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Fusion reactor start-up by RF cavity mode heating of a gas

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Abstract. The application of high power microwaves to a gas-filled resonant cavity may be an ideal way to start up Tokamak, mirror or other thermonuclear reactors. RF coupling problems are avoided by input of microwave power prior to wave cut-off.

Keywords: Fusion; plasma heating; radio frequency power; cavity mode heating; plasma model.

1. Introduction

The application of radio frequency power (Jones 1978) has long been considered one of the best means of supplying auxiliary heating for fusion reactor start-up. Difficulties arise, however, in trying to achieve penetration of the RF waves into the core of a hot, dense pre-existing discharge (Bellan and Porkolab 1974).

2. Cavity mode heating of a gas

Resonant cavities have been employed since the earliest days of controlled thermonuclear research as a means of localizing substantial (vacuum) RF field energy for plasma heating (Kapitza 1965 and Jones *et al* 1977). The (time-averaged) energy stored in such a cavity E is related to the power dissipated in the cavity walls P_w and that input to the plasma P according to the relation:

$$P + P_w = \omega E / Q, \tag{1}$$

where ω is the RF cavity eigenfrequency and Q is the usual quality factor which can be expressed in the form (Jackson 1962):

$$Q = G \frac{\mu V}{\mu_w S \delta}, \tag{2}$$

where G is a parameter determined by the cavity geometry, μ and μ_w are the permeabilities of the dielectric fill and cavity walls respectively, S is the cavity surface area, V is the cavity (and plasma) volume, and δ is the RF skin depth:

$$\delta = c / (2\pi \mu_w \omega b)^{1/2}, \tag{3}$$

with c the speed of light and b the wall electrical conductivity. In radio frequency plasma circuitry used todate Q has typically been limited to values of $\sim$2000 (Motz and Watson 1967). The use of superconductors could increase this.

The energy deposition profile for such devices is dependent upon the particular cavity geometry and mode of excitation chosen. As a specific example we might choose a right circular cylindrical cavity oriented along the z axis having a length d and radius R. For the lowest order TM_{010} mode the fields will be of the form (Jackson 1962):

$$E_z = E_0 J_0 (2{\cdot}405\, \rho/R) \exp(-i\,\omega t), \tag{4}$$

$$B_\phi = -i(\mu\,\epsilon)^{1/2} E_0 J_1 (2{\cdot}405\, \rho/R) \exp(-i\,\omega t), \tag{5}$$

where J is the Bessel function, ϕ, ρ, and z are the usual cylindrical coordinates, and t is the time. The resonant frequency is just:

$$\omega_{010} = 2{\cdot}405\, c/(\mu\,\epsilon)^{1/2} R, \tag{6}$$

where ϵ is the conventional permittivity. Since substantial gas ionization could adversely perturb the deposition profile suggested by equations (4) and (5) we consider instead the addition of an (initially) unionized neutral gas to the cavity. This gas will subsequently be ionized at a rate given by (Drawin 1967):

$$\dot{n}_e \Big|_{\text{ionize}} = \frac{371\, n_e\, n_n \exp(-15{\cdot}6/T_e)}{(15{\cdot}6 + T_e/T_e)(T_e)^{1/2}} \left[\frac{T_e}{15{\cdot}6 + 20\,T_e} + \ln\left(\frac{19{\cdot}5 + 1{\cdot}25\,T_e}{15{\cdot}6}\right)\right], \tag{7}$$

in hydrogen gas, where n_e and n_n are the (time-varying) plasma and neutral gas densities respectively (in 10^9 cm^{-3}), and T_e is the electron plasma temperature (in eV).

The presence of dissipation in the cavity gives rise to a decline in stored RF energy of the form:

$$E(t) = E(t=0) \exp(-\omega t/Q), \tag{8}$$

where Q/ω is the time constant of the decay. Provided that Q/ω is not too small compared to the heating time a substantial fraction of the input RF energy will go into the plasma. The RF might be self excited by suitable beam-plasma instabilities if need be.

3. The plasma model

The power P that must be input to form the thermonuclear plasma is obtained from a simple zero-dimensional ('point model') computer code (Sprott and Strait 1976). The plasma density n_e is obtained as a function of time, by integration of,

$$\dot{n}_e = \dot{n}_e \Big|_{\text{ionize}} - \dot{n}_e \Big|_{\text{diffuse}} - \dot{n}_e \Big|_{\text{end loss}}, \tag{9}$$

with $n_n(t)$ the (time-dependent) neutral gas density given by:

$$n_n(t) = n_n(t = 0) - n_e(t), \tag{10}$$

for times short compared to the reactor particle confinement time (typically $\geqslant 1$ sec).

The cross-field diffusion loss is given by (Kovrizhnykh 1969):

$$\dot{n}_e \Big|_{\text{diffuse}} = \frac{\frac{1}{3} n_e^2}{B^2 R^2 T_e^{1/2}} + \frac{n_e n_n T_e}{B^2 R^2} 10^{-3}. \tag{11}$$

Modifications to the parameters in (11) permit the modelling of the usual pseudo-classical wave induced 'anomalous' plasma transport (Artsimovich 1972 and Jones 1980). (Wave induced reductions in the plasma resistivity may also reduce the crossfield diffusion if $D_{\perp B} \propto \eta_{\perp B}$).

End loss (if any) along open field lines is given by (Jones 1979):

$$\dot{n}_e \Big|_{\text{end loss}} = \frac{2 n_e A (T_e + T_i)^{1/2}}{V} \times 10^5, \tag{12}$$

where T_i is the ion plasma temperature (in eV) and A is the loss area (in cm^2).

The electron temperature at each time step is found by integration of:

$$\dot{T}_e = \frac{1}{n_e} \left(\frac{2}{3} \dot{U}_e - T_e \dot{n}_e \right), \tag{13}$$

where the electron power density is just:

$$\dot{U}_e = \dot{U}_e \Big|_{\text{DT}} + \dot{U}_e \Big|_{\text{RF}} - \dot{U}_e \Big|_{\text{ionize}} - \dot{U}_e \Big|_{\text{brem}} - \dot{U}_e \Big|_{\text{ions}} - \dot{U}_e \Big|_{\text{diffuse}} - \dot{U}_e \Big|_{\text{end loss}} - \dot{U}_e \Big|_{\text{impurity}}. \tag{14}$$

The RF power input is:

$$\dot{U}_e \Big|_{\text{RF}} = 2 \times 10^9 \frac{P}{V} \tag{15}$$

where P is expressed in watts. Fusion alpha heating enters through:

$$\dot{U}_e \Big|_{\text{DT}} = \tfrac{1}{4} n_e^2 \langle \sigma v \rangle_{\text{DT}} E_{\text{F}}, \tag{16}$$

where E_{F} is the fusion energy release (3·52 MeV for the DT alpha particle, the energetic neutron escapes the plasma and heats the blanket) and the rate coefficient is:

$$\langle \sigma v \rangle_{\text{DT}} = 2{\cdot}87 \times 10^{-16} - 8{\cdot}78 \times 10^{-20} T_i + 1{\cdot}02 \times 10^{-23} T_i^2 - 3{\cdot}74 \times 10^{-28} T_i^3 + 4{\cdot}93 \times 10^{-33} T_i^4. \tag{17}$$

We assume here that the alphas give up their energy to electrons (by classical or anomalous processes) on a time scale faster than the times of interest.

Impurity radiation loss has been modelled for an (equilibrium) iron contaminant (a pessimistic assumption):

$$\dot{U}_e \Big|_{\text{impurity}} = 2 \times 10^9 G n_e n_I, \tag{18}$$

where:

$$\begin{aligned} G = {} & 3{\cdot}3 \times 10^{-28}\, T^{1/2} \\ & + 1{\cdot}4 \times 10^{-27} \times 1{\cdot}21 \times \exp -0{\cdot}194(13{\cdot}12\,(|\log (T/13{\cdot}0)|)^{1{\cdot}43} + 1)^{0{\cdot}701} \\ & + 1{\cdot}9 \times 10^{-27} \times 2{\cdot}11 \times \exp -0{\cdot}744(17{\cdot}94\,(|\log (T/13{\cdot}0)|)^{2{\cdot}24} + 1)^{0{\cdot}447} \\ & \times (1 - \exp[-(T/6{\cdot}4)^{2{\cdot}3}]) \\ & + 5{\cdot}1 \times 10^{-27} \times 2{\cdot}75 \times \exp -1{\cdot}01(114{\cdot}6\,(|\log (T/6{\cdot}0)|)^{2{\cdot}95} + 1)^{0{\cdot}340} \\ & + 6{\cdot}6 \times 10^{-27} \times 3{\cdot}79 \times \exp -1{\cdot}33(23{\cdot}41\,(|\log (T/1{\cdot}4)|)^{2{\cdot}17} + 1)^{0{\cdot}460} \\ & + 4{\cdot}0 \times 10^{-26} \times 14{\cdot}50 \times \exp -2{\cdot}67(60{\cdot}69\,(|\log (T/0{\cdot}45)|)^{3{\cdot}56} + 1)^{0{\cdot}281}, \end{aligned} \tag{19}$$

and $T = T_e \times 10^{-3} \geqslant 0{\cdot}4$ with n_I the iron density in cm^{-3}.

Ionization losses are accounted for by:

$$\dot{U}_e \Big|_{\text{ionize}} = \dot{n}_e \left(E_I + e\, V_p + \frac{3}{2} T_e \right), \tag{20}$$

where E_I is the gas ionization energy, e is the electronic charge, and V_p is the plasma potential. For most applications we can take:

$$eV_p = T_e \ln\, [(m_i/m_e)^{1/2}], \tag{21}$$

where m_i/m_e is the ion-to-electron mass ratio (for hydrogen).

The Bremsstrahlung power loss is included by:

$$\dot{U}_e \Big|_{\text{brem}} = 10^{-4}\, n_e^2\, T_e^{1/2}, \tag{22}$$

and the electron-ion collisional power transfer gives:

$$\dot{U}_e \Big|_{\text{ions}} = \frac{2{\cdot}3\, n_e^2\, (T_e - T_i)}{T_e^{3/2}} \ln \left[\frac{5{\cdot}2 \times 10^{11}\, T_e^3}{n_e\, (40 + T_e)} \right]. \tag{23}$$

The cross-field diffusion loss is given by:

$$\dot{U}_e \Big|_{\text{diffuse}} = 2{\cdot}5\, T_e\, \dot{n}_e \Big|_{\text{diffuse}}, \tag{24}$$

and the end loss is just:

$$\dot{U}_e\Big|_{\text{end loss}} = 2\,T_e\,\dot{n}_e\Big|_{\text{end loss}}. \tag{25}$$

The ion temperature T_i is obtained, as a function of time by integration of:

$$\dot{T}_i = \frac{1}{n_e}\left(\frac{2}{3}\,\dot{U}_i - T_i\,\dot{n}_e\right). \tag{26}$$

(By plasma neutrality $n_e = n_i$ the ion plasma density.) The ion power density is given by:

$$\dot{U}_i = \dot{U}_e\Big|_{\text{ions}} - \dot{U}_i\Big|_{\text{CX}} - \dot{U}_i\Big|_{\text{diffuse}} - \dot{U}_i\Big|_{\text{end loss}}, \tag{27}$$

where the charge exchange loss is:

$$\dot{U}_i\Big|_{\text{CX}} = 0{\cdot}0732\; n_e\, n_n\, (T_i)^{3/2}\,(1 + 0{\cdot}00585\; T_i^{3/2})\,\exp\,(-\,0{\cdot}0582\; T_i^{1/2}), \tag{28}$$

and the cross-field diffusion loss is:

$$\dot{U}_i\Big|_{\text{diffuse}} = 2{\cdot}5\; T_i\,\dot{n}_e\Big|_{\text{diffuse}}. \tag{29}$$

Any end loss is accounted for by:

$$U_i\Big|_{\text{end loss}} = 2\,T_i\,\dot{n}_e\Big|_{\text{end loss}}. \tag{30}$$

The model equations (7) and (9) through (30) define a dependence of P, the plasma power required by equation (1), on the heating time (which should not greatly exceed the time constant defined by equation (8)).

4. Discussion and conclusions

The model equations described above have been exercised using a variety of different confinement assumptions but always subject to the required final plasma parameters

$$n_e \cong 10^{14}\ \text{cm}^{-3},$$

and $$T_e \cong 10{,}000\ \text{eV}.$$

It is impossible here to discuss details of the time development and radio frequency power required without specializing our results.

Garelis (1974) suggested the possibility of RF-driven inertial confinement fusion wherein RF heating of a droplet occurs (in a very high Q superconducting cavity) in a time short compared to the pellet's hydrodynamic dissassembly time ($\sim 10^{-9}$ sec). For such inertial confinement applications very high wave frequency (very short wavelength microwaves on the order of 1 mm or less) and cavity Q were required.

In magnetic confinement applications, on the other hand, (Mega gauss cusps for instance) the results are rather more encouraging. Heating is easily accomplished on the (much longer) plasma confinement time scale (often > 1 sec) and is, instead, limited by the cavity loss rate which is established by the Q/ω decay time. With magnetic confinement in mind it is adequate to heat only the electrons on this cavity time scale since ion-electron equilibration can occur subsequently (by classical or other processes, on a time scale closer to the energy confinement time τ_E previously mentioned, just as it occurs in conventional RF, beam, and turbulent heating of many magnetic fusion experiments).

Figure 1 shows the time development of plasma density and temperature for a hypothetical closed toroidal confinement system having $\tau_E = 1$ sec and with an RF input power (at 30 GHz) of 75 MW/cm^{-3}. For this calculation the neutral fill gas density was set at 10^{14} cm^{-3} (hydrogen) and the RF input was held constant until the plasma density reaches wave cut-off, at which time the RF is set to zero and the gas is allowed to burn itself out without further assistance.

We see from such numerical calculations that the plasma density has to exponentiate (from an arbitrarily established low initial level and electron temperature of 5 to 10 eV) a number of times before cut-off is reached. The energy which is deposited during this time is sufficient to subsequently burn out the gas, forming a fusion grade plasma (figure 1). The RF coupling is not affected by the (sufficiently tenuous) plasma below cut-off, hence justifying our neglect of the usual plasma wave attenuation phenomena (Jones 1978). Cut-off is typically reached in times short enough to ensure that a reasonable fraction of the incident power ($\sim$10%) appears as plasma energy (the rest being lost in the cavity walls). Modest levels of impurity ions (but $n_I \ll n_e$) are not seen to alter these conclusions. Alpha particle energy input is negligible. It might be possible to reduce P and rapidly heat ions electrostatically by using aerosols rather than a pure gas fill.

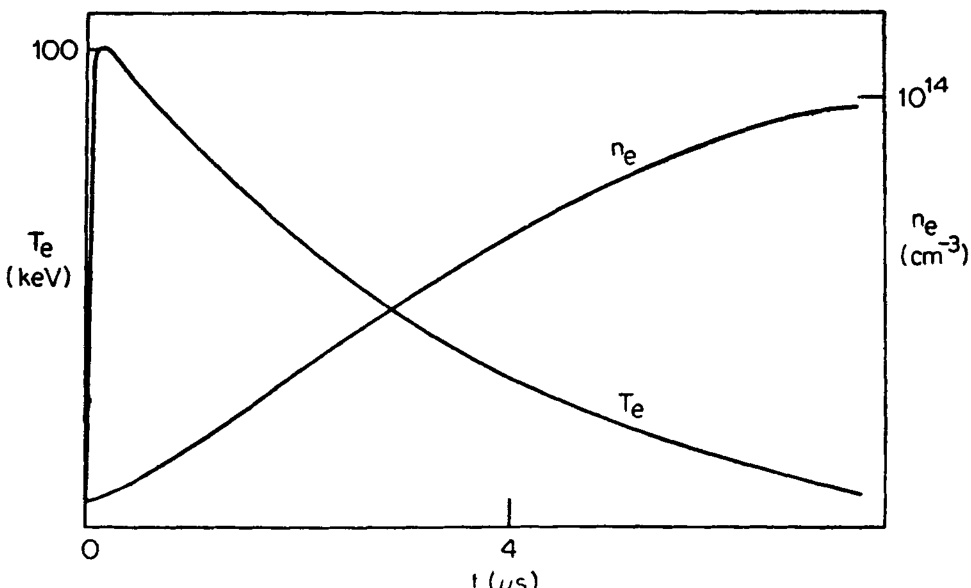

Figure 1. Evolution of the plasma electron density and temperature as a function of time.

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