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Reduction of wave function associated with electromagnetic fields for imaginary mass system to standard helicity representation of Lorentz group

B S RAJPUT and K D PUROHIT*
Department of Physics, Garhwal University, Srinagar 246 174, Garhwal, UP
*Department of Physics, GGI College, Srinagar 246 174, Garhwal, UP

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Abstract. Reduced expansions for electromagnetic fields associated with spin-1 particles of imaginary mass (tachyons) have been derived in terms of standard helicity representations of inhomogeneous Lorentz group. The effects of wave equation and reality condition on these reduced expansions to satisfy Maxwell's field equations have been derived. The reduced expansions of charge and current source densities associated with these fields have also been derived and it has been shown that these fields cannot satisfy the Maxwell's equations in the absence of both the current and charge source densities.

Keywords. Reduced expansion; imaginary mass; Poincaré group; superluminal frames; standard helicity representation.

1. Introduction

There has been growing interest in recent years in the study of possible existence of faster than light particles. Due to negative results given by experiments so far, it has become necessary to develop an appropriate quantum field theory of these particles so as to study their quantum mechanical properties. Feinberg (1967) and Dhar and Sudarshan (1968) restricted their theories to spin-zero tachyons only while Bandukwala and Shay (1974), Lemke (1976) and others (Shay 1975 and Schroer 1971) developed the theory for tachyons of spin-$\frac{1}{2}$ and arbitrary spin. There are yet many controversies in the formulation of a consistent quantum theory for these particles. Keeping these in view, we derived in an earlier paper (Rajput and Purohit 1979a) the reduced expansion for the wave-function which transforms as field associated with spin-zero tachyons in purely group theoretical manner using the general technique given by Lomont and Moses (1967). Study of invariance of Maxwell's field equations under real superluminal transformations of Antippa (1975) and imaginary superluminal transformations of Recami and Mignani (1973) was also undertaken (Rajput and Purohit 1979b).

In the present paper we extend our theory to tachyons of spin-1. The reduced expansion for the wave-function associated with spin-1 tachyons in terms of the standard helicity representation of proper, orthochronous, inhomogeneous Lorentz group has been derived. Wave-equation for imaginary mass and Lorentz condition has been used on the reduced expansion to eliminate some of the modes in the reduction of wave-function. Conditions for reduced expansion of this wavefunction to satisfy Maxwell's equations have been discussed. Reduced expansions for charge and current source densities to prescribe the desired fields have been derived and it has been shown that in the absence of both charge and current source densities simultaneously reduction for electromagnetic fields is not possible. Finally, we have derived the reduced expansions for electric and magnetic fields associated with tachyons.

2. Reduction of wave-function

In order to find the reduced expansions for the wave-function which transforms as electromagnetic field for imaginary mass system, let us consider the following transformation relation between the bases in which it is represented by the function $f(\xi)$, where ξ collectively denotes $\mathbf{x}$ and t and that in which it is represented by a complex function $f(c, \epsilon, \mathbf{p}, \lambda)$ [c is the eigen value of mass operator, ϵ is the sign of energy ($\epsilon = \pm 1$), $\mathbf{p}$ has three components of momentum vector $p = |\mathbf{p}| \geqslant c$ and λ is the helicity of the system ($\lambda = 0, \pm 1$)

$$f(\xi) = \sum_{\epsilon=\pm 1} \sum_{\lambda=0,\pm 1} \int dc \int \frac{dp}{\omega(c,p)} \langle \xi | c, \epsilon, \mathbf{p}, \lambda \rangle f(c, \epsilon, \mathbf{p}, \lambda). \quad (1)$$

Transformation function $\langle \xi | c, \epsilon, \mathbf{p}, \lambda \rangle$ for imaginary mass system is given by

$$\langle \xi | c, \epsilon, \mathbf{p}, \lambda \rangle = [\exp\{i\,\omega \cdot \hat{\mathbf{J}}\} \exp\{i\,\beta \cdot \hat{\mathbf{Z}}\}\, U]^{\xi}\, g(\xi; c, \lambda), \quad (2)$$

where $\hat{\mathbf{J}}$ and $\hat{\mathbf{Z}}$ are the infinitesimal generators corresponding to rotation and pure superluminal Lorentz transformations respectively, superscript ξ denotes that the operators inside the bracket act on the function through ξ; U is a unitary matrix which diagonalise the matrix corresponding to the infinitesimal generator of rotation about the direction of motion and the functions $g(\xi; c, \lambda)$ are the linearly independent solutions of the following equations for tachyons moving along z-direction, (Lomont-Moses 1967 and Moses 1968a);

$$\hat{P}_i^{\xi}\, g(\xi; c, \lambda) = 0, \quad (i = 1, 2),$$

$$\hat{P}_3^{\xi}\, g(\xi; c, \lambda) = k\, g(\xi; c, \lambda), \; (k = |c|^{1/2}),$$

$$H^{\xi}\, g(\xi; c, \lambda) = 0. \quad (3)$$

The general solution of these equations can be written as

$$g(\xi; c, \lambda) = C(c, \lambda) \exp\{ikx_3\},$$

where $C(c, \lambda)$ has been taken as an integration constant. Substituting equation (4) in equation (2), we get

$$\langle \xi | c, \epsilon, \mathbf{p}, \lambda \rangle = C(c, \lambda)\, [\exp\{i\,\vec{\omega} \cdot \hat{\mathbf{J}}\} \exp\{i\,\nu\,\hat{Z}_3\}\, U]^{\xi} \exp\{ikx_3\}. \quad (5)$$

A comparison of equation (3) for imaginary mass and corresponding equation for real non-zero mass (Moses 1968a) shows an interchange in the role of energy and the component of momentum in the direction of motion which automatically incorporates the interchange in the role of time and space component in the direction of motion ($t \to x_3'$ and $x_3 \to t'$). Keeping in mind this interchange under superluminal Lorentz transformation, we can write equation (5) in the following form

$$\langle \xi \,|\, c, \epsilon, \mathbf{p}, \lambda \rangle = C(c, \lambda) \, [\exp\{i \vec{\omega} \cdot \hat{M}\} \exp\{i \nu N_3\} \, U] \\ \exp\{i(p_1 x_1 + p_2 x_2 + p_3 t - \omega x_3)\}. \tag{6}$$

In deriving equation (6) we have also used the relations

$$p = k \cosh \nu \tag{7a}$$

and

$$x_3' = x_3, \tag{7b}$$

$$t' = t \cos \omega - (\omega_1 x_2 - \omega_2 x_1) \frac{\sin \omega}{\omega}, \text{ (under rotation).}$$

Substituting equation (6) in equation (1) and using the vector notations we can write the wave-function as follows:

$$\vec{\psi}(\mathbf{x}, t) = \sum_{\epsilon = \pm 1} \sum_{\lambda = 0, \pm 1} \int dM(c, \lambda) \int \frac{dp}{\omega(c, p)} \, [f(c, \epsilon, \mathbf{p}, \lambda) \, \chi(c, \epsilon, \mathbf{p}, \lambda) \\ \times \exp\{i(\mathbf{p}.\mathbf{x} - \omega x_3)\}], \tag{8}$$

where $dM(c, \lambda) = c(c, \lambda) dc$ is a measure function, $\mathbf{p}.\mathbf{x} = p_1 x_1 + p_2 x_2 + p_3 t$ and $\chi(c, \epsilon, \mathbf{p}, \lambda)$ is defined as,

$$\chi(c, \epsilon, \mathbf{p}, \lambda) = [\exp\{i\omega . \hat{M}\} \exp\{i\nu \hat{N}_3\} U]. \tag{9}$$

Matrices M_i and N_i of equation (9) may be written as $M_i = S_i$ and $N_i = iS_i$, where S_i are spin matrices and U is the unitary matrix which diagonalizes M_3. Here we have taken the same matrices as used for non-zero mass system (Rajput 1969) but in deriving the expressions for $\chi(c, \epsilon, \mathbf{p}, \lambda)$ we have used relation (7a) keeping in view the change in the role of x_3 and t. Substituting the values of M_i, N_3 and U in equation (9), we get

$$\chi(c, \epsilon, \mathbf{p}, \lambda) = \frac{\lambda}{2^{1/2} k} \begin{bmatrix} \dfrac{p_1\{(p_1 p - \epsilon\omega p_2) + i\lambda(\epsilon\omega p_1 + p_2 p)\}}{p(p + p_3)} - (p + i\lambda\epsilon\omega) \\[2ex] \dfrac{p_2\{(p_1 p - \epsilon\omega p_2) + i\lambda(\epsilon\omega p_1 + p_2 p)\}}{p(p + p_3)} + (\epsilon\omega - i\lambda p) \\[2ex] \dfrac{\{(p_1 p - \epsilon\omega p_2) + i\lambda(\epsilon\omega p_1 + p_2 p)\}}{p} \end{bmatrix}$$

$$= \frac{\lambda}{2^{1/2} k} \sigma(c, \epsilon, \mathbf{p}, \lambda), \text{ (for } \lambda = \pm 1), \tag{10}$$

where $\sigma(c, \epsilon, \mathbf{p}, \lambda)$ is the matrix part in $\chi(c, \epsilon, \mathbf{p}, \lambda)$. The corresponding expression for $\lambda = 0$ can be written as

$$\chi(c, \epsilon, \mathbf{p}, 0) = \mathbf{p}/p. \tag{11}$$

Substituting equations (10) and (11) in equation (8), we get the following reduced expansion for the wave-function:

$$\begin{aligned}
\vec{\psi}(\mathbf{x}, t) &= \int dM(c, 0) \int \frac{d\mathbf{p}}{\omega(c, p)} \left[f(c, \mathbf{p}, 0) \frac{\mathbf{p}}{p} \exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} \right] \\
&- \int dN(c, 0) \int \frac{d\mathbf{p}}{\omega(c, p)} \left[h^*(c, \mathbf{p}, 0) \frac{\mathbf{p}}{p} \exp\{-i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} \right] \\
&+ \sum_{\lambda=\pm 1} \frac{\lambda}{2^{1/2}k} \int dM(c, \lambda) \int \frac{d\mathbf{p}}{\omega(c, p)} [f(c, \mathbf{p}, \lambda)\, \sigma(c, \mathbf{p}, \lambda) \\
&\exp\{-i(\mathbf{p}\mathbf{x} - \omega x_3)\}] - \sum_{\lambda=\pm 1} \frac{\lambda}{2^{1/2}k} \int dN(c, \lambda) \int \frac{d\mathbf{p}}{\omega(c, p)} [h^*(c, \mathbf{p}, \lambda) \\
&\times \sigma^*(c, \mathbf{p}, \lambda) \exp\{-i(\mathbf{p}\mathbf{x} - \omega x_3)\}],
\end{aligned} \tag{12}$$

where we have used the relations

$$\chi(c, +1, +\mathbf{p}, 0) = -\chi(c, -1, -\mathbf{p}, 0),$$

$$\zeta(c, +1, +\mathbf{p}, \lambda) = \exp\{2i\lambda\phi\}\, \chi^*(c, -1, -\mathbf{p}, \lambda),$$

$$\exp\{(2i\lambda\phi)\} = \left\{ \frac{(p_1 p - \epsilon\omega p_2) + i\lambda(\epsilon\omega p_1 + p_2 p)}{(p_1 p + \epsilon\omega p_2) + i\lambda(\epsilon\omega p_1 - p_2 p)} \right\}, \tag{13a}$$

and $\tan\phi = p_2/p_1$,

which give $h(c, +1, +\mathbf{p}, \lambda) = \exp\{-2i\lambda\phi\} f(c, -1, -\mathbf{p}, \lambda)$,

and
$$\zeta(c, \epsilon, \mathbf{p}, \lambda) = -\frac{\lambda}{2^{1/2}k}\, \sigma(c, \epsilon, \mathbf{p}, \lambda). \tag{13b}$$

Let us now impose the requirement that the components of $\vec{\psi}(\mathbf{x}, t)$ satisfy the wave-equation

$$[\Box' - k^2]\, \vec{\psi}(\mathbf{x}, t) = 0, \tag{14}$$

where $\Box' = \dfrac{\partial^2}{\partial x_3^2} - \dfrac{\partial^2}{\partial x_1^2} - \dfrac{\partial^2}{\partial x_2^2} - \dfrac{\partial^2}{\partial t^2}$, in the superluminal frame.

The reduced expansion (12) then becomes

$$\vec{\psi}(x) = C^0 \int \frac{d\mathbf{p}}{\omega(k, p)} \left[\frac{\mathbf{p}}{p} f(k, \mathbf{p}, 0) \exp \{ i(\mathbf{p}\cdot\mathbf{x} - \omega x_3) \} \right]$$

$$- D^0 \int \frac{d\mathbf{p}}{\omega(k, p)} \left[\frac{\mathbf{p}}{p} h^*(k, \mathbf{p}, 0) \exp \{ - i(\mathbf{p}\cdot\mathbf{x} - \omega x_3) \} \right]$$

$$+ \sum_{\lambda = \pm 1} \frac{\lambda}{2^{1/2} k} C \int \frac{d\mathbf{p}}{\omega(k, p)} \left[f(k, \mathbf{p}, \lambda)\, \sigma\, (k, \mathbf{p}, \lambda) \exp \{ i(\mathbf{p}\cdot\mathbf{x} - \omega x_3) \} \right]$$

$$- \sum_{\lambda = \pm 1} \frac{\lambda}{2^{1/2} k} D \int \frac{d\mathbf{p}}{\omega(k, p)} \left[h^*(k, \mathbf{p}, \lambda) \sigma^*(k, \mathbf{p}, \lambda) \exp \{ -i(\mathbf{p}\cdot\mathbf{x} - \omega x_3) \} \right], \tag{15}$$

where C^0, D^0, C and D are the positive constants which may be chosen such that the canonical formalism in terms of Hamiltonian density agrees with the particle interpretation. The Hamiltonian density which yields the wave-equation (14) can be written in the following form:

$$H(x) = \sum_{j=1}^{3} [\tilde{\partial}_0 \psi_j(x)\, \tilde{\partial}_0\, \psi_j^*(x) + \tilde{\nabla} \psi_j(x) \cdot \tilde{\nabla} \psi_j^*(x) - k^2\, \psi_j(x)\, \psi_j^*(x)], \tag{16}$$

where $\tilde{\partial}_0 = \partial / \partial x_3$,

and $$\tilde{\nabla} = i \frac{\partial}{\partial x_1} + j \frac{\partial}{\partial x_2} + k \frac{\partial}{\partial t}. \tag{17}$$

Substituting the reduced expansion (15) for different modes respectively in equation (16), the constants may be calculated to have the following values;

$$C^0 = D^0 = (2)^{-1/2}\, (2\pi)^{-3/2},$$

$$C = D = (2)^{-1/2}\, (2\pi)^{-3/2}\, k\, (\omega^2 + p^2)^{-1/2}. \tag{18}$$

Then equation (15) becomes

$$\vec{\psi}(x) = \vec{\psi}^L(x) + \vec{\psi}^T(x), \tag{19}$$

where the reduced expansions of longitudinal part $\vec{\psi}^L(x)$ and transverse part $\vec{\psi}^T(x)$ are given by

$$\vec{\psi}^L(x) = \frac{1}{4\pi^{3/2}} \int \frac{d\mathbf{p}}{\omega(k, p)} \frac{\mathbf{p}}{p} \left[f(k, \mathbf{p}, 0) \{ \exp i\, (\mathbf{p}\cdot\mathbf{x} - \omega\, x_3) \} \right.$$

$$\left. - h^*(k, \mathbf{p}, 0) \exp \{ -i\, (\mathbf{p}\cdot\mathbf{x} - \omega\, x_3) \} \right], \tag{19a}$$

and
$$\vec{\psi}^T(x) = \frac{1}{4\pi^{3/2}} \sum_{\lambda=\pm 1} \frac{\lambda}{2^{1/2}} \int \frac{d\mathbf{p}}{\omega(k, p)(\omega^2 + p^2)^{1/2}} [f(k, \mathbf{p}, \lambda)\, \sigma(k, \mathbf{p}, \lambda)$$
$$\times \exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} - h^*(k, \mathbf{p}, \lambda)\, \sigma^*(k, \mathbf{p}, \lambda)$$
$$\exp\{-i(\mathbf{p}\,\mathbf{x} - \omega x_3)\}], \tag{19b}$$

from which it is obvious that

$$\tilde{\nabla} \times \vec{\psi}^L(x) = 0, \tag{20a}$$

and

$$\tilde{\nabla} \cdot \vec{\psi}^T(x) = 0, \tag{20b}$$

where the operator $\tilde{\nabla}$ has been defined by equation (17) and the following properties of matrix $\chi(k, \mathbf{p}, \lambda)$ have been used;

$$\tilde{\nabla}\cdot\left[\frac{\mathbf{p}}{p}\exp\{i\,\mathbf{p}\,\mathbf{x} - \omega x_3)\}\right] = i\,p \exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\},$$
$$\tilde{\nabla}\cdot[\sigma(k, \mathbf{p}, \lambda)\exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}] = 0,$$
$$\tilde{\nabla}\times\left[\frac{\mathbf{p}}{p}\exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}\right] = 0,$$
$$\tilde{\nabla}\times[\sigma(k, \mathbf{p}\cdot\lambda)\exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}] = \lambda\, p\, \sigma(k, \mathbf{p}\cdot\lambda)$$
$$\exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}. \tag{21}$$

Equations (20a) and (20b) warrant the longitudinal nature of $\vec{\psi}^L(x)$ and the transverse nature of $\vec{\psi}^T(x)$ because these equations reveal that $\vec{\psi}^L(x)$ may be expressed as the gradient of a scalar potential and $\vec{\psi}^T(x)$ as the curl of a vector potential.

These reduced expansions of wave-function may be shown to satisfy the field equations

$$\tilde{\nabla} \cdot \vec{\psi}(x) = 4\pi\rho, \tag{22a}$$

and

$$\tilde{\nabla} \times \vec{\psi}(x) = -i\frac{\partial\, \vec{\psi}(x)}{\partial x_3} - 4\pi i\, \mathbf{J}, \tag{22b}$$

where the reduced expansions for the charge and current source densities to prescribe these fields are given as follows:

$$\rho = \frac{i}{16\pi^{5/2}} \int \frac{d\mathbf{p}}{\omega(k, p)}\, p \left[f(k, \mathbf{p}, 0)\exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3) + h^*(k, \mathbf{p}, 0)\right.$$
$$\left.\exp\{-i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}\right], \tag{23}$$

and
$$\mathbf{J}(x) = \frac{i}{16\pi^{5/2}} \int \frac{d\mathbf{p}}{\omega(k,p)} \frac{\mathbf{p}}{p} \Big[f(k, \mathbf{p}, 0) \exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} + h^*(k, \mathbf{p}, 0) \exp\{-i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} \Big]$$
$$+ \frac{i}{16\pi^{5/2}} \sum_{\lambda=\pm 1} \frac{1}{2^{1/2}} \int \frac{d\mathbf{p}}{\omega(k,p)(\omega^2 + p^2)^{1/2}} \Big[(p + \lambda\omega) f(k, \mathbf{p}, \lambda)\, \sigma(k, \mathbf{p}, \lambda) \exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} - (p - \lambda\omega) h^*(k, \mathbf{p}, \lambda)\, \sigma^*(k, \mathbf{p}, \lambda) \exp\{-i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} \Big]. \tag{24}$$

Field equations (22a) and (22b) appear similar to Maxwell's equations except that the operator $\tilde{\nabla}$ is given by equation (17) which changes the forms of expressions for divergence and curl in superluminal frame. The electric and magnetic fields **E** and **H** associated with the wavefunction $\vec{\psi}(x)$ may be defined as

$$\vec{\psi}(x) = \mathbf{E} - i\mathbf{H}. \tag{25}$$

Assuming both these fields to be real, equation (22a) reduces to

$$\tilde{\nabla} \cdot \mathbf{E}^L = 4\pi\, \rho,$$

and
$$\mathbf{H}^L = 0, \tag{26}$$

which shows that in the presence of electric charge source density the longitudinal electric field associated with tachyons is non-vanishing and the longitudinal magnetic field is vanishing, making magnetic field purely transverse.

In case charge source density is made vanishing, equation (23) gives $f(k, \mathbf{p}, 0) = h^*(k, \mathbf{p}, 0) = 0$ and then longitudinal component of electric field also vanishes making both the fields purely transverse. In this case the reduced expansion (24) for current source density becomes

$$\mathbf{J}(x) = \frac{i}{16\pi^{5/2}} \sum_{\lambda=\pm 1} \frac{1}{2^{1/2}} \int \frac{d\mathbf{p}}{\omega(k,p)(\omega^2 + p^2)^{1/2}} [(p + \lambda\omega) f(k, \mathbf{p}, \lambda)\, \sigma(k, \mathbf{p}, \lambda) \exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} - (p - \lambda\omega) h^*(k, \mathbf{p}, \lambda)\, \sigma^*(k, \mathbf{p}, \lambda) \exp\{-i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}] \tag{27}$$

If the current source density is also made zero in addition to vanishing charge density then we get, $p = \pm\lambda\omega$, which is impossible for a non-vanishing mass and therefore it is not possible to reduce the fields associated with imaginary mass in the absence of both charge and current source densities.

In the absence of charge source density, the reduced expansion for purely transverse electric and magnetic fields may be derived in the following form;

$$\mathbf{E} = \frac{1}{8\pi^{3/2}} \sum_{\lambda=\pm 1} \frac{\lambda}{2^{1/2}} \int \frac{d\mathbf{p}}{\omega(k,p)(\omega^2+p^2)^{1/2}} [g(k,\mathbf{p},\lambda)\,\sigma(k,\mathbf{p},\lambda)$$

$$\exp\{i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\} + g^*(k,\mathbf{p},\lambda)\,\sigma^*(k,\mathbf{p},\lambda)$$

$$\exp\{-i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}] \tag{28}$$

and

$$\mathbf{H} = \frac{i}{8\pi^{5/2}} \sum_{\lambda=\pm 1} \frac{\lambda}{2^{1/2}} \int \frac{d\mathbf{p}}{\omega(k,p)(\omega^2+p^2)^{1/2}} [g'(k,\mathbf{p},\lambda)\,\sigma(k,\mathbf{p},\lambda)$$

$$\exp\{i(-\mathbf{p}\,\mathbf{x} - \omega x_3)\}$$

$$-g'^*(k,\mathbf{p},\lambda)\,\sigma^*(k,\mathbf{p},\lambda)\exp\{-i(\mathbf{p}\cdot\mathbf{x} - \omega x_3)\}], \tag{29}$$

where

$$g(k,\mathbf{p},\lambda) = [f(k,\mathbf{p},\lambda) - h(k,\mathbf{p},\lambda)],$$

$$g'(k,\mathbf{p},\lambda) = [f(k,\mathbf{p},\lambda) + h(k,\mathbf{p},\lambda)].$$

The reduced expansion for electromagnetic field derived here couples to tachyons according to Maxwell's equations in a superluminal frame, where tachyons appear as bradyons. The superluminal electromagnetic field when viewed upon from a bradyonic frame certainly does not satisfy the usual Maxwell's equations. Other properties of these fields have already been discussed in detail in Rajput and Purohit (1979b).

3. Discussion

Reduction of wavefunction has been derived in terms of standard helicity representation of Poincaré group with superluminal Lorentz transformations which have been shown to interchange the role of time with the spatial coordinate along the motion. Such a change of roles has already been discussed by Lemke (1976) for developing quantum field theory of spin-$\frac{1}{2}$ tachyons. We have chosen here standard helicity representation for reduction of wave function not only because this method can be used for all masses (real non-zero, zero and imaginary) but also because for the tachyons the invariant integrals (a scalar for integral spin and a pseudoscalar for odd half-integral spin) depend on helicity and are unaffected by the little group transformations (Shay 1975). As such, for these particles (tachyons), helicity and not just charge or particle number is important. It is clear from equation (10) that in our case the transformation matrix is different from that derived by Moses (1968b) for zero-mass system and consequently the transformation relations for changing the negative energy particle to positive energy ones are also given in a different form (equation (13a)). Equation (17), for the operator $\tilde{\nabla}$ is justified for superluminal frame, since we believe that all the quantum mechanical properties (inner quantum numbers) of a tachyon in its own frame will be identical to corresponding quantum mechanical properties of a bradyon in a bradyonic frame. But if these quantities are observed from a subluminal frame of reference then they may not be identical for the same observer. We have shown by equations (22), (25), and (27) that Maxwell's equations in superluminal frame appear similar in form as Maxwell's equations in subluminal frame, if the form of $\tilde{\nabla}$ operator given in equation (17) is used. We therefore conclude that corresponding to a superluminal frame there is a superluminal electromagnetic field, reduced expansion of which is given by equations (28) and (29), which satisfy Maxwell's field equations (22a) and (22b). Finally, it may be emphasised that changing of roles between time and space coordinates does not mean to crossing the light barrier. We have already shown in an earlier paper (Rajput and Purohit 1979b) that not only the space and time coordinates get interchanged but the same interchange is incorporated between the energy and momentum, scalar and vector potential, and charge and current source densities also. These interchanges are the result of applying superluminal transformation on any of these physical quantities and are independent of the nature of the reference frame. It will be also shown in a forthcoming paper that these transformations also change the role of creation and annihilation operators.

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