac{\pi}{2}\,y f_2\right)$$

$$+\rho\left\{-\frac{G_R}{(2s)^{1/2}}A(-\beta)-b_0\,a_0 f_2+b_1\,a_1\left(\frac{\pi}{2}f_1-y f_2\right)\right.$$

$$\left.+b_2\,a_2\left(\frac{\pi}{2}y f_1-y^2 f_2+\frac{\pi^2}{4}f_2\right)\right\}\Bigg]\Big/\left[\sigma_T(1+\rho^2)\right], \tag{26}$$

where $A(\beta)=K(1-\beta)+\frac{\pi}{2}\,\alpha'_R\,(1+\beta)-\beta\,a_4,$

$$y=\ln s/\mu^2,\quad K=a_R+\alpha'_R\ln s, \tag{27}$$

$$f_1=\text{real } F(Z)=b_0+b_1 y+b_2\left(y^2-\frac{\pi^2}{4}\right),$$

$$f_2=\text{Im } F(Z)=-\frac{\pi}{2}\,b_1-\pi\,b_2\,y. \tag{28}$$

Equations (22) to (28) are the main results of this paper. The corresponding results for $p\bar{p}$ can be obtained by substituting $-\beta$ for β.

3. Numerical results and comparison with experimental data

The total cross-section for pp and $p\bar{p}$ are plotted in figure 3. The value of $G_R\,\beta$ is obtained from the experimental data on the difference of $p\bar{p}$ and pp total cross-sections. The σ_T for pp is obtained for different values of β. The best agreement is obtained for $\beta=1$ signifying the presence of strong exchange degeneracy. This is also a strong evidence for the Harari-Freund duality. For this case the proton-proton total cross-section is completely independent of the Regge contribution. We have

$$\sigma_T=b_0+b_1\ln\frac{s}{\mu^2}+b_2\left[\left(\ln\frac{s}{\mu^2}\right)^2-\frac{\pi^2}{4}\right]$$

where $b_0=80{\cdot}00$ mb, $b_1=-9{\cdot}25$ mb and $b_2=0{\cdot}5296$ mb.
In figure 3 we also plot σ_T for the case of $\beta=0{\cdot}4$. The other parameters are $G_R=103$ mb, $b_0=22{\cdot}855$ mb, $b_1=0{\cdot}4097$ mb and $b_2=0{\cdot}1021$ mb.

The results of Collins *et al* (1974) and Amaldi *et al* (1977) are also reproduced for comparison. The agreement is very good. The data are from Amaldi *et al* (1977, 1978), Amendolia *et al* (1973), Ayers *et al* (1977), Carrol *et al* (1973, 1974, 1979) and Akerlof (1976). The data at much higher energies are also available from cos-

