Pramāṇa, Vol. 15, No 2, August 1980, pp. 145-151. © Printed in India.

Nuclear reaction rate in Debye-Hückel electrolytic plasma in stellar interiors

H L DUORAH and A E MD KHAIROZZAMAN*
Department of Physics, Gauhati University, Gauhati 781 014, India
*Permanent Address: Department of Physics, B. N. College, Dhubri, India

MS received 14 February 1980; revised 4 June 1980

Abstract. The transmission probability of charged particle reactions in a stellar plasma is examined by considering the effective potential between the interacting charges as having an exponential factor in the Coulomb term arising out of the spherical distribution of electrons with its uniform background of positive charges (Debye-Hückel electrolyte). The expression for the ratio of this transmission probability to that due to pure Coulomb field, called the enhancement factor, is then obtained. Thermonuclear reactions of astrophysical interest such as $^{12}C(\alpha, \gamma)^{16}O$ and $^{16}O(\alpha, \gamma)^{20}$ Ne are then considered. Our enhancement factor in the $^{12}C(\alpha, \gamma)^{16}O$ reaction rate is found to agree reasonably well with those calculated from the expression given by Alastuey and Jancovici.

Keywords. Thermonuclear reactions; enhancement factor; stellar interiors; nuclear reaction rate; Debye-Hückel electrolyte.

1. Introduction

Charged particle reaction rate in vacuum depends on the penetrability of the Coulomb barrier. The situation is different in the case of stellar interiors. The Coulomb barrier is modified by the presence of polarising cloud of electrons surrounding the positive ions. The potential seen by a reacting particle (positive ion) is generally found to be smaller than the Coulomb potential and quantum-mechanical tunnelling of the barrier becomes easier. As a result, the reaction rate is augmented in stellar interiors. Salpeter (1954) calculated the accelerating factor in nuclear reaction rates and found that it was not satisfactory for both low and high density regimes at the same time. Attempts were made later by a number of workers to improve upon his work. Recently Mitler (1977) considered a two-fluid approximation which was an improvement over the Salpeter model. Ito *et al* (1977), on the other hand, calculated the enhancement factor in thermonuclear reaction rate due to strong screening effect through explicit consideration of correlations between ions. Alastuey and Jancovici (1978) considered the short-range behaviour of the pair correlation function in a dense one-component plasma. The result was an expression to be solved numerically and was an improvement over earlier studies. We wish to investigate here the enhancement factor in charged particle reaction rate in a plasma, likened to a Debye-Hückel electrolyte.

Most of the atoms are found in ionised states in the stellar interiors. Electrons and ions in stellar interiors from a plasma. Electronic plasma is a collection of electrons with a uniform background of positive charges. This medium shows a collective response to charge fluctuation leading to plasma oscillation. Here the electrons surround a test charge Ze such that they screen out the Coulomb field at a distance of the order of the screening radius $r_D(=K_D^{-1})$. Such a screening was first studied by Debye and Hückel in their theory of electrolytes. The effect of polarisation of the charge clouds is the thinning of the barrier leading to the enhancement of the nuclear reaction rate in these situations.

When a plasma is in thermal equilibrium in an electrostatic potential ϕ then the spatial density of electrons is given by (Jackson 1978)

$$n(x) = n_0 \exp(-e\phi/kT), \tag{1}$$

where n_0 is the equilibrium number density of electrons. The potential can be obtained from the Poisson's equation for the spherical distribution of electrons around a test charge Ze given by

$$\nabla^2\phi - K_D^2\phi = -4\pi Ze\,\delta(x), \tag{2}$$

where k_D is the Debye wave number and $\delta(x)$ represents the delta function distribution of charges. The spherically symmetric solution of this equation is given by

$$\phi(r) = Ze \exp(-K_D.r)/r. \tag{3}$$

Then the effective interaction energy between nuclei of charges Z_1 and Z_2 is given by

$$Z_1Z_2e^2\frac{\exp(-K_Dr)}{r}. \tag{4}$$

The Coulomb repulsive force as seen by the interacting particles appears to be less due to the screening cloud.

In § 2, we calculate the transmission probability arising out of this screened potential and then obtain the ratio of this penetrability to that obtained by Cox and Giuli (1968) for a pure Coulomb field. Numerical calculations leading to the determination of the enhancement factor for two specific reactions of astrophysical interest are given in § 3.

2. Transmission probability

The low energy cross-section for exothermic charged particle reaction is given by (Cox and Giuli 1968)

$$\sigma = \text{const}\; E^{1/2}\, P_l(E) \text{ for } E \to 0. \tag{5}$$

The penetration factor for a charged particle with angular momentum l relative to the nucleus in question is given by (Blatt and Weisskopf 1952)

$$P_l \approx \frac{K_l(R)}{k}\exp\left[-2\int_R^{R_c} K_l(r)\,dr\right], \tag{6}$$

where $$K_l(r) \equiv \left(\frac{2m}{\hbar^2} U_l(r) - k^2\right)^{1/2}. \tag{7}$$

$U_l(r)$ is the potential barrier given by·

$$U_l(r) = V(r) + \frac{l(l+1)\hbar^2}{2mr^2}, \tag{8}$$

and $$V(r) = \frac{Z_1 Z_2\, e^2}{r} \exp(-K_D . r), \tag{9}$$

with $$K_D^{-1} = 6{\cdot}91\, (T/n_0)^{1/2}\ \text{cm}. \tag{10}$$

We define R as the interaction radius and R_c as the classical radius of closest approach i.e.,

$$R = r_0\, (A_1^{1/3} + A_2^{1/3})\ \text{fm}, \tag{11}$$

where r_0 is a constant expressed in fm and A_1 and A_2 are the mass numbers of the interacting charges and

$$R_c = \frac{Z_1 Z_2\, e^2}{E + Z_1 Z_2\, e^2\, K_D}. \tag{12}$$

For low energy interactions $l = 0$ the penetration probability is given by

$$P_0 = \left(\frac{B}{E} - 1\right)^{1/2} \exp(-\mathscr{T}), \tag{13}$$

where $$B = V(R) = Z_1 Z_2\, e^2/R, \tag{14}$$

$$E = \hbar^2\, k^2/2m, \tag{15}$$

and $$\mathscr{T} = 2 \int_R^{R_c} \left(\frac{2m\, Z_1 Z_2\, e^2}{r\hbar^2} \exp(-r/r_D) - k^2\right)^{1/2} dr. \tag{16}$$

Expanding the exponential term under the radical after taking only upto the second order term for r/r_D ($\ll 1$) we have

$$\mathscr{T} \approx 2\, k \left(1 + \frac{Z_1 Z_2\, e^2}{E\, r_D}\right)^{1/2} \int_R^{R_c} \left(\frac{R_c}{r} - 1\right)^{1/2} dr. \tag{17}$$

Substituting ξ^2 for r/R_c, we finally obtain after some algebra

$$\mathscr{F} \approx \frac{4\,k\,Z_1\,Z_2\,e^2}{E\left(1+\dfrac{Z_1\,Z_2\,e^2}{E\,r_D}\right)^{1/2}}$$

$$\left\{\frac{\pi}{4}-\frac{1}{2}\,(R/R_c)^{1/2}\left(1-\frac{R}{R_c}\right)^{1/2}-\frac{1}{2}\sin^{-1}\,(R/R_c)^{1/2}\right\}. \tag{18}$$

Since $R/R_c \ll 1$, the term within the curly bracket can be expanded to give

$$\mathscr{F} \approx \zeta\left(2\pi\eta - 8\eta\alpha + \frac{4}{3}\,\eta\alpha^3 + \frac{1}{5}\,\alpha^5\eta + \frac{1}{14}\,\eta\alpha^7 + \ldots\right), \tag{19}$$

where $$\zeta = \frac{1}{\left(1+\dfrac{Z_1\,Z_2\,e^2}{E\,r_D}\right)^{1/2}}, \tag{20}$$

$$\alpha = (R/R_c)^{1/2}, \tag{21}$$

and $$\eta = (m/2)^{1/2}\,\frac{Z_1\,Z_2\,e^2}{\hbar\,E^{1/2}}. \tag{22}$$

With this expression for $\mathscr{F}$, the penetration probability is approximately given by

$$P_0 \approx \left(\frac{B}{E}\right)^{1/2} \exp(-\zeta)\,\exp\left[\eta\,\alpha\left(8-\frac{4}{3}\,\alpha^2-\frac{1}{5}\,\alpha^4-\frac{1}{14}\,\alpha^6-\ldots\right)\right]$$

$$\exp\,(-2\,\pi\,\eta). \tag{23}$$

The approximate expression for the penetrability for a pure Coulomb field as obtained by Cox and Giuli (1968) is given by

$$P_0 \approx \left\{B^{1/2}\,\exp\left[\frac{8e}{\hbar}\left(\frac{m\,Z_1\,Z_2\,R}{2}\right)^{1/2}\right]\right\}\,E^{-1/2}\exp\,(-2\pi\,\eta). \tag{24}$$

Thus we define the ratio

$$\mathscr{R} = \frac{P_0}{P_0^{CG}} = \exp\,(-\zeta-\chi)\,\exp\left(\eta\,\alpha\left(8-\frac{4}{3}\,\alpha^2-\frac{1}{5}\,\alpha^4-\frac{1}{14}\,\alpha^6-\ldots\right)\right), \tag{25}$$

where $$\chi = \frac{8e}{\hbar}\,(\tfrac{1}{2}mZ_1Z_2R)^{1/2}. \tag{26}$$

3. Numerical calculations

For a thermonuclear reaction in stars, the effective thermal energy is given by (Fowler and Hoyle 1964)

$$E = 0{\cdot}122\,(Z_1^2 Z_2^2 A)^{1/3}\, T_9^{2/3}\ \text{MeV}, \tag{27}$$

where Z_1 and Z_2 are the atomic numbers of the interacting nuclei and A is the reduced mass number of the pair and the temperature is expressed in $10^9 K$ unit.

Considering charge neutrality of the plasma medium consisting mainly the species with charge Z_1 and mass number A_1, we can write the electron density as

$$n_0 = Z_1 n_1, \tag{28}$$

where n_1 is the number density of the ions constituting the plasma. But

$$n_1 = \rho X_1 / A_1 H, \tag{29}$$

where ρ is the density of the medium, X_1 is the fractional abundance by weight of the main constituent, $H^{-1} = N_A$, the Avogadro's number.

Thus, the expression for K_D^{-1} in equation (10) can be written as

$$K_D^{-1} = r_D = 6{\cdot}91 \left(16{\cdot}60531 \frac{A_1}{Z_1 X_1} \frac{T_9}{\rho_6}\right)^{1/2} \times 10^{-11}\ \text{cm}, \tag{30}$$

where the density is expressed in 10^6 g cm^{-3} units. We further write

$$Z_1 Z_2 e^2 = 14{\cdot}4 \times 10^{-14}\, Z_1 Z_2\ \text{MeV-cm}. \tag{31}$$

Using these expressions from equations (27) through (31) in equation (20), we get

$$\zeta = 1/[1 + 4{\cdot}192 \times 10^{-3} Z_1 Z_2 \rho_6^{1/2} (Z_1/A_1)^{1/2} X_1^{1/2} / (Z_1^2 Z_2^2 A)^{1/3}\, T_9^{7/6}]. \tag{32}$$

By knowing the various quantities like ζ, χ, η, and α, we can calculate the ratio $\mathscr{R}$ which is the enhancement factor in nuclear reaction rate in dense stellar plasma.

Next, we consider a few reactions of astrophysical interest. In a helium plasma, during the advanced evolutionary stages of a star, the $^{12}C(\alpha, \gamma)^{16}O$ reaction is important and so also the $^{16}O(\alpha, \gamma)^{20}Ne$ reaction. Numerical solutions for $\mathscr{R}$'s are obtained for these reactions at various temperature and density conditions. Some of the results are shown in table 1. The temperature and density range considered in table 1 are likely to prevail in the white dwarf stages.

The enhancement factor in the rate of nuclear reactions in dense stellar matter (pycnonuclear reactions) has recently been considered by Alastuey and Jancovici (1978). We have calculated their enhancement factor for $^{12}C(\alpha, \gamma)^{16}O$ reaction and compared them with our result as shown in figure 1.

Table 1. Enhancement factors for $^{12}C(\alpha, \gamma)^{16}O$ and $^{16}O(\alpha, \gamma)^{20}Ne$ reactions as a function of temperature (T_9) for density $\rho_6 = 1{\cdot}0$

T_9 \ Reaction	$^{12}C(\alpha, \gamma)^{16}O$	$^{16}O(\alpha, \gamma)^{20}Ne$
0·003	0·8236 $E+08$	0·2036 $E+11$
0·005	0·8865 $E+05$	0·3235 $E+07$
0·008	0·1118 $E+04$	0·1181 $E+05$
0·010	0·2333 $E+03$	0·1572 $E+04$
0·030	0·2703 $E+01$	0·4965 $E+01$
0·050	0·1076 $E+01$	0·1505 $E+01$
0·100	0·5365 $E+00$	0·6109 $E+00$
0·500	0·2492 $E+00$	0·2318 $E+00$

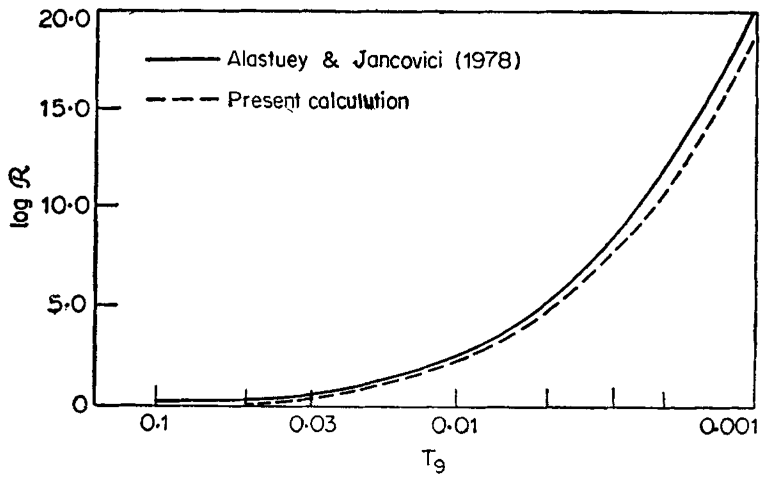

Figure 1. Logarithm of the enhancement factor as a function of temperature (T_9) for $^{12}C(\alpha, \gamma)^{16}O$ reaction at $\rho = 10^6$ g cm^{-3}.

4. Results, discussions and conclusions

When the density is low, the gas behaves as an ideal gas. At a very high temperature, electron degeneracy disappears. In the intermediate range of high density and low temperature, the density effect on nuclear reaction rate becomes much more important than the temperature effect. At this stage, the reaction is known as pycnonuclear reaction which only will be effective in the energy generation processes in stars characterised by low temperature and high density conditions.

The present calculation has been done from the simple conception of the Coulomb potential being screened off at a distance of Debye radius. Comparison of our result with that of Alastuey and Jancovici (1978) for the $^{12}C(\alpha, \gamma)^{16}O$ reaction (figure 1) clearly shows that it compares well with their result. An order of magnitude difference in the lower range of temperatures may be due to certain simplifying assumptions such as charge, atomic mass and abundance of the constituents of the plasma.

Duorah and Kushwaha (1963) showed that $^{16}O(\alpha, \gamma)^{20}Ne$ reaction rate is insignificant as compared to the $^{12}C(\alpha, \gamma)^{16}O$ reaction during the advanced red-giant stage of evolution of a star. But table 1 shows that at low temperature, the former rate is more enhanced than the latter. Thus, it is likely that the enhanced rates of helium burning rates may assume importance at some stage of stellar evolution like in pre-nova situations.

References

Alastuey A and Jancovici B 1978 *Astrophys. J.* **226** 1034

Blatt J M and Weisskopf V 1952 *Theoretical Nuclear Physics* (New York: John Wiley and Sons, 1952)

Cox J P, and Giuli T R 1968 *Principles of Stellar Structure* (New York: Gordon and Breach)

Duorah H L and Kushwaha R S 1963 *Astrophys. J.* **137** 566

Fowler W A and Hoyle F 1964 *Astrophys. J. Suppl. No.* **9** 121

Ito N, Totsuji H and Ichimaru S 1977 *Astrophys. J.* **218** 477

Jackson J D 1978 *Classical Electrodynamics* (New Delhi: Wiley Eastern Ltd.)

Mitler H 1977 *Astrophys. J.* **212** 513

Salpeter E E 1954 *Aust. J. Phys.* **7** 343