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Simple Regge pole model for the reaction $\pi^- p \to \eta' n$

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Abstract. The most recently measured differential cross-section data for $\pi^- p \to \eta' n$ has been fitted by using a simple Regge pole model with phenomenological residue functions. It has also been observed that this inelastic process has the scaling property.

Keywords. Regge pole model; residue functions; scaling property.

Recently, Apel *et al* (1979) have measured differential cross-sections for the reaction $\pi^- p \to \eta'$ (958) n at momenta of 15, 20, 25, 30 and 40 GeV/c. This reaction has already been investigated at momenta up to 50 GeV/c by Harvey *et al* (1971), Apel *et al* (1972, 1973) and Bolotov *et al* (1974 a), but at a statistical level of only 50 decays of $\eta' \to 2\gamma$. In the experiment of Apel *et al* (1979), a total of 6000 decays were recorded. This made it possible to widen the range of the determination of differential cross-sections to $-t \approx 1{\cdot}8$ (GeV/c)^2 and also to study the region of small momentum transfers, where, in contradiction with the data of Edwards *et al* (1976) a marked drop in the cross-section is seen for $t \to 0$. In order to improve the statistics, the data at momenta of 20, 25 and 30 GeV/c were combined by Apel *et al* (1979), including a weighting for the relative differential cross sections $d\sigma/dt$. The results for the differential cross-sections thus obtained are shown in figure 1. The sharp drop in the differential cross-section near $t=0$ points to the dominating contribution of the spin flip amplitude to the process. In this paper we will confine ourselves to the results of Apel *et al* (1979) and fit them by using a simple Regge pole model.

The main characteristics of the high energy angular distribution for this reaction are:

(i) The differential cross-section data show a turnover near $t=0$.

(ii) The differential cross-section decreases with an increase in energy.

(iii) For $0{\cdot}2 \lesssim -t < 0{\cdot}8$ (GeV/c)^2, the $d\sigma/dt$ decreases exponentially with a slope of about 10 (GeV/c)^2.

(iv) The $d\sigma/dt$ shows a change of slope near $-t = 0{\cdot}8$ (GeV/c)^2. For $-t \gtrsim 0{\cdot}8$ (GeV/c)^2 the errors are so large that $d\sigma/dt$ may be considered as falling or rising slowly.

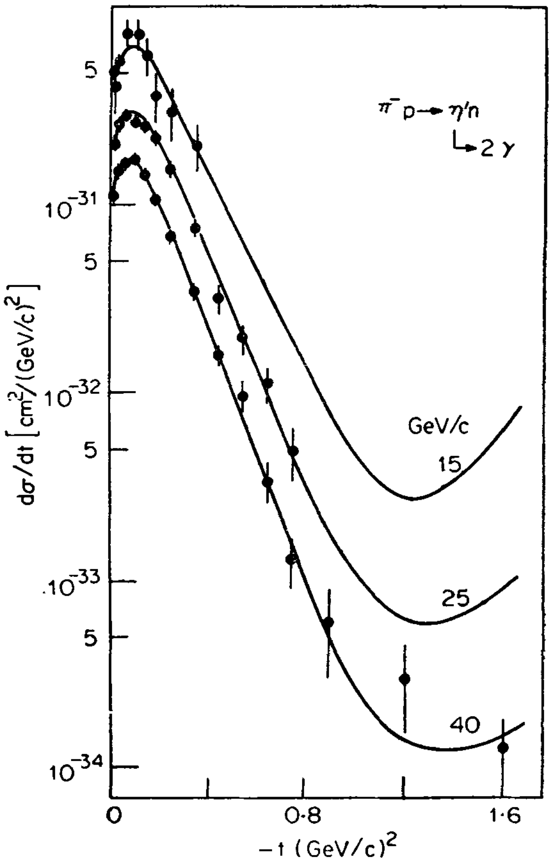

Figure 1. The differential cross-sections of reaction $\pi^- p \to \eta' n$ at $P_{\rm lab} = 15, 25$ and 40 GeV/c. The experimental points have been taken from Apel *et al* (1979). The curves represent the theoretical results obtained by using the simple Regge pole model.