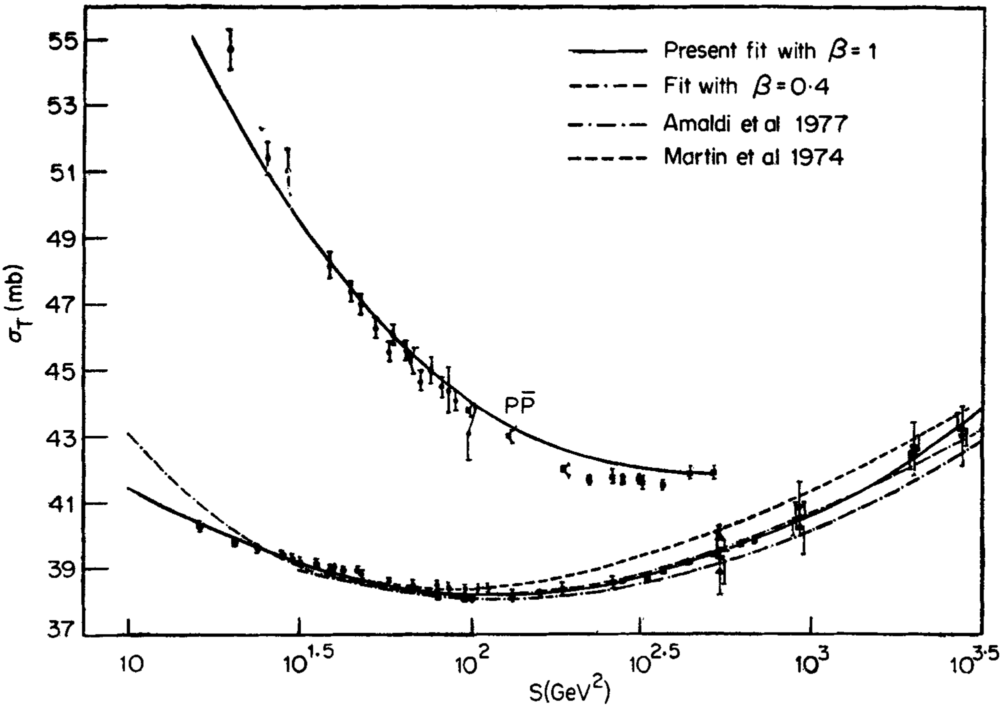

Figure 3. Proton-proton and proton-antiproton total cross-sections.

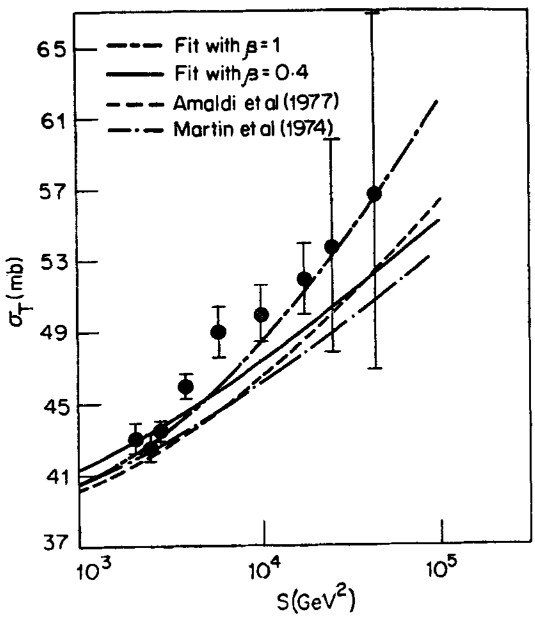

Figure 4. Proton-proton total cross-section at cosmic ray energies.

mic ray experiments (Nam *et al* 1977). The present results agree with these data as is shown in figure 4.

With the same set of parameters the ratio of real-to-imaginary part of the forward scattering amplitude is given in figure 5. This is a prediction of the theory. The agreement is again very good.

The forward slope parameter for pp is shown in figure 6 (Leith 1975). We take $\alpha_R(0) = 0{\cdot}5$ and $\alpha'_R = 1$. The other fitted parameters are

$$a_R = 2{\cdot}9292\ \text{GeV}^{-2},\ a_4 = 10{\cdot}1416\ \text{GeV}^{-2},$$

$$\alpha_0 = 0{\cdot}061\ \text{GeV}^{-2},\ \alpha_1 = 0\ \text{GeV}^{-2},\ \alpha_2 = 0{\cdot}024\ \text{GeV}^{-2}.$$

The $B(s, 0)$ for $p\bar{p}$ is shown in figure 7. The data for this case are widely varying. However a general trend can be extracted from these data which shows a fast de-

