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Theory of radiation from relativistic positrons moving in the ⟨110⟩ axial channels of f.c.c. (diamond) crystal

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Abstract. The emission of radiation from relativistic positrons moving in the ⟨110⟩ axial channels of an f.c.c. (diamond) crystal has been studied. An expression for the radiation intensity has been obtained for the general case of positron motion. This expression has been simplified for the particular case of well-collimated incident beam. Enhancement of the radiation over (ordinary) bremsstrahlung has been discussed.

Keywords. Channeling radiation; axial channeling; electromagnetic radiation; relativistic positrons; radiation intensity; f.c.c. diamond crystal; incident beam.

1. Introduction

In recent years emission of electromagnetic radiation from channeling particles has been of great theoretical interest. Possibility of such a radiation was first explored by Kumakhov (1976) using classical theory, and was later described by Kumakhov and Wedell (1976) using quantum mechanical theory. Since then much theoretical progress has been made towards the understanding of the characteristics of the channeling radiation (Kumakhov 1977; Kumakhov and Wedell 1977; Terhune and Pantell 1977; Bazylev and Zhevago 1977; Wedell 1978; Pantell and Alguard 1979; Lal and Joshi 1980). However, the characteristics of the radiation emitted by axially channeled relativistic positrons, have not been studied while experimental measurements have been made of the axial spectrum (Alguard *et al* 1979). It was felt worthwhile to study the axial radiation properties of the channeling positrons.

In § 2, we consider the motion of the positrons moving along the ⟨110⟩ rows of an f.c.c. (diamond) crystal. We obtain in § 3 a general expression for the photon intensity. This expression is simplified in § 4, for the particular case of an one-directional collimated beam. General features of the radiation characteristics have also been discussed.

Throughout the paper we have taken $\hbar=c=1$, where $2\pi\hbar$ is the Plank's constant, and c is the photon speed in vacuum.

2. Motion of the channeled positrons

Projection of the ⟨110⟩ rows of the f.c.c. (diamond) crystal on a (110) transverse plane is shown in figure 1. The z-component of the 4-coordinate (t, x, y, z), where t

is time, has been assumed along the $\langle 110\rangle$ rows. In the transverse space (x, y), the x-axis has been assumed containing the rows R_1 and R_4 (see figure 1). The origin $(x, y) \equiv (0, 0)$ in the transverse space is assumed on the middle point 0 of the rows R_1 and R_4.

We assume that the potential of the atoms of a ($\langle 110\rangle$) row is described (at the point (x, y)) by a continuum scalar potential $V_i(x, y)$, where the subscript i serves to let V denote (continuum) potential of the ith row. Then the crystal potential is given by (Gemmell 1974)

$$U(x, y) = \sum_{i=\text{all rows}} V_i(x, y) - U^{\min}, \tag{1}$$

where $U^{\min}$ is a constant whose value is taken such that the minimum value of $U(x, y)$ is zero. The crystal potential defined in this way is periodic in the transverse space (x, y). The periodicity is such that $U(x, y)$ remains invariant in the translations

$$x \to x + Md, \quad y \to y + N(d/\sqrt{2}), \tag{2}$$

where $M, N \equiv 0, \pm 1, \pm 2, \ldots$, and d is the lattice constant (see figure 1, $d = 4\, R_2R_3$). Moreover, the crystal potential is invariant under the reflections

$$\begin{aligned} &\text{(i) } x \to -x \\ &\text{(ii) } y \to -y \\ &\text{(iii) } x \to -x, y \to -y. \end{aligned} \tag{3}$$

2.1. *General motion*

The field operator $\psi(t, \mathbf{r})$ of the positron in the field $U(x, y)$ may be written as $[\mathbf{r} \equiv (x, y, z)]$:

$$\psi(t, \mathbf{r}) = \sum [a_s \phi_s^{(+)}(\mathbf{r}) \exp(-i\epsilon^{(+)}t) + b_s^+ \phi_s^{(-)}(\mathbf{r}) \exp(i\epsilon^{(-)}t)], \tag{4}$$

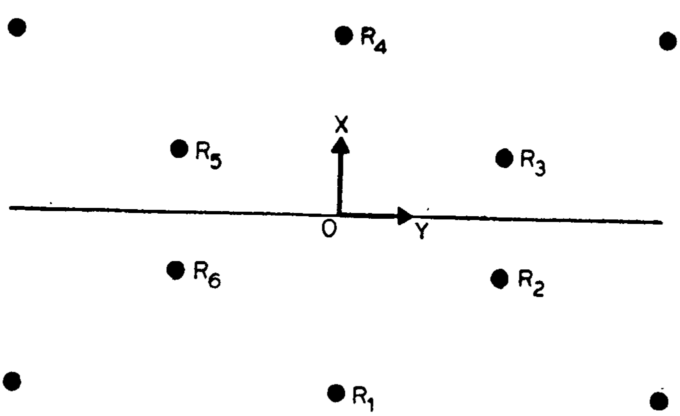

Figure 1. Projection of $\langle 110\rangle$ rows of an f.c.c. (diamond) crystal on the (110) transverse plane. The origin $O = (0, 0)$ of the (x, y) transverse space is the centre of a circle passing through the six neighbouring rows R_1, R_2, R_3, R_4, R_5 and R_6.

where $\phi_s^{(\pm)}$ and $\epsilon_s^{(\pm)} = p_0 \pm (-m)$ are the wave functions and energy levels of the electron and the positron (+ for electron, — for positron) respectively, which satisfy Dirac's equation in the field $U(x, y)$. That is

$$(\gamma^\mu p_\mu - m - \gamma^o e\, U)\, \phi_s\, (\mathbf{r}) = 0. \tag{5}$$

Here γ^μ are Dirac matrices; m is the positron mass; p_μ is the 4-momentum operator: and e is the electronic charge.

Under channeling conditions the transverse energy E_{s_t} of the positron would be considerably smaller than the longitudinal energy $E_{s_l}^{(-)}$. Also, the transverse energy will be conserved. Therefore $\phi_s^{(-)}$ may be broken up in two parts—one corresponding to the longitudinal relativistic motion, say $\psi_{sl}^{(-)}(z)$, while the other corresponding to the transverse non-relativistic motion, say $\chi_{s_t}(x, y)$. [Here s_l and s_t are quantum numbers of the longitudinal and transverse motions respectively.] In this sense,

$$\phi_s^{(-)} = \chi_{s_t}(x, y)\, \psi_{s_l}^{(-)}(z). \tag{6}$$

Here the wave functions χ_{s_t} and $\psi_{s_l}^{(-)}$ are solutions of

$$\left(\gamma^\circ E_{s_l}^{(-)} - \gamma^3 p_z\right) \psi_{s_l}^{(-)}(z) = 0, \tag{7}$$

$$\left(E_{s_t} + \frac{1}{2\gamma m} \nabla^2 - e\, U\right) \chi_{s_t}(x, y) = 0. \tag{8}$$

In equation (8), γ is $(1 - v^2)^{-1/2}$, where v is the positron velocity. $\nabla^2 \equiv (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ is the two-dimensional Laplacian.

The hamiltonian H corresponding to the motion of the positron described by (7) and (8) is given by

$$H = \sum_{s \equiv (s_l,\, s_t)} (E_{s_l}^{(-)} + E_{s_t}) b_s b_s^+. \tag{9}$$

Corresponding to the relativistic and non-relativistic longitudinal and transverse motions of the positron, the component $j^3_{s's}(t, \mathbf{r})$ of the transition 4-current $j^\mu_{s's}(t, \mathbf{r})$ would be relativisitic, while the components $j^{1,2}_{s's}(t, \mathbf{r})$ would be non-relativisitic. That is to say, $j^3_{s's}(t, \mathbf{r}) = \bar{\phi}_s^{(-)} \exp(-iHt)\, \gamma^3 \exp(iHt)\, \phi_s^{(-)}$, (10)

and $$j^{1,2}_{s's}(t, \mathbf{r}) = \mathrm{Re}\, [\bar{\phi}_s^{(-)} \exp(-iHt)\, v^{1,2} \exp(iHt)\, \phi_s^{(-)}]. \tag{11}$$

Here $v^{1,2}$ are the x- and y- components of the 4-velocity operator v^μ, i.e.,

$$v^1 = -\frac{i}{\gamma m} \frac{\partial}{\partial x},\; v^2 = -\frac{i}{\gamma m} \frac{\partial}{\partial y}. \tag{12}$$

2.2. *Solutions of* (7) and (8)

Solutions of equation (7) may be written as

$$\psi_{p_z}^{(-)(r)}(z) = N u^{(r)}(-p_z) \exp(-ip_z z), \tag{13}$$

where the bispinor $u^{(r)}(-p_z)$, with (r) as the spin index, satisfies

$$(\gamma^\circ E_l^{(-)} - \gamma^3 p_z)\, u^{(r)}(-p_z) = 0. \tag{14}$$

We specify the normalization constant N in (13) by requiring

$$\int dz\, \bar{\psi}_{p_z}^{(-)(r)}(z)\, \psi_{p'_z}^{(-)(s)}(z) = \delta_{rs}\, \delta(p_z - p'^z). \tag{15}$$

The exact solution of (8) is difficult to obtain so that there is a need for some approximation. From a practical viewpoint we must have χ_{s_t} as a product of two functions—one (say $f_\mu(x)$) depending upon x only, while the other (say $g_\nu(y)$) depending upon y only. That is to say, we must have

$$\chi_{s_t}(x, y) \approx f_\mu(x)\, g_\nu(y). \tag{16}$$

Here μ and ν are the quantum numbers of the x-space and y-space, respectively.

Equation (16) implies that the potential $U(x, y)$ must be approximated by a separable form, say $[U_1(x)+U_2(y)]$. Since channeling radiation occurs when the positron makes transitions among its bound states (cf. for example, Kumakhov and Wedell 1976), $U_1(x)$ and $U_2(y)$ must correspond to bound state motion of the positron. Such a motion is possible (in the unit cell containing the origin 0, cf. figure 1) about the x-and y-axes only. This is because the potential sources (the $\langle 110 \rangle$ rows) are symmetrically situated about these two axes. But, confinement of the positron in an oscillatory motion about the x-axis (cf. figure 1) will be significantly weaker than in the motion about the y-axis. There are two reasons for this: Firstly, in the case of the motion about the y-axis pairs of nearest neighbouring rows (R_2, R_3 or R_5, R_6) make the potential more suitable for an oscillatory motion (cf. figure 2 of Esbensen *et al* 1977) than that made in the case of the motion about x-axis, by the pairs of neighbouring rows R_3, R_5 or R_2, R_6. [Notice that $R_2 R_6 = 2\sqrt{2}\, R_2 R_3$.] Secondly, confinement of the positron in an oscillatory motion will be about the whole y-axis since there are no rows on the y-axis (cf. appendix also), while along the x-axis confinement of motion will not be about the whole x-axis as there lie rows (like R_1, R_4) on the x-axis.

The foregoing discussion leads us to the conclusion that the region about the y-axis is considerably more suitable for emission of channeling radiation. So we limit ourselves only to this region. We also assume that the incident beam is well collimated in the x-direction so that the positron motion is confined to small x ($x \ll d/4$) values only. Then it will be reasonable to approximate $U_2(y)$ by $U(0, y)$ so that $g_\nu(y)$ will be a solution of the equation

$$\left[-\frac{1}{2\gamma m} \frac{\partial^2}{\partial y^2} + e\, U(o, y) \right] g_\nu(y) = E_\nu\, g_\nu(y), \tag{17}$$

where E_ν is the energy corresponding to the motion in y-space.

In order to determine $U_1(x)$ we keep in mind that $x \ll d/4$ and expand $U(x, y)$ about $x = 0$ and obtain

$$U(x, y) \approx U(o, y) + \xi(y)\, x^2, \tag{18}$$

where
$$\xi(y) = \frac{1}{2} \frac{\partial^2 U}{\partial x^2}\bigg|_{x=0}. \tag{19}$$

Since the first term of (18) has been taken equal to $U_2(y)$, we determine $U_1(x)$ from the second term of (18). In order to incorporate the effect of the motion in y-space on the positron's motion in x-space, we define the potential $U_1(x)$ as the potential $\xi(y)x^2$ calculated according to the quantum-mechanical probability given by the y-space wave function $g_\nu(y)$. In mathematical language, $U_1(x) = \bar{\xi}x^2$, where

$$\bar{\xi} = \int g_\nu^*(y)\, \xi(y)\, g_\nu(y)\, dy. \tag{20}$$

The wave function $f_\mu(x)$ is now a solution of

$$\left[-\frac{1}{2\gamma m} \frac{\partial^2}{\partial x^2} + e\bar{\xi}x^2\right] f_\mu(x) = E_\mu f_\mu(x), \tag{21}$$

where $E_\mu\ (= E_{s_t} - E_\nu)$ is the energy corresponding to the motion in x-space.

The value of $\bar{\xi}$ given by (20) warrants some discussion. If $g_\nu(y)$ is a unbound state wave function $\bar{\xi}$, in a unit transverse cell, would be zero. Consequently $f_\mu(x)$ would be a free wave. The minimum value of $\bar{\xi}$, say $\bar{\xi}^{bs}_{\min}$, corresponds to the minimum value of the y-component E_ν of the transverse energy E_{s_t}. [Here the superscript bs indicates that $\bar{\xi}_{\min}$ is the minimum value of $\bar{\xi}$ when g_ν's are bound state functions. The true minimum value of $\bar{\xi}$ is, as discussed above, zero; but it corresponds to unbound function g_ν.] For minimum $E_\nu(<0)$, $g_\nu(y)$ would be a one-loop function with maximum at $y=0$, i.e., $|g_\nu(0)| > |g_\nu(y)|$, $y \neq 0$. This, when combined with the fact that (see table 1 of the appendix) $\xi(y)$ varies slowly near $y=0$, leads to

$$\bar{\xi}^{bs}_{\min} \sim \xi(0) > 0. \tag{22a}$$

Similarly the maximum value of $\bar{\xi}$, say $\bar{\xi}_{\max}$ would correspond to the bound state (in the y-space) of maximum E_ν. This, when combined with the fact that maximum value of $\xi(y)$ is $\xi\left(\pm \frac{d}{2\sqrt{2}}\right)$ (see table 1 of appendix) leads to

$$\bar{\xi}_{\max} < \xi\left(\pm \frac{d}{2\sqrt{2}}\right). \tag{22b}$$

Moreover, we note that $\xi(y)$, being a distribution, would be uncertain by an amount

$$\triangle \xi = [(\overline{\xi^2}) - (\bar{\xi})^2]^{1/2}. \tag{23}$$

Since $\bar{\xi}$ is positive for all E_ν belonging to bound states (see 22a), solution of equation (21) would be that of a harmonic oscillator. That is,

$$f_\mu(q) = [2^\mu \mu! \sqrt{\pi}]^{-1/2} \exp(-\tfrac{1}{2} q^2) H_\mu(q) \tag{24}$$

where $q = x(\gamma_m w_x)^{1/2}$; $H_\mu(q)$ are hermite polynomials; and, in the laboratory frame,

$$w_x = (2\bar{\xi}/\gamma m)^{1/2}. \tag{25}$$

The energy eigenvalues E_μ are given by

$$E_\mu = (\mu + \tfrac{1}{2}) w_x. \tag{26}$$

Since, $\xi(y)$ is uncertain (cf. 23), w_x will also be uncertain by an amount

$$\triangle w_x = w_x (\triangle \xi / \bar{\xi})^{1/2}. \tag{27}$$

3. Emission of photons

In the preceding section we have studied the motion of relativistic positrons in the $\langle 110 \rangle$ axial channels of an f.c.c. (diamond) crystal. In this section we study the radiation characteristics of these positrons.

The per unit time per unit frequency probability $d^2W/dwdt$ of emission of a photon of energy $w(>0)$ from the positron corresponding to the $s \rightarrow s'$ transition is given by (Yakimets 1965; Zhevago 1977):

$$\frac{d^2 W_{s's}}{dw\, dt} = \frac{e^2}{2^5 \pi^7} \operatorname{Im} \iiint_0^\infty L^{\lambda\rho}_{s's}(\tau, \mathbf{k}, \mathbf{k}') D_{\lambda\rho}(w, \mathbf{k}, \mathbf{k}')\, d\tau\, d\mathbf{k}\, d\mathbf{k}', \tag{28}$$

where $\tau = t - t'$.

The tensor $L^{\lambda\rho}$ is connected with the matrix elements of the Fourier transform of the transition 4-current $j^\lambda_{s's}(t, \mathbf{r})$ by the relation (Zhevago 1977)

$$L^{\lambda\rho}_{s's}(\tau, \mathbf{k}, \mathbf{k}') = \exp(-iw\tau)\, j^\lambda_{ss'}(t, \mathbf{k})\, j^{\rho *}_{s's}(t, \mathbf{k}'), \tag{29}$$

where $$j^\lambda_{ss'}(t, \mathbf{k}) = \int j^\lambda_{ss'}(t, \mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r})\, d\mathbf{r}, \tag{30}$$

is the transition 4-current in the momentum representation.

The tensor

$$D_{\lambda\rho}(w, \mathbf{k}, \mathbf{k}') = \int D_{\lambda\rho}(\tau, \mathbf{r}, \mathbf{r}') \exp i\,[(w\tau - \mathbf{k} \cdot \mathbf{r} + \mathbf{k}' \cdot \mathbf{r}')] \times d\tau\, d\mathbf{r}\, d\mathbf{r}' \tag{31}$$

is the photon propagator in the crystal in the momentum representation. **k** is the momentum of the photon.

We neglect the effect of the crystal medium on the photon propagator, and therefore assume the space as homogeneous. This assumption leads to the following simplified form of the photon propagator.

$$D_{\lambda\rho}(w, \mathbf{k}, \mathbf{k}') = D_{\lambda\rho}(w, \mathbf{k}, \mathbf{k}-\mathbf{k}')$$
$$= D_{\lambda\rho}(w, \mathbf{k})(2\pi)^3\,\delta(\mathbf{k}-\mathbf{k}'). \tag{32}$$

Moreover, we take the photon propagator in the gauge wherein only its space components are non-zero, i.e., we take

$$D_{ij}(w, \mathbf{k}) = -\frac{4\pi}{w^2-k^2}\left(\delta_{ij}-\frac{k_i k_j}{w^2}\right) \tag{33}$$

where $i, j = 1, 2, 3$; and δ_{ij} is the Kronecker delta symbol.

Putting (32), (33) and (29) in (28) and using the fact that (28) is gague-invariant we get the following expression for the emission probability per unit frequency, per unit time, per unit solid angle

$$\frac{d^3 W_{s's}}{dw\,dt\,d\hat{k}} = -\frac{e^2}{\pi^3}\,\mathrm{Im}\int\int_0^\infty \frac{k^2}{w^2-k^2}\left(\delta_{ij}-\frac{k_i k_j}{w^2}\right)$$
$$\times\, j^i_{ss'}(t, \mathbf{k})\, j^{j*}_{s's}(t', \mathbf{k})\exp(-iw\tau)\,d\tau\,dk. \tag{34}$$

We now separate out the t-dependence of the transition currents by using the relation

$$j^i(t, \mathbf{k}) = \exp(-iHt)\, j^i(\mathbf{k})\exp(iHt), \tag{35}$$

where H is defined by (9). Equation (35) gives

$$j^i_{ss'}(t, \mathbf{k}) = \exp\left[-i\left(E^{(-)}_s - E^{(-)}_{s'}\right)t\right] j^i_{ss'}(\mathbf{k}), \tag{36}$$

where $E^{(-)}_s = E_{s_t} + E^{(-)}_{s_l}$.

When equation (36) and an analogous expression of $j^{j*}_{s's}(t, \mathbf{k})$ are used in (34), τ- integration may be carried out, thereby providing

$$\frac{d^3 W_{s's}}{dw\,dt\,d\hat{k}} = -\frac{e^2}{\pi^2}\,\mathrm{Im}\int\frac{k^2}{w^2-k^2}\,[\mathbf{j}_{ss'}(\mathbf{k})\cdot\mathbf{j}_{s's}(\mathbf{k})$$
$$-\frac{1}{w^2}(\mathbf{k}\cdot\mathbf{j}_{ss'}(\mathbf{k}))(\mathbf{k}\cdot\mathbf{j}_{s's}(\mathbf{k}))]\,dk\,\delta(w-w_{s's}) \tag{37}$$

where $w_{s's} = E^{(-)}_{s'} - E^{(-)}_s$.

Using dipole approximation,

$$\exp(i\mathbf{k}\cdot\mathbf{r}) \approx 1, \tag{38}$$

in the expressions of the transition currents, we obtain

$$\mathbf{j}_{ss'}(\mathbf{k}) = \mathbf{j}_{ss'}(0) = -iw_{s's}\mathbf{r}_{ss'}. \tag{39}$$

The dipole approximation (38) is justified when the de Broglie wavelength associated with the emitted photon is significantly longer than the de Broglie wavelength of the positron. Under the dipole approximation, (37) reduces, in the positron rest frame, to

$$\frac{d^3 W_{s's}}{dw\, dt\, d\hat{k}} = \frac{e^2 w^3}{2\pi} \left[|\mathbf{r}_{s's}|^2 - (\hat{k}\cdot\mathbf{r}_{ss'})(\hat{k}\cdot\mathbf{r}^*_{s's}) \right] \delta(w - w_{s's}), \tag{40}$$

where $\mathbf{r}_{s's} = \mathbf{x}_0\, x_{s's} + \mathbf{y}_0\, y_{s's} + \mathbf{z}_0\, z_{s's}$. (41)

In (41), $\mathbf{x}_0$, $\mathbf{y}_0$, $\mathbf{z}_0$ are unit vectors along the $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ axes respectively.

Since the longitudinal motion is assumed as a free motion (cf. equation (7)) we have

$$x_{s's} = x_{\mu'\nu',\,\mu\nu} = \int \chi^*_{\mu'\nu'}\, x\, \chi_{\mu\nu}\, dx\, dy, \tag{42a}$$

$$y_{s's} = y_{\mu'\nu',\,\mu\nu} = \int \chi^*_{\mu'\nu'}\, y\, \chi_{\mu\nu}\, dx\, dy, \tag{42b}$$

$$z_{s's} = \frac{i}{w_{s's}} \frac{p_z}{m} \delta_{\mu'\mu}\, \delta_{\nu'\nu}\ (w_{s's} z_{s's} = 0). \tag{42c}$$

4. Results and discussion

Under the approximation (16) for $\chi_{\mu'\nu'}$, $x_{s's}$ and $y_{s's}$ become

$$x_{s's} \approx x_{\mu'\mu}\, \delta_{\nu'\nu}, \tag{43}$$

$$y_{s's} \approx \delta_{\mu'\mu}\, y_{\nu'\nu},$$

where $x_{\mu'\mu} = \int f^*_{\mu'}\, x f_\mu\, dx,$ (44)

$$y_{\nu'\nu} = \int g^*_{\nu'}\, y g_\nu\, dy.$$

According to (43), dipole emission of photons is possible when, at one time, only one of the two quantum numbers μ and ν is changed, i.e., when either $\mu' \neq \mu$, $\nu' = \nu$ or $\mu' = \mu$, $\nu' \neq \nu$, but not $\mu' \neq \mu$, $\nu' \neq \nu$.

Taking the case $\mu' \neq \mu$, $\nu' = \nu$, (40) is simplified (in the positron rest frame) to

$$\frac{d^3 W_{\mu'\mu}}{dw\ dt\ d\hat{k}} = \frac{e^2 w^3}{2\pi} \mid x_{\mu'\mu} \mid^2 [1 - \sin^2 \theta_k \cos^2 \phi_k]\ \delta\,(w - w_{\mu'\mu}), \tag{45}$$

where θ_k, ϕ_k are the polar angles of the photon momentum **k** (in the positron rest frame).

Expression (45) is valid only for the special case where transverse motion of the positron is independent along the x- and y- directions. In the case of well-collimated beam in the x-direction—the case when $f_\mu(x)$ has the solution (24)—(45) appears formally the same as for the planar channeling radiation emission probability (see, for example, equation (1) of Pantell and Alguard 1979). However, there is a difference between (45) and its counterpart in the planar channeling case. The frequency $w_{\mu'\mu} = (E_{\mu'} - E_\mu)$ is a difference of two frequencies which are uncertain (cf. 25-27) so that even if all line broadening mechanisms (including natural line broadening mechanism) are absent, $w_{\mu'\mu}$ will have a mean square deviation $\triangle w_{\mu'\mu}$. That is, there will be (in the axial channeling case) an extra line broadening mechanism, as compared to the case of planar channeling. The origin of this extra line broadening mechanism lies in the transverse motion (cf. (20)).

The frequency of the emitted radiation given by formula (45), in the laboratory frame, is:

$$\wedge_\theta = w^{(r)}_{\mu'\mu}/[\gamma\,(1 - v \cos \theta)], \tag{46}$$

where θ is the angle of the emitted photon in the laboratory frame (measured with respect to the incident positron direction). $w^{(r)}_{\mu'\mu}$ is, in the positron rest frame, the frequency separation between the positron transverse (x-direction only) states μ and μ'. According to (25) and (26)

$$w^{(r)}_{\mu'\mu} = (2\gamma\bar{\xi}/m)^{1/2} \qquad (\mu' = \mu \pm 1), \tag{47}$$

Therefore, for $\theta = 0$

$$\wedge_0 = 2\gamma^{3/2}\,(2\bar{\xi}/m)^{1/2} \tag{48}$$

For 56 MeV positrons, using (22a) and (22b) we find that $\wedge_0$ has the following values:

$$[\xi(0)]^{1/2} \approx (\wedge_0/9{\cdot}14) < [\xi\,(\pm d/2(2)^{1/2}]^{1/2}, \tag{49}$$

where $\xi(y)$ is in the units of eV—Å^{-2}, while $\wedge_0$ is in keV. Using table 1 (for silicon crystal) we obtain

$$10{\cdot}05\text{ - keV} \approx \wedge_0 < 98{\cdot}96\text{ - keV}. \tag{50}$$

Equation (50) implies that the axial channeling radiation of 56 MeV positrons, in the case of silicon $\langle 110 \rangle$ rows, will have energy from 10·05 - keV to (less than)

98·86 keV. [No channeling emission will be at 98·86 keV.] The specific value of the photon energy depends upon the value of the critical channeling angle ψ^c_{yz} in the (yz) plane. For 56 MeV positrons the critical channeling angle ψ^c_{yz} required for emission of the axial radiation (being discussed here) is $\approx 0{\cdot}7$ mrad. Thus positrons having $\psi^c_{yz} > 0{\cdot}7$ mrad will not emit axial channeling (but bremsstrahlung) radiation.

We now consider the conditions under which Alguard *et al* (1979) measured the $\langle 110 \rangle$ axial spectrum of 56 MeV positrons. Alguard *et al* (1979) used a positron beam whose divergence was 3 mrad $\times$ 9 mrad. The value 0·7 mrad of ψ^c_{yz} is significantly less than this beam divergence. Consequently, in the experiment of Alguard *et al* only, at most, 15% beam would contribute to the axial channeling radiation, and (due to large beam divergence) the contribution of the beam would nearly be equal to all photon energies lying in the range 10·05 keV to (less than) 98·86 keV. It is, therefore, clear that the axial channeling radiation is incapable of causing the enhancement (nearly 37%) observed by Alguard *et al* (1979). Moreover, the axial channeling radiation fails to predict even qualitative aspects of the observation of Alguard *et al.* In particular, the two peaks—one near 45 keV, the other near 100 keV—cannot be understood in terms of the axial channeling radiation.

5. Conclusions

In this paper we have studied emission of photons from axially channeled positrons. It has been found that (see § 3) the $\langle 110 \rangle$ rows of an f.c.c. (diamond) crystal, under the special case of well beam collimation in the x-direction, become a source of axial channeling radiation whose characteristics are different from the (ordinary) bremsstrahlung. In fact the axial channeling radiation has all the characteristics of the planar channeling radiation (see after (45)) with the difference that the former type of radiation has poor monochromaticity (large line width). This means that axial channels cannot be used as a source of monochromatic radiation. However, the radiation emitted by positrons while moving in these channels is useful in investigating crystal structure and properties of the motion of the radiator (positron).

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Appendix

We consider the unit (transverse) cell $-\frac{d}{2} \leqslant x \leqslant \frac{d}{2}, -\frac{d}{2\sqrt{2}} \leqslant y \leqslant \frac{d}{2\sqrt{2}}$. In this cell the crystal potential $U(x, y)$ may be approximated by the contributions of the six neighbouring rows $R_1, R_2, \ldots R_6$ (see figure 1). That is to say,

$$U(x, y) \approx \sum_{i=1}^{6} V_i(x, y) - U(0, 0) \tag{A-1}$$

With this potential the function $\xi(y)$ defined by equation (19) becomes

$$\xi(y) = A\,[F_{14}(y) + F_{23}(y) + F_{56}(y)], \tag{A-2}$$

where $$A = 2\sqrt{2}Z\, e^2/d, \tag{A-3}$$

and $$F_{ij}(y) = \frac{1}{A}\left.\frac{\partial^2 V_i}{\partial x^2}\right|_{x=0} = \frac{1}{A}\left.\frac{\partial^2 V_j}{\partial x^2}\right|_{x=0}. \tag{A-4}$$

In equation (A-4), i, j refer to rows, and take values such that $(i, j)=(1, 4), (2, 3), (5, 6)$. The rows i and j lie at equal distances from the y-axis but on opposite sides. This is why $F_{ij}(y)$ has two equivalent forms, (A-4).

The rows (2, 3) and (5, 6) lie at equal distances from the x-axis but on opposite sides (see figure 1). Therefore

$$F_{56}(y) = F_{23}(-y). \tag{A-5}$$

Making Lindhard's continuum field approximation for the V_i's (cf. equation (2·15) and 2·19) of Gemmell 1974), we obtain

$$F_{14}(y) = \frac{1}{(1+cy^2)(1+ey^2)}\left[\frac{a(1+by^2)}{(1+dy^2)(1+ey^2)} - \frac{fy^2}{(1+cy^2)}\right], \tag{1A-6}$$

and $$F_{23}(y) = \frac{1}{(1-ky+ly^2)(1-py+qy^2)}\left[\frac{g(1-hy+jy^2)}{(1-my+ny^2)(1-py+qy^2)} - \frac{r(1-sy+ty^2)}{(1-ky+ly^2)}\right]. \tag{A-7}$$

Here for the silicon crystal, the coefficient $a, b, c, \ldots$ etc. are:

$a = 0{\cdot}0188$	$g = 0{\cdot}0021$	$n = 0{\cdot}2423$
$b = 0{\cdot}2431$	$h = 0{\cdot}9179$	$p = 0{\cdot}9016$
$c = 0{\cdot}2418$	$j = 0{\cdot}2401$	$q = 0{\cdot}2358$
$d = 0{\cdot}2424$	$k = 0{\cdot}9266$	$r = 0{\cdot}0057$
$e = 0{\cdot}2359$	$l = 0{\cdot}2417$	$s = 1{\cdot}0420$
$f = 0{\cdot}0015$	$m = 0{\cdot}9262$	$t = 0{\cdot}2719$

Using (A-6) and (A-7) with these coefficient values, some $F_{ij}(y)$ and $\xi(y)$ have been calculated and are tabulated in the table 1.

Table 1. Values of F_{14}, F_{23} and $\xi(y)$ for the silicon crystal. F_{14} and F_{23} are in Å^{-2}; $\xi(y)$ is in eV$-$Å^{-2}.

y	$F_{14}(y)$	$F_{23}(y)$	$F_{23}(-y)$	$\xi(y)$
0·00	0·0188	−0·0036	−0·0036	1·22
0·76	0·0126	0·0000	−0·0014	1·18
1·80	0·0023	+0·9703	−0·0000	101·85
1·92	0·0017	+1·1143	−0·0000	117·00

We now give the range or positive F_{ij}'s.

$$F_{14}(y) > 0 \quad \text{for} \quad -3{\cdot}43 < y < 3{\cdot}43 \qquad \text{(A-8a)}$$
$$F_{23}(y) > 0 \quad \text{for} \quad +0{\cdot}76 < y < 3{\cdot}08 \qquad \text{(A-8b)}$$
$$F_{56}(y) > 0 \quad \text{for} \quad -3{\cdot}08 < y < -0{\cdot}76 \qquad \text{(A-8c)}$$

It is clear from equation (A-8) that $F_{14}(y)$ is positive in the whole unit cell which is being considered here, and that $(F_{23}+F_{56})$ is negative for $-0{\cdot}76 < y < 0{\cdot}76$. But according to table 1, minimum value of $(F_{23}+F_{56})$ in $-0{\cdot}0072$ whose magnitude is less than $F_{14}(0{\cdot}76)$. So that (see equation A-2) $\xi(y)$ would be positive for all y (belonging to the unit cell being considered here).

Similar calculations may be carried out for other f.c.c. (diamond) crystals (C, Ge etc.) also. And, in particular, it may be seen that $\xi(y)$ would be positive for all y belonging to a unit (transverse) cell.

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