Assuming that the differential cross-section rises slowly after the break, we will show that all these characteristics can be described by using a simple Regge pole model with phenomenological residue functions.

From a theoretical point of view, the reaction $\pi^- p \to \eta' n$ is very interesting as it is believed to be dominated through the exchange of a single Regge trajectory *viz.* A_2 and therefore can play a significant role in arriving at a conclusion about the nature of this trajectory. According to the simple Regge pole model the independent helicity non-flip and helicity flip amplitudes T_{++} and T_{+-} for this reaction involving the exchange of a single Regge trajectory A_2 are given by

$$T_{++}(s, t) = \gamma_{++}(t)\ \xi(t)\ (s/s_0)^{\alpha(t)}\ \sqrt{\mu b}\ \text{GeV},$$

$$T_{+-}(s, t) = \sqrt{-t}\ \gamma_{+-}(t)\ \xi(t)\ (s/s_0)^{\alpha(t)},\ \sqrt{\mu b}\ \text{GeV},$$

where $\alpha(t)$ stands for the Regge trajectory A_2, $\gamma_{++}(t)$ and $\gamma_{+-}(t)$ are the residue functions which are unknown theoretically, $\xi(t)$ is the signature factor and s is the square of the CM energy. The scaling factor s_0 has been introduced to make s/s_0 dimensionless. The differential cross section $(d\sigma/dt)$ and the polarisation P are given by

$$d\sigma/dt = 1/sp^2\{\,|\,T_{++}(s, t)\,|^2 + |\,T_{+-}(s, t)\,|^2\}\ \mu b/(\mathrm{GeV}/c)^2,$$

$$P = 2\ \mathrm{Im}\ \{T_{++}(s, t)\ T^*_{+-}(s, t)\}/sp^2\ (d\sigma/dt),$$

where p is the CM momentum of the incident pion. For the exchange of a single trajectory, these expressions for $d\sigma/dt$ and P give

$$d\sigma/dt = \frac{s^{2\alpha(t)-1}}{p^2}\{\,|\,\gamma_{++}(t)\,|^2 - t\,|\,\gamma_{+-}(t)\,|^2\}\ \mu b/(\mathrm{GeV}/c)^2,\ p=0,$$

where s_0 has been taken as $1(\mathrm{GeV})^2$ and the factor $1/\sin \frac{1}{2}(\pi\alpha)$ has been absorbed in γ's.

Following Saleem and Aleem (1978) we assign phenomenological values to the residue functions. We find that for $d\sigma/dt$ a very good fit with experiment is obtained by the following choice of residue functions:

$$\gamma_{++}(t) = 73\ e^{1 \cdot 685t}\sqrt{\mu b}\ \mathrm{GeV},$$

$$\gamma_{+-}(t) = 438\ e^{1 \cdot 685t + 0 \cdot 465t^4}\ c\sqrt{\mu b}.$$

The equation for the A_2 exchange trajectory has been taken as $\alpha_{A_2}(t) = 0{\cdot}37 + 0{\cdot}8t$ and the branching ratio factor has been absorbed in the residue functions. Figure 1 shows $d\sigma/dt$ plotted versus $-t$. The agreement between calculated and measured values of $d\sigma/dt$ is very good. We have also predicted the $d\sigma/dt$ for $p_{\mathrm{lab}} = 15$ and 25 GeV/c beyond $-t = 0{\cdot}35$ and $0{\cdot}75$ $(\mathrm{GeV/c})^2$ respectively. The polarisation measurements have not yet been made but if they come out as non-zero, the aforementioned model has to be modified either by introducing another trajectory or by taking the A_2 exchange as complex.

The above analysis gives valuable information regarding the A_2 trajectory. Two-component duality coupled with non-exoticity of reasonances leads to the exchange degