Pramāna, Vol. 15, No. 3, September 1980, pp. 231–244. © Printed in India.

Effect of destabilizing fields on hydrodynamic instabilities in nematics

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MS received 14 May 1980; revised 1 August 1980

Abstract. The effect of destabilizing fields on the roll instability (RI) threshold for shear flow and on the homogeneous instability (HI) threshold for plane Poiseuille flow of nematic HBAB ($\mu_3 > 0$) is studied on the basis of the continuum theory of nematics for flow cells of infinite lateral width. It turns out that the critical shear rate and wave vector at RI threshold decrease with increasing destabilizing field but do not approach zero at the Freedericksz transition. However calculations show that beyond the Freedericksz threshold HI may be favourable over a range of destabilizing field with shear in the stabilizing role. For plane Poiseuille flow a similar analysis points to the existence of a HI threshold in the presence of destabilizing field beyond the Freedericksz threshold again with shear acting as a stabilizing field. These results are compared with theoretical results obtained previously for MBBA.

Keywords. Roll instability; nematics; MBBA; HBAB; homogeneous instability; shear flow; plane Poiseuille flow.

1. Introduction

Homogeneous (HI) and roll instabilities (RI) have been the subject of theoretical and experimental study. Pieranski and Guyon (1973) observed and measured the HI threshold for shear flow in MBBA and gave a simple theoretical analysis based on the continuum theory. Subsequently they found (Pieranski and Guyon 1974a) that in the presence of large stabilizing fields RI is more favourable than HI and gave a simple theoretical picture of the RI involving *hydrodynamic focussing*. Leslie developed rigorous solutions for HI and independently Manneville and Dobois-Violette (1976a) also did the same but extended their study to RI. They pointed out that HI cannot occur in a nematic with $\mu_3 > 0$ (μ_3 is an Ericksen-Leslie coefficient) and also studied effects of stabilizing fields on HI and RI. More recently the effects of destabilizing fields on HI were studied (Kini 1978) on the basis of the approach taken by Leslie (1976). Approximate solutions obtained by Dubois-Violette and Manneville (1978) for RI and HI in cylindrical Couette flow re-emphasized the possibility of observing RI in nematics with $\mu_3 > 0$. However effects of destabilizing fields on RI in such nematics have not been studied.

Pieranski and Guyon (1974b, 1975) reported theoretical and experimental studies on the plane Poiseuille flow of MBBA. They indicated theoretically the existence of the Twist and Splay-modes of which they observed only the former. Janossy *et al* (1976) gave a simple analysis of the HI and experimentally established the occurrence of net secondary flow with the Twist mode. Manneville and Dubois-Violette

(1976b) gave rigorous solutions for HI and established that the Twist mode is always more favourable in the presence of stabilizing fields. From these studies it became clear that HI cannot be excited in nematics with $\mu_3 > 0$ for the field-free case or in the presence of stabilizing fields. Subsequent theoretical study on MBBA indicated (Kini 1978) that the Splay mode can become more favourable in the presence of destabilizing fields. However effects of destabilizing fields on the HI of nematics with $\mu_3 > 0$ have not been studied.

In the present communication the differential equations for RI in shear flow are solved exactly by power series. Calculations are presented for nematic HBAB ($\mu_3 > 0$) including effects of destabilizing magnetic fields. For destabilizing fields higher than the Freedericksz threshold HI is shown to occur. The HI in plane Poiseuille flow is studied by the Fourier series technique developed by Manneville and Dubois-Violette (1976b). In the case of HBAB the HI is shown to occur in the presence of destabilizing fields beyond the Freedericksz threshold. Results for HBAB are compared with those for MBBA.

2. Differential equations for shear flow

Consider an incompressible nematic sheared between two infinite plane parallel plates $z = \pm 1$, the plate $z = +1$ moving along the $+y$ direction with respect to the plate $z = -1$ with a constant velocity V. (Both z and x are assumed to be scaled by a the semisample thickness.) The director is initially aligned along x. Solutions are sought for the director and velocity fields in the form:

$$n_x = 1; \qquad n_y = \phi\,(x, z, t); \qquad n_z = \theta\,(x, z, t);$$

$$v_x = v_1\,(x, z, t); \qquad v_y = v^0\,(z) + v_2\,(x, z, t); \qquad v_z = v_3\,(x, z, t);$$

where ϕ, θ, v_1, v_2, v_3 are first order perturbations imposed on the steady state director and velocity fields. A magnetic field

$$H_x = H_1; \qquad H_y = H_2; \qquad H_z = H_3$$

is also applied such that *at a time only one component exists and the other two are zero. This will be understood to be valid throughout the discussion whenever the effect of* **H** *are considered.* Effects of high frequency electric fields are not considered. From the continuum theory of nematics one can obtain the following differential equations:

$$2K_{33}\theta_{,xx} + 2K_{11}\theta_{,zz} + a\,(\lambda_1 + \lambda_2)\,v_{1,z} + a\,(\lambda_2 - \lambda_1)\,v_{3,x}$$

$$+ Sa^2\,(\lambda_1 + \lambda_2)\,\phi + 2\lambda_1\;a^2\,\dot{\phi} + 2\,(\Delta\chi)\,a^2\,(H_3^2 - H_1^2)\theta = 0, \qquad (1)$$

$$2K_{33}\phi_{,xx} + 2K_{22}\phi_{,zz} + a\,(\lambda_2 - \lambda_1)\,v_{2,x} + Sa^2\,(\lambda_2 - \lambda_1)\,\theta$$

$$+ 2\,(\Delta\chi)\,(H_2^2 - H_1^2)\,a^2\,\phi + 2\,\lambda_1 a^2\,\dot{\theta} = 0, \qquad (2)$$

$$v_{2,xx}(\mu_5+\mu_4-\mu_2)+aS(\mu_5-\mu_2)\theta_{,x}+\mu_4 v_{2,zz}$$
$$-2\rho Sa^2 v_3 = 2\rho a^2 \dot{v}_2 - 2a\mu_2\dot{\phi}_x, \tag{3}$$

$$2v_{1,xx}(\mu_1+\mu_4+\mu_5+\mu_6)+v_{1,zz}(\mu_3+\mu_4+\mu_6)$$
$$+v_{3,xz}(\mu_6+\mu_4-\mu_3)+aS(\mu_3+\mu_6)\phi_{,z}$$
$$=2ap_{,x}+2\rho a^2\dot{v}_1-2a\mu_3\dot{\theta}_z, \tag{4}$$

$$v_{1,xz}(\mu_2+\mu_4+\mu_5)+v_{3,xx}(\mu_5+\mu_4-\mu_2)$$
$$+aS(\mu_2+\mu_5)\phi_{,x}+2\mu_4 v_{3,zz}=2ap_{,z}+2a^2\rho\dot{v}_3-2\mu_2 a\dot{\theta}_x, \tag{5}$$

where K_{ii} are elastic constants, μ_k the viscosity coefficients, $\lambda_1=\mu_2-\mu_3$, $\lambda_2=\mu_5-\mu_6$, p is the pressure, $\Delta\chi$ the diamagnetic anisotropy, a comma denotes differentiation with z and a dot denotes $\partial/\partial t$. S is the constant steady shear rate imposed on the liquid with $v^0=S(z+1)a$. It should be kept in mind that the discussion of the problem is unaffected if the plates $z=\pm 1$ move in opposite directions with velocities $\pm V/2$. Equations (1)–(5) are supplemented by the equation of incompressibility

$$v_{1,x}+v_{3,z}=0. \tag{6}$$

Equations (1)–(6) are sought to be solved with boundary conditions

$$\phi(x,\pm 1,t)=\theta(x,\pm 1,t)=v_1(x,\pm 1,t)$$
$$=v_2(x,\pm 1,t)=v_3(x,\pm 1,t)=0. \tag{6$'$}$$

Equations (1)–(6) support solutions of the form $f(x,z)\,e^{wt}$. To seek solutions at threshold w is equated to zero by adopting the principle of exchange of instabilities and hence all time derivatives in (1)–(5) are ignored. The differential equations for HI can be recovered from (1)–(5) by putting $v_2=0=v_3$ and taking only z dependence for all quantities.

$$2K_{11}\theta_{,zz}+a(\lambda_1+\lambda_2)v_{1,z}+Sa^2(\lambda_1+\lambda_2)\phi$$
$$+2a^2(\Delta\chi)(H_3^2-H_1^2)\theta=0, \tag{7}$$

$$2K_{22}\phi_{,zz}+Sa^2(\lambda_2-\lambda_1)\theta+2(\Delta\chi)(H_2^2-H_1^2)a^2\phi=0, \tag{8}$$

$$v_{1,zz}(\mu_3+\mu_4+\mu_6)+aS(\mu_3+\mu_6)\phi_{,z}=0. \tag{9}$$

Equation (5) describes the z dependence of p. Since detailed physical descriptions of hydrodynamic torques have been given in many of the papers quoted (see for example Manneville and Dubois-Violette 1976a) this will not be discussed here.

From the torque equations (7) and (8) by following Pieranski and Guyon (1974a) one can get a simple formula for the HI threshold including effects of destabilizing fields:

$$S_H^2 = [2(\Delta\chi)a^2(H_3^2 - H_1^2) - K_{11}\pi^2/2]\,[2(\Delta\chi)a^2(H_2^2 - H_1^2) - K_{22}\pi^2/2]/[a^4(\lambda_2^2 - \lambda_1^2)]. \tag{10}$$

One can conclude from (10) that (A) if $\mu_3 < 0$, S_H is defined for any field H_1, for $H_2 \leqslant H_{F_2}$ or for $H_3 \leqslant H_{F_3}$ where H_{F_2} and H_{F_3} are respectively the Twist and Splay Freedericksz thresholds. (B) if $\mu_3 > 0$, S_H is undefined for $\boldsymbol{H} = 0$ or for H_1; however S_H is defined for $H_2 \geqslant H_{F_2}$ or for $H_3 \geqslant H_{F_3}$. *Thus HI appears to be favourable in a material like HBAB beyond the Freedericksz thresholds H_{F_2} or H_{F_3}.*

The treatment of Manneville and Dubois-Violette (1976a) is more rigorous. After ascertaining that (1) to (6) support two decoupled modes they consider the mode (say mode 1) for which ϕ, θ, v_2 and v_3 are symmetric since mode 2 for which ϕ, θ, v_2 and v_3 are antisymmetric involves a larger elastic energy and is not more favourable. In addition to providing exact numerical calculation of the RI threshold they present an approximate formula which can be rewritten including effects of destabilizing fields in the form

$$S_R^2 a^4 = f_y f_z \beta_2 / [\mu_2 \eta_3 (q_x^2 + q_z^2)\mu_3 - \mu(\mu_2 q_x^2 - \mu_3 q_z^2)/f], \tag{11}$$

where $$f_y = K_{22}q_z^2 + K_{33}q_x^2 + (\Delta\chi)a^2(H_1^2 - H_2^2),$$

$$f_z = K_{11}q_z^2 + K_{33}q_x^2 + (\Delta\chi)a^2(H_1^2 - H_3^2).$$

and the other quantities are defined as in the reference. These authors have found that in the case of MBBA equation (11) describes the RI threshold fairly accurately.

3. Calculations

3.1 *Roll instability*

In the appendix an exact solution of equations (1)-(6) by power series is described. Model calculations have been presented for HBAB with the parameters given in table 1 (For relevant references to the literature see Kini 1980). At the RI threshold there exist both a critical shear rate S_c and the corresponding critical wave vector q_{xc}. For MBBA parameters one can recover the results obtained by Manneville and Dubois-Violette (1976a).

Figure 1 shows the neutral stability curve for the field free case. Equation (13) gives slightly smaller values for S_c and q_{xc} and is not meaningful for $q_x \lesssim 1{\cdot}7$. For HBAB, $S_c = 2{\cdot}16$ sec^{-1} and $q_{xc} = 2{\cdot}31$ in the absence of fields for a sample thickness of 200μm. Figure 2 illustrates the increase of S_c and q_{xc} with a stabilizing magnetic field H_1. Since H_1 can influence both the Splay and Twist angles [equations (1) and (2)] it is meaningful to use the quantity $(h_1 h_2)^{1/2}$ (as has been done by Manneville and

Table 1. Material constants of MBBA and HBAB used in the present calculations.

	K_{11}	K_{22}	K_{33}	$\Delta\chi$	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
	$\times 10^{-7}$ dynes			$\times 10^{-7}$ cgs			poise			
MBBA	5·8	3·0	7·0	1·14	0·065	−0·775	−0·012	0·832	0·463	−0·324
HBAB	8·44	4·78	10·87	0·745	0·06	−0·327	0·0034	0·2746	0·2218	−0·1018

Semisample thickness $a=100$ μm. $\rho=1{\cdot}088$ g cm^{-3} for both fluids.

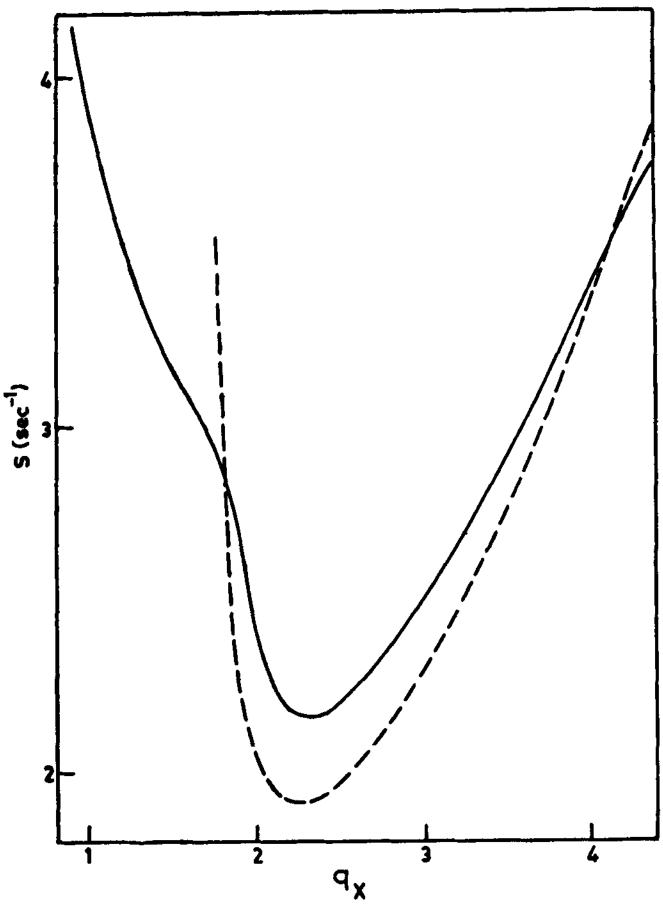

Figure 1. Neutral stability curves for RI in shear flow for HBAB. The dashed curve has been calculated from equation (11).

Dubois-Violette 1976a) to measure the effect of H_1 where $h_1=(\Delta\chi)H_1^2a^2/K_{11}$, $h_2=(\Delta\chi)H_1^2a^2/K_{22}$. The values of S_c and q_{xc} calculated from (11) are found to be generally less than those calculated by the exact method. At sufficiently high fields the value of S_c (approximate) exceeds S_c(exact) but q_{xc}(approximate) always remains less than the exact value. Thus for HBAB also equation (11) is found to give a sufficiently accurate value of the RI threshold at moderate stabilizing fields.

The effect of destabilizing fields is more interesting. Since the fields H_2 and H_3 occur in dimensionless combinations such as $(\Delta\chi)H^2a^2/K_{ii}$ the quantities $h_2=(\Delta\chi)H_2^2a^2/K_{22}$ and $h_1=(\Delta\chi)H_3^2a^2/K_{11}$ have been used to describe the strengths of the

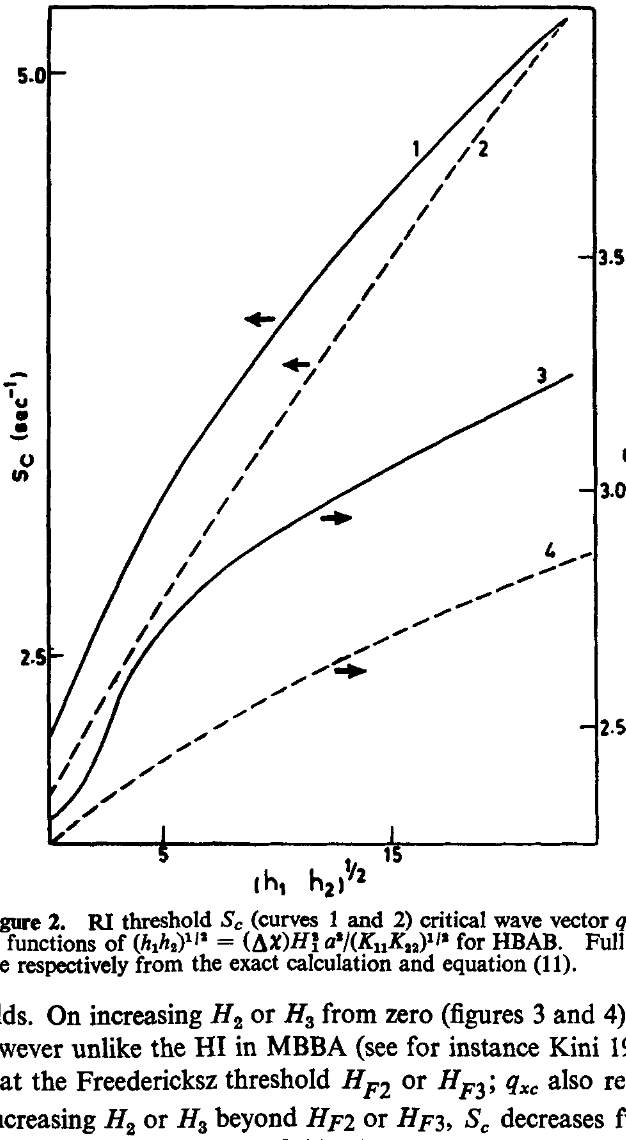

Figure 2. RI threshold S_c (curves 1 and 2) critical wave vector q_{xc} (curves 3 and 4) as functions of $(h_1h_2)^{1/2} = (\Delta\chi)H_1^2 a^2/(K_{11}K_{22})^{1/2}$ for HBAB. Full and dashed curves are respectively from the exact calculation and equation (11).

respective fields. On increasing H_2 or H_3 from zero (figures 3 and 4) both S_c and q_{xc} decrease. However unlike the HI in MBBA (see for instance Kini 1978) S_c does not become zero at the Freedericksz threshold H_{F2} or H_{F3}; q_{xc} also remains non-zero. On further increasing H_2 or H_3 beyond H_{F2} or H_{F3}, S_c decreases further becoming zero in a small range of destabilizing field. The reason for q_{xc} or S_c not becoming zero at the Freedericksz threshold is clearly because the Freedericksz threshold does not become a hydrostatic limit for the type of disturbances which cause the RI. There is no way in which the destabilizing torques which come into play in RI can become zero at the Freedericksz threshold. But once the destabilizing field has crossed the Freedericksz threshold the type of solutions that were sought for RI will cease to be meaningful since a deformation can exist even in the absence of imposed-shear. Thus the dashed parts of curves in figures 3 and 4 which describe decrease of S_c and q_{xc} beyond the Freedericksz threshold have no significance. Equation (11) amply describes the field H_2 or H_3 at which S_c becomes zero. Putting $S_R = 0$ in (11) one gets for a field H_3

$$(\Delta\chi)\, H_3^2 a^2/K_{11} = K_{33}\, q_x^2/K_{11} + q_z^2 = h_{10}, \tag{12}$$

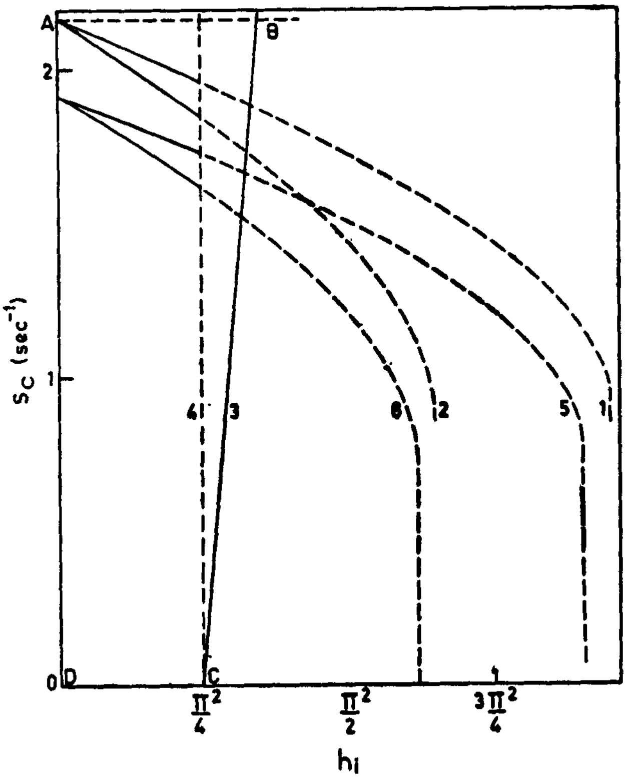

Figure 3. Threshold shear rates for RI and HI as functions of destabilizing field for HBAB. Variation of RI threshold S_c with $h_2=(\Delta\chi)H_2^2 a^2/K_{22}$: curve (1) by exact calculation; curve (5) by equation (11). Variation of S_c with $h_1=(\Delta\chi)H_1^2 a^2/K_{11}$: curve (2) by exact calculation; curve (6) by equation (11). Variation of HI threshold S_H with h_1 (or h_2) is given by curve 3. The vertical dashed line 4 corresponds to the Freedericksz threshold h_1 (or h_2)$=\pi^2/4$. The dashed parts of 1, 2, 5 and 6 are of only academic interest.

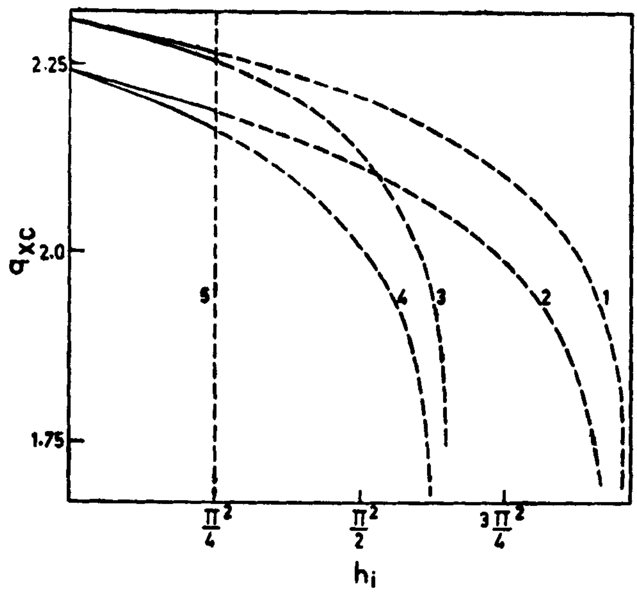

Figure 4. Critical wave vector q_{xc} at RI threshold as a function of destabilizing field for HBAB. Variation of q_{xc} with $h_2=(\Delta\chi)H_2^2 a^2/K_{22}$; curve (1) by exact calculation; curve (2) by equation (11). Variation of q_{xc} with $h_1=(\Delta\chi)H_1^2 a^2/K_{11}$; curve (3) by exact calculation; curve (4) by equation (11). The dashed line 5 corresponds to the Freedericksz threshold h_1 (or h_2)$=\pi^2/4$. Dashed parts of 1, 2, 3 and 4 are of academic interest.

and for a field H_2

$$(\Delta\chi)\, H_2^2 a^2/K_{22} = K_{33}\, q_x^2/K_{22} + q_z^2 = h_{20}. \tag{13}$$

From figures 3 and 4 one finds that for H_3, $q_x = 1{\cdot}78$ and for H_2, $q_x = 1{\cdot}7$. Putting $q_z = \pi/2$ one finds from (12) and (13) that $h_{10} = 6{\cdot}55$ and $h_{20} = 9{\cdot}0$ in good agreement with figures 3 and 4. One can also see from figures 3 and 4 that the decrease of S_c and q_{xc} with H_2 or H_3 is described fairly accurately by (11). Finally one must point out that the dashed parts of curves in figures 3 and 4 were obtained mainly because the steady state solutions $\mathbf{n} = (1,0,0)$ and $\mathbf{v} = (0, v, 0)$ are assumed to hold good initially regardless of the magnetic field or imposed shear rate and the perturbations are assumed to act on *this* steady state solution.

3.2 *Homogeneous instability*

One can now investigate the possibility of exciting an instability when a destabilizing field has a value greater than the Freedericksz threshold. Suppose one has applied a shear rate $S_I < S_c$ the RI threshold. Then RI will not occur and the shear rate will try to stabilize the initial orientation $\mathbf{n} = (1, 0, 0)$ against HI in the case of a material like HBAB. It is now clear from (10) that *one can expect HI to set in if a destabilizing field greater than the Freedericksz threshold is applied.* To solve equations (7)–(9), following Leslie (1976) equation (9) is integrated so that

$$v_{1,z}\,(\mu_3 + \mu_4 + \mu_6) + aS\,(\mu_3 + \mu_6)\,\phi = b, \tag{14}$$

where b is the constant transverse stress in the xz plane. For a field H_2 one can write

$$\theta_{,zz} + m_1\,(\phi + \delta) = 0, \tag{15}$$

$$\phi_{,zz} + m_2\,\theta + h_2\,\phi = 0, \tag{16}$$

where $m_1 = Sa^2\,(\lambda_1 + \lambda_2)\,\mu_4/[2K_{11}\,(\mu_3 + \mu_4 + \mu_6)]$;
$m_2 = Sa^2(\lambda_2 - \lambda_1)/(2K_{22})$, and $\delta = b/Sa\mu_4$. Here for HBAB ($\mu_3 > 0$) one can define $E = (-m_1 m_2)^{1/2}$ as the Ericksen number. Of the two possibilities that arise the case $h_2^2 < 4E^2$ does not yield any solution for the HI threshold. But for $h_2^2 > 4E^2$ one gets from 14, 15 and 16 the compatibility condition

$$(\mu_3 + \mu_4 + \mu_6)\,(k_1^2 - k_2^2) + (\mu_3 + \mu_6)\,[(k_2^2 \tan k_1)/k_1 - (k_1^2 \tan k_2)/k_2] = 0, \tag{17}$$

where $k_{1,2}^2 = [h_2 \mp (h_2^2 - 4\,E^2)^{1/2}]$.

For a field H_3 one can write

$$\theta_{,zz} + m_1\,(\phi + \delta) + h_1\,\theta = 0, \tag{18}$$

$$\phi_{,zz} + m_2\,\theta = 0. \tag{19}$$

Again for the case $h_1^2 > 4E^2$, with $k_{1,2}^2 = [h_1 \mp (h_1^2 - 4E^2)^{1/2}]/2$ one gets formally the same compatibility condition as equation (17). Thus for both the fields H_2 and H_3 a plot of S_H the HI threshold as a function of h_2 or h_1 will result in the same curve 3 in figure 3. Though the h_1 and h_2 values are the same it should be clear that the H_3 or H_2 field applied will be different. S_H increases with increasing h_1 or h_2. But curve 3 ceases to be significant once $S_H \geqslant S_c$ the RI threshold. Because once a shear $S = S_c$ is applied a RI will be excited even in the absence of any field. Thus for a material like HBAB, in the presence of destabilizing fields the variation of critical shear rate for RI or HI gets confined to the portion ABCD of figure 3.

4. Homogeneous instability in plane Poiseuille flow

In this case the plates $z = \pm 1$ are fixed and the nematic initially oriented with $\mathbf{n} = (1, 0, 0)$ flows along $+y$ under the action of a constant pressure gradient $p_{,y}$ with a velocity $\mathbf{v} = (0, v^0, 0)$ with $v^0 = p_{,y}\, a^2\, (z^2 - 1)/\mu_4$. With a homogeneous perturbation the director and velocity fields become

$$n_x = 1, \qquad n_y = \phi(z), \qquad n_z = \theta(z)$$

$$v_x = v_1(z) \qquad v_y = v^0(z), \qquad v_z = 0.$$

The differential equations governing the disturbances are (see for instance Kini 1978)

$$\phi_{,zz} + m_2\, z\theta + (\Delta\chi)a^2\, (H_2^2 - H_1^2)\, \phi/K_{22} = 0, \tag{20}$$

$$\theta_{,zz} + m_1\, (\delta + \phi z) + (\Delta\chi)\, a^2\, (H_3^2 - H_1^2)\theta/K_{11} = 0 \tag{21}$$

$$(\mu_3 + \mu_4 + \mu_6)\, v_{1,z}/(2p_{,y}\, a^2) = \delta - (\mu_3 + \mu_6)\, z\phi/\mu_4, \tag{22}$$

with $m_1 = (\lambda_1 + \lambda_2)\, p_{,y}\, a^3/[K_{11}\, (\mu_3 + \mu_4 + \mu_6)]$,
$m_2 = (\lambda_2 - \lambda_1)\, p_{,y}\, a^3/[\mu_4\, K_{22}]$, $\delta = b/ap_{,y}$ where b is the constant transverse stress in the zx plane. As has been pointed out by earlier authors there are two decoupled modes. For the Twist mode ϕ and v_1 are even but θ is odd in z, whereas for the Splay mode the reverse is true. By using the Fourier series method developed by Manneville and Dubois-Violette (1976a) equations (20) to (22) are solved and the critical Ericksen number E_c studied as a function of field strength. Manneville and Dubois-Violette (1976a) have shown that for a material like HBAB ($\mu_3 > 0$) HI cannot occur in plane Poiseuille flow. These authors have however studied the case of MBBA ($\mu_3 < 0$) and have established that the Twist mode is always more favourable than the Splay mode in the presence of stabilizing fields. Later calculations (Kini 1978) have indicated that in the presence of a destabilizing field H_3 close to the Freedericksz transition the Splay mode can become more favourable. This can be seen clearly in figure 5 which is similar to the one presented in Kini (1978). For MBBA the Ericksen number E is defined as $(m_1 m_2)^{1/2}$. With a field H_2 the Splay mode is always less favourable than the Twist mode. The critical Ericksen number E_c for the Splay mode goes to zero only at $(\Delta\, \chi)\, H_2^2\, a^2/K_{22} = h_2 = \pi^2$ corresponding to twice the Freedericksz field (figure 5)

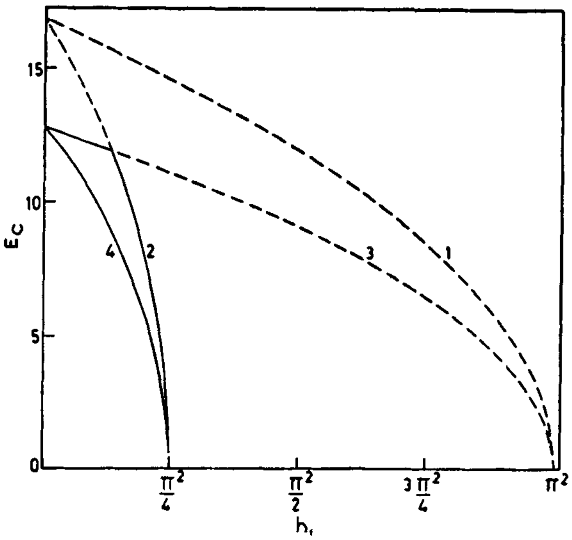

Figure 5. Variation of critical Ericksen number E_c of HI with destabilizing field in plane Poiseuille flow of MBBA. E_c as a function of $h_2=(\Delta\chi)H_2^2\, a^2/K_{22}$; curve (1) for splay mode; curve (4) for twist mode. E_c as a function of $h_1=(\Delta\chi)H_3^2\, a^2/K_{11}$; curve (2) for splay mode; curve (3) for twist mode. Dashed portions of the curves are of only theoretical interest.

which is a consequence of ϕ being antisymmetric. But since ϕ is symmetric for the Twist mode E_c decreases with increasing H_2 and goes to zero at the Freedericksz threshold $h_2 = \pi^2/4$. With a field H_3 the roles of θ and ϕ are reversed so that the Splay mode is more favourable in the vicinity of the Freedericksz threshold $(\Delta\,\chi)\, H_3^2\, a^2/K_{11} = h_1 = \pi^2/4$. For lower values of the field H_3 the Twist mode is more favourable.

In the case of HBAB the behaviour is different (figure 6). The Ericksen number is defined as $E = (-m_1 m_2)^{1/2}$. In this case an imposed shear rate will have a stabilizing influence on the initial orientation $\mathbf{n} = (1, 0, 0)$. A critical Ericksen number E_c corresponding to HI cannot exist either for the field-free case, or for a field H_1 or for destabilizing fields below the Freedericksz threshold.

However for H_2 or H_3 fields above the Freedericksz threshold one can excite HI (figure 6). With a field H_2, E_c for the Twist mode increases from its zero value at the Freedericksz threshold $h_2 = \pi^2/4$. However for the Splay mode, since ϕ is antisymmetric the HI can be excited only for $h_2 \geqslant \pi^2$, $h_2 = \pi^2$ corresponding to the hypothetical Freedericksz transition involving an antisymmetric twist angle. Thus the Splay mode would never be observed for a material like HBAB with a field H_2. This result is similar to that for MBBA (figure 5).

With a field H_3 the roles of Splay and Twist modes are reversed. Since θ for the Splay mode is symmetric, E_c for the Splay mode increases (figure 6) from a zero value at the Freedericksz transition $h_1 = \pi^2/4$ when H_3 is increased beyond the Freedericksz transition. Since for the Twist mode θ is antisymmetric the Twist mode can be excited only when h_1 is greater than π^2, where $h_1 = \pi^2$ is the hypothetical Freedericksz field which corresponds to an antisymmetric θ. Thus the Twist mode can never be observed in a material like HBAB in the presence of a field H_3. One can thus see by comparison with figure 5 that there is a marked difference in behaviour of the HI threshold for MBBA and HBAB.

Finally as in § 3 it should be pointed out that the curves for HI in figure 6 will cease

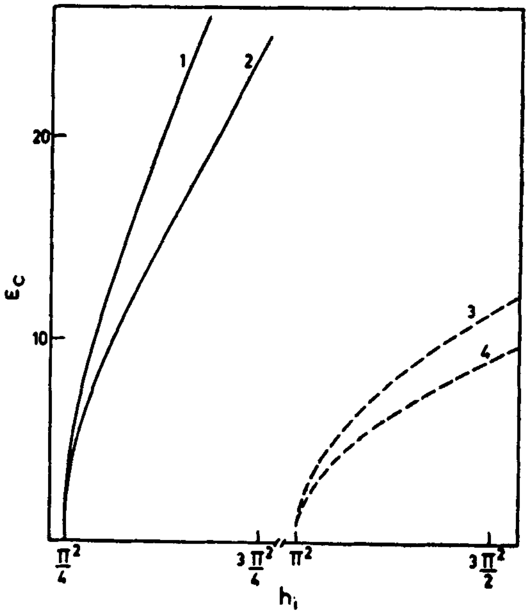

Figure 6. Variation of critical Ericksen number E_c of HI with destabilizing field in plane Poiseuille flow of HBAB. E_c as a function of $h_2=(\Delta\chi)H_2^2\, a^2/K_{22}$; curve (2) for Twist mode; curve (3) for Splay mode. E_c as a function of $h_1=(\Delta\chi)H_1^2\, a^2/K_{11}$; curve (1) for Splay mode; curve (4) for Twist mode. Curves 3 and 4 are only of academic interest.

to be meaningful when the imposed shear rate crosses the RI threshold for plane Poiseuille flow. However no quantitative estimate of the RI has been made in this paper.

Acknowledgement

The author thanks Professor S Chandrasekhar for his kind interest in this work and a referee for helpful comments in improving the manuscript.

Appendix

Equations (1)—(5) can be written with the help of (6) as

$$(D^2 + B_1)\,\phi + iB_2\, v_2 + B_3\,\theta = 0, \tag{A1}$$

$$(D^2 + B_4)\,\theta + iB_5\, D^2 v_3 + iB_6\, v_3 + B_7\,\phi = 0, \tag{A2}$$

$$(D^2 + B_8)\, v_2 + iB_9\,\theta + B_{10}\, v_3 = 0, \tag{A3}$$

$$i\,(D^4 + B_{11}\, D^2 + B_{12})\, v_3 + B_{13}\, D^2\phi + B_{14}\,\phi = 0, \tag{A4}$$

$$iq_x\, v_1 + D\, v_3 = 0 \tag{A5}$$

by putting an x dependence of exp $(iq_x\, x)$ for all quantities.

D stands for d/dz;

$$B_1 = (\Delta\chi)(H_2^2 - H_1^2)a^2/K_{22} - K_{33}\, q_x^2/K_{22};$$

$$B_2 = q_x(\lambda_2 - \lambda_1)\, a/2K_{22};$$

$$B_3 = Sa^2(\lambda_2 - \lambda_1)/2K_{22};$$

$$B_4 = (\Delta\chi)(H_3^2 - H_1^2)/K_{11} - K_{33}\, q_x^2/K_{11};$$

$$B_5 = a(\lambda_1 + \lambda_2)/2q_x K_{11};$$

$$B_6 = aq_x(\lambda_2 - \lambda_1)/2\, K_{11};$$

$$B_7 = Sa^2(\lambda_1 + \lambda_2)/2\, K_{11};$$

$$B_8 = -q_x^2(\mu_5 + \mu_4 - \mu_2)/\mu_4;$$

$$B_9 = a\, Sq_x(\mu_5 - \mu_2)/\mu_4;$$

$$B_{10} \equiv -2\,\rho\, Sa^2/\mu_4;$$

$$B_{11} \equiv -q_x^2(2\mu_1 + 2\mu_4 + \mu_3 - \mu_2 + \mu_5 + \mu_6)/\eta_1;$$

$$B_{12} = q_x^4(\mu_5 + \mu_4 - \mu_2)/\eta_1;$$

$$B_{13} = aq_x\, S(\mu_3 + \mu_6)/\eta_1;$$

$$B_{14} = a\, Sq_x^3(\mu_2 + \mu_5)/\eta_1, \text{ with } \eta_1 = \mu_3 + \mu_4 + \mu_6.$$

On combining (A1) to (A5) one gets

$$[D^{10} + c_4 D^8 + c_3 D^6 + c_2 D^4 + c_1 D^2 + c_0](\phi, \theta, v_2, v_3) \equiv 0, \qquad \text{(A6)}$$

where

$$c_4 = B_1 + B_4 + B_8 + B_{11},$$

$$c_3 = B_{12} + B_8 B_{11} + (B_1 + B_4)(B_8 + B_{11}) + B_1 B_4 + B_3 B_5 B_{13} - B_3 B_7,$$

$$\begin{aligned} c_2 = {} & B_8 B_{12} + (B_{12} + B_8 B_{11})(B_1 + B_4) + B_1 B_4 (B_8 + B_{11}) \\ & + B_3 B_{13}(B_6 + B_5 B_8) + B_3 B_5 B_{14} - B_3 B_7 (B_8 + B_{11}) \\ & - B_2 B_7 B_9 + B_2 B_{10} B_{13} + B_2 B_5 B_9 B_{13}, \end{aligned}$$

$$\begin{aligned} c_1 = {} & B_8 B_{12}(B_1 + B_4) + B_1 B_4 (B_{12} + B_8 B_{11}) + B_3 B_6 B_8 B_{13} \\ & + B_3 B_{14}(B_6 + B_5 B_8) - B_3 B_7 (B_{12} + B_8 B_{11}) - B_2 B_7 B_9 B_{11} \\ & + B_2 B_{10}(B_{14} + B_4 B_{13}) + B_2 B_9 (B_6 B_{13} + B_5 B_{14}), \end{aligned}$$

$$\begin{aligned} c_0 = {} & B_1 B_4 B_8 B_{12} + B_3 B_6 B_8 B_{14} - B_3 B_7 B_8 B_{12} - B_2 B_7 B_9 B_{12} \\ & + B_2 B_4 B_{10} B_{14} + B_2 B_6 B_9 B_{14}. \end{aligned}$$

For the mode under study, since ϕ, θ, v_2 and v_3 are symmetric in z solutions are sought in the form $(\phi, \theta, v_2, v_3) = \sum_{r=0}^{\infty} [P_r, T_r, V_r^{(2)}, V_r^{(3)}]\, z^{2r}$. Using (A6) all coefficients of a given kind for $r > 4$ can be expressed as linear combinations of the coefficients $r = 0, 1, 2, 3, 4$. For instance,

$$T_r = \sum_{i=0}^{4} M_{ri}\, T_i,$$

where $M_{ri} = \delta_{ri}\ (r, i = 0 \text{ to } 4)$ and

$$M_{r+5,\,i}\,(2r+10)! = -\sum_{k=0}^{4} c_i\, M_{r+k,\,i}\,(2r+2k)!$$
$$r \equiv 0, 1, \ldots \tag{A7}$$

The boundary conditions for v_3 and v_1 can be written down using (A5) and (6)′ in the form

$$\sum_{i=0}^{4} \Big[\sum_{r=0}^{\infty} M_{ri}\Big] V_i^{(3)} = 0, \tag{A8}$$

$$\sum_{i=0}^{4} \Big[\sum_{r=0}^{\infty} (2r+2) M_{r+1,i}\Big] V_i^{(3)} = 0. \tag{A9}$$

By using (A4) one can express P_0—P_4 in terms of $V_0^{(3)} - V_4^{(3)}$ as $P_j = \sum_{i=0}^{4} X_{ji} V_i^{(3)}$ so that the boundary condition for ϕ becomes

$$\sum_{i=0}^{4} \Big[\sum_{r=0}^{\infty} \sum_{j=0}^{4} M_{rj}\, X_{ji}\Big] V_i^{(3)} = 0. \tag{A10}$$

With (A2) T_0—T_4 can be expressed in the form $T_j = \sum_{i=0}^{4} N_{ji} V_i^{(3)}$ and the boundary condition for θ becomes

$$\sum_{i=0}^{4} \Big[\sum_{r=0}^{\infty} \sum_{j=0}^{4} M_{rj}\, N_{ji}\Big] V_i^{(3)} = 0. \tag{A11}$$

Finally from (A1) $V_0^{(2)} - V_4^{(2)}$ can be expressed as $V_j^{(2)} = \sum_{i=0}^{4} L_{ji}\, V_i^{(3)}$ and the boundary condition on v_2 can be written as

$$\sum_{i=0}^{4} \Big[\sum_{r=0}^{\infty} \sum_{j=0}^{4} M_{rj}\, L_{ji}\Big] V_i^{(3)} \equiv 0. \tag{A12}$$

Equations (A8) to (A12) define the 5×5 determinant the vanishing of which constitutes the compatibility condition. For a given q_x and H the value of the shear rate satisfying this condition is determined. By varying q_x, for a given H the threshold will be given by that $q_x = q_{xc}$ for which the shear rate is minimum. Summation over r is terminated at a suitable integer which gives proper convergence.

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