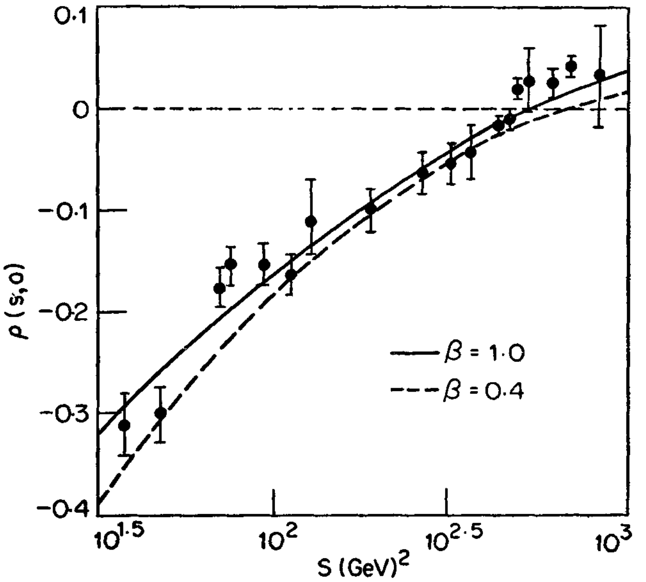

Figure 5. Ratio of real to imaginary part of forward scattering amplitude for pp.

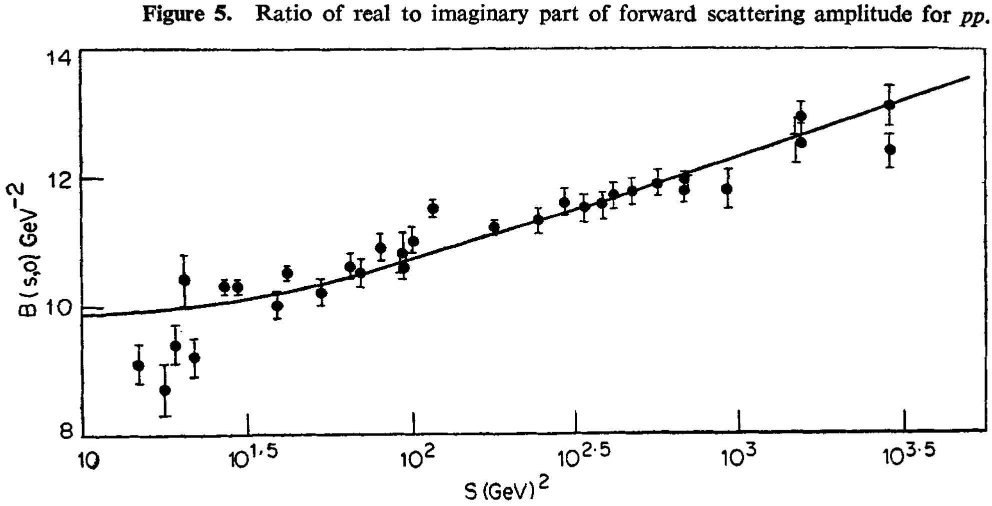

Figure 6. Forward slope parameter for pp.

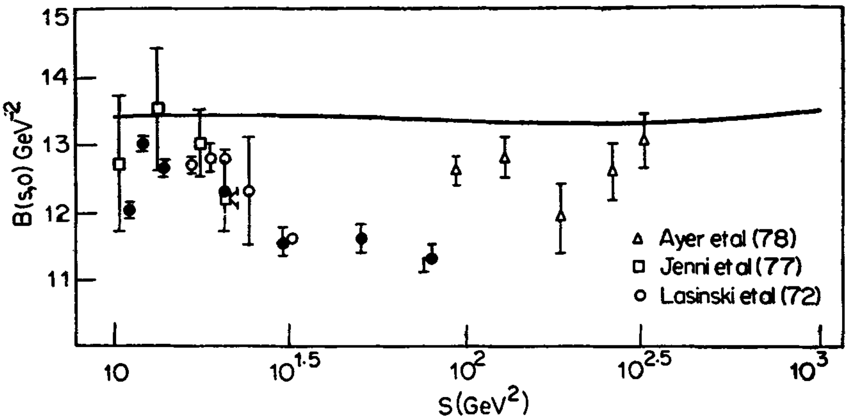

Figure 7. Forward slope parameter for $p\bar{p}$.

crease and then a slight increase. Although our results conform to this pattern of behaviour the general agreement is bad. We attribute this to the inadequacy of the Regge part of the amplitude.

4. Conclusions

In this paper we have constructed a model for the scattering amplitude for small values of t. The amplitude incorporates Froissart bound and also some other bounds on the behaviour of scattering amplitude at $t=0$. The explicit s-dependence has been incorporated through analyticity. The experimental agreement is very good. Our results on σ_T are much better than others showing explicit violation of Froissart bound.

Our results favour strong exchange degeneracy and is consistent with the Harari-Freund duality.

Acknowledgement

One of the authors (RCB) is thankful to the University Grants Commission, New Delhi and the Sambalpur University for a fellowship. The authors also thank Prof. R K Satpathy for encouragement and Dr J K Mahapatra for help in computation.

References

Abarbanel H D I 1976 *Rev. Mod. Phys.* **48** 435
Akerlof C W *et al* 1976 *Phys. Rev.* **D14** 2864
Amaldi U *et al* 1977 *Phys. Lett.* **B66** 390
Amaldi U *et al* 1978 *Nucl. Phys.* **B145** 367
Amendolia *et al* 1973 *Phys. Lett.* **B44** 119
Ayers D S *et al* 1977 *Phys. Rev.* **D15** 3105

Carrol A S 1973 *Phys. Lett.* **B61** 303
Carrol A S 1974 *Phys. Rev. Lett.* **33** 928
Carrol A S 1979 *Phys. Lett.* **B80** 423
Chou T T and Yang C N 1979 *Phys. Rev.* **D19** 3268
Ciulli S 1969 *Nuovo Cimento* **A61** 787
Collins P D B, Gault F D and Martin A 1974a *Nucl. Phys.* **B80** 135
Collins P D B, Gault F D and Martin A 1974b *Nucl. Phys.* **B83** 252
Cutkosky R E and Deo B B 1970 *Phys. Rev.* **D1** 2547
Denisov S P 1971 *Phys. Lett.* **B36** 415
Erdelyi A 1953 *Higher transdental functions* (New York: McGraw Hill) Vol. 2, ch. 10
Foley 1967 *Phys. Rev. Lett.* **19** 857
Frazer W R 1961 *Phys. Rev.* **123** 2180
Freund P G O 1968 *Phys. Rev. Lett.* **20** 235
Froissart M 1961 *Phys. Rev.* **123** 1053
Greenberger D M and Margolis B 1961 *Phys. Rev. Lett.* **6** 310
Harari H 1968 *Phys. Rev. Lett.* **20** 235
Kasman J 1976 *Phys. Rev.* **D14** 2273
Kinoshita T 1966 Lectures at Boulder Summer School
Leader E and Moar U 1973 *Phys. Lett.* **B43** 505
Leith D W G S 1975 *Recent development in diffraction studies*, SLAC-Pub-1646
Levinger J S and Peierls R F 1964 *Phys. Rev.* **134B** 1341
Martin A 1963 *Phys. Rev.* **129** 1432
Mishra S P and Maharana J 1976 *Phys. Rev.* **D14** 133
Nam 1977 *Proc. 15th Int. Conf. Cosmic Rays,* Plovdiv Bulgaria **7** 104
Parida M K 1979 *Phys. Rev.* **D19** 150, 164
Popov N S and Mur V D 1965 Yardern Fiz 3 1966 *Sov. J. Nucl. Phys.*
Ross W A 1973 *A model for low energy pion nucleon scatterings* SLAC-Pub-1180
Roy S M 1972 *Phys. Rep.* **C5** 128
Sidhu D P and Wang L L 1975 *Phys. Rev.* **D11** 1954
Singh V and Roy S M 1970a *Phys. Rev. Lett.* **29** 28
Singh V and Roy S M 1970b *Phys. Rev.* **1D** 2638
Torgerson R and Kamal A N 1975 *Phys. Rev.* **D12** 1974