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Coherent bremsstrahlung from relativistic channelled positrons

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Abstract. Poor beam collimation leads to overcompensation of suppression of coherent bremsstrahlung in the axial channelling case. This fact has been used to study the effect of beam divergence on the radiation emitted by relativistic (axially) channelled positrons. It has been found that due to beam divergence in the experiment of Alguard and co-workers in which radiation of 56 MeV positrons channelling along ⟨110⟩ rows of a silicon crystal is observed, coherent bremsstrahlung becomes an important contributing factor to the high frequency part of the observed spectrum.

Keywords. Coherent bremsstrahlung; channelling; beam divergence; positrons; axial spectrum.

1. Introduction

When relativistic positrons (or electrons) move along major axes or between major planes of a crystal they emit a radiation which is highly directional, highly polarised and considerably more intense (than ordinary bremsstrahlung). This radiation (channelling radiation) was first predicted by Vorobiev *et al* (1975), and later by Kumakhov (1976). Considerable studies have been made towards the understanding of the characteristics of the planar channelling radiation of relativistic positrons (see, for example, Pantell and Alguard 1979 and references cited therein.) In particular, the characteristics of the planar chanelling radiation (like peak energy) have been understood (Alguard *et al* 1979). But the ($\approx 37\%$) enhancement of the radiation of 56 MeV positrons observed under axial channelling conditions by Alguard *et al* (1979) near 100 keV is not theoretically understood. This prompted us to undertake this investigation.

From the viewpoint of the incident angle, the condition of axial channelling is somewhat similar to the condition of occurrence of intense coherent bremsstrahlung in the sense that intense coherent bremsstrahlung and axial channelling both require the positrons (or electrons) incident in a direction which differs from a major axis only by a small angle (see Palazzi 1968 for coherent bremsstrahlung). [One consequence of this fact is that intense coherent bremsstrahlung cannot be expected under planar channelling conditions, since in planar channelling case the incident direction differs significantly from a major axis.] Therefore, coherent bremsstrahlung would compete with the axial channelling radiation. The purpose of the present paper is to study the contribution of coherent bremsstrahlung under axial channelling conditions.

2. Coherent bremsstrahlung under channelling conditions

We consider a cubic crystal. Let ($\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$) be a triplet of basis vectors (major axes) of the reciprocal lattice. We choose the axis along which the positrons beam is incident as $\mathbf{b}_1$ axis. Suppose that a positron of the beam which makes an angle θ with the $\mathbf{b}_1$ axis and which has energy and momentum E and $\mathbf{p}$ respectively collides with an atom in the crystal and emits a bremsstrahlung photon of energy (momentum) $k(\mathbf{k})$, and then moves with energy E' and momentum $\mathbf{p}'$. Then the momentum transferred to the atomic nucleus is

$$\mathbf{q} = \mathbf{p} - \mathbf{p}' - \mathbf{k}. \tag{1}$$

The condition for coherent bremsstrahlung is (Palazzi 1968)

$$\mathbf{q} = \mathbf{g}, \tag{2}$$

where $$\mathbf{g} = h_1\mathbf{b}_1 + h_2\mathbf{b}_2 + h_3\mathbf{b}_3, \tag{3}$$

is a reciprocal lattice vector.

The longitudinal (parallel to $\mathbf{p}$) and transverse components of $\mathbf{q}$, q_l and q_t, are given by

$$q_e = g_1 + \theta(g_2 \cos \alpha + g_3 \sin \alpha),$$
$$q_t^2 = g_2^2 + g_3^2. \tag{4}$$

Here α is the angle between the planes ($\mathbf{b}_1\mathbf{p}_2$) and ($\mathbf{b}_1\mathbf{b}_2$), and $g_i = h_i\, b_i$ (see (3)).

Due to the condition (1), for $k \ll E$, $E \gg 1$, q_l and q_t satisfy the following kinematical inequalities (§ 2.2 of Timm 1969):

$$k/2E^2 < q_l \leqslant 2/E, \quad 0 \leqslant q_t \leqslant 4. \tag{5}$$

Here we have used normalised units ($\hbar = m = c = 1$, $\hbar$ = Planck's constant divided by 2π, m = positron rest mass, c = velocity of light).

An exact theory of coherent bremsstrahlung is lacking. However, there is a formula based upon kinematical considerations which enables one to approximately find the energy peaks of the bremsstrahlung spectrum. This formula (for $k \ll E$, $E \gg 1$) reads (Timm 1969)

$$k \simeq (2Eq_l - q_t^2)E \,/\, (1 + 2Eq_l). \tag{6}$$

In order to find the value of k from (6), we consider beam divergence and channelling effect. To incorporate beam divergence we use the Gaussian function

$$P(\theta) = (2\pi\sigma^2)^{-1/2} \exp\,(-\,\theta^2/2\sigma^2), \tag{7}$$

where σ (half width at half maximum (HWHM)) denotes the beam divergence of the incident positron beam. In general σ would depend upon α, the angle between the planes ($\mathbf{b}_1$ $\mathbf{b}_2$) and ($\mathbf{b}_1$ $\mathbf{p}$). And, therefore, the portion of the beam occupied by positrons moving θ to $\theta+d\theta$ radians off the $\mathbf{b}_1$-axis and α to $\alpha+d\alpha$ radiations off the plane ($\mathbf{b}_1$ $\mathbf{b}_2$) would be $P(\theta)\, d\theta\, d\alpha/2\pi$.

Let ψ_c (HWHM) denote the critical angle for axial channelling of positrons moving along the major axis b_1 of the crystal. Since all the positrons for which $\theta \leqslant \psi_c$ will not channel, the channelling fraction of the beam would be less than

$$P_{\text{chan}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{-\phi_c}^{\phi_c} P(\theta)\, d\theta\, d\alpha. \tag{8}$$

If σ is independent of α, P_{chan} would involve error function.

Due to the channelling effect, bremsstrahlung from the channelling portion of the beam which is somewhat less than P_{chan} will be suppressed (Walker *et al* 1970, 1975; Grishaev *et al* 1977). However, if the beam divergence is such that the portion $P_{\text{brem}} = 1 - P_{\text{chan}}$ of the beam is comparable to P_{chan}, suppression of the bremsstrahlung will be to some extent overcompensated, thereby making coherent bremsstrahlung, to compete with axial channelling radiation. Since for $\theta \leqslant \psi_c$, most of the positrons are channelled, from the viewpoint of the intensity of the coherent bremsstrahlung θ values larger than ψ_c will only be important.

For $\theta > \psi_c$, contribution of one reciprocal lattice vector to (6) would lead to a distribution of k values; k increasing with θ. But, for a fixed lattice vector in the distribution of k values the minimum k value (pertaining to $\theta = \psi_c$) would correspond to the highest intensity (Timm 1969). The distribution function (7) would also cause reduction in the intensity of higher k ($\theta > \psi_c$) values. Therefore, for a fixed reciprocal lattice vector formula (6) will provide only one peak, namely that which corresponds to $\theta = \psi_c$, the effect of higher θ values being to broaden the bremsstrahlung line.

The foregoing consideration was based upon one reciprocal lattice vector only. But many reciprocal lattice vectors contribute to the intensity (see expression 36 of Palazzi 1968) of the bremsstrahlung. However, from a practical viewpoint (see Walker *et al* 1975) only that peak is important for which g^2 is minimum. This is because the expression of intensity contains factors like

$$\exp(-Ag^2)\, g^2\, F(g^2), \tag{9}$$

(see, for example, equation (106) of Timm 1969). In equation (9), A is the mean-square thermal temperature displacement of the atoms, and $F(g^2)$ is the atomic form factor corresponding to the momentum transfer $\mathbf{g}$. The structure of the form factor $F(g^2)$ is such that (Timm 1969) $g^2\, F(g^2)$ decreases with increasing g^2. Consequently (9) is a rapidly decreasing function of g^2. So only the reciprocal lattice vector of minimum length contributes to the intensity significantly. In view of (4) this means

$$g_1 = 0,\; g_2^2 + g_3^2 = \text{minimum}. \tag{10}$$

Notice that those values of g_2, g_3 are taken in (10) for which the structure factor is nonvanishing (see after (11)).

Let us consider an example, namely the $\langle 110\rangle$ axis of a silicon crystal. In this case the above discussion leads to the following value of k (cf. (6)).

$$k \simeq 2E^2 \psi_c b_2 (\cos \alpha + \sqrt{2} \sin \alpha), \tag{11}$$

with $b_2=4\cdot 4\times 10^{-8}$. Here we have used $h_1=0$, $h_2=h_3=1$, since these values of h_1, h_2, h_3 correspond to minimum length of g ($g^2=2$) for the non-vanishing structure factor. The next higher value of g^2 for which the structure factor is non-vanishing is $g^2=10$ ($h_1=0$, $h_2=1$, $h_3=3$) whose contribution to the intensity of coherent bremsstrahlung, according to the argument preceding equation (10), would be significantly lower than that for $g^2=2$. Therefore the photon energy peak of the coherent bremsstrahlung in the case of $\langle 110\rangle$ axial channelling of relativistic positrons in a silicon crystal would occur at the k value given by (11).

3. Axial spectrum

We now consider the axial spectrum of Alguard *et al* (1979) measured in the case of channelling of 56-MeV positrons in the $\langle 110\rangle$ channels of silicon crystal. We have to first estimate the value of ψ_c and for this purpose we use the fact that a proton and a positron whose energies are such that the quantity

$$(\epsilon + 2M)\, \epsilon \,/\, (2\epsilon + 2M), \tag{12}$$

is the same for both (ϵ=energy, M=mass, $\hbar=m=c=1$) have nearly the same critical angle for axial channelling (cf. figure 99 of Gemmell 1974). Our problem of finding the value of ψ_c for positrons reduces to finding the value of ψ_c for protons. We use figure (6) of Varelas and Sizmann (1972). We standardize that figure by experimental values of ψ_c for protons of energies 2·0 MeV and 3·0 MeV. [The experimental values of ψ_c are given in table IV of Varelas and Sizmann 1972.] After an extrapolation of the results of figure (6) of Varelas and Sizmann (1972) we find that for channelling of 28·6 MeV protons (equivalent to value 56 MeV positrons by equation (12)) along the $\langle 110\rangle$ rows of a silicon crystal, the value of ψ_c is 1·7 mrad (HWHM). Corresponding to this value of ψ_c, we find that in the experiment of Alguard *et al* (1979) a fraction of the beam (between the limits 25·7% and 70·6%) will fall outside the channelling area, thereby causing significant coherent bremsstrahlung. [The above limiting values of beam portion which will not channel are obtained by taking in equation (8) $\sigma=1\cdot 5$ mrad (HWHM) and $\sigma=4\cdot 5$ mrad (HWHM) as taken by Alguard *et al* (1979), and using the fact that for σ independent of α, equation (8) is an error function. The exact value of P_{chan} requires evaluation of the integral

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \operatorname{erf}\left(\frac{\psi_c}{2\sigma(\alpha)}\right) d\alpha.$$

However, the limiting values suffice to lead us to the conclusion that a considerable part of the beam is available to coherent bremsstrahlung.]

For $E = 56$ MeV and $\psi_c = 1{\cdot}7$ mrad, equation (11) leads to

$$k = 93{\cdot}6\ (\cos\alpha + \sqrt{2}\sin\alpha)\ \text{keV}. \tag{13}$$

This implies that in the axial spectrum of Alguard *et al* (1979), there must be a coherent bremsstrahlung peak at 93·6 keV to $93{\cdot}6\sqrt{2}$ keV. For small α, the peak energy given by (13) agrees well with one of the experimentally observed peak (≈ 100 keV). This agreement along with the result of the preceding paragraph that more than 25·7% of the positron beam is available to coherent bremsstrahlung (against 37% enhancement of the observed radiation) lead us to the conclusion that coherent bremsstrahlung is an important contributing factor to the ($\approx 37\%$) enhancement observed near 100 keV by Alguard *et al* (1979) in the motion of 56 MeV positrons in the channels of the ⟨110⟩ rows of silicon crystal.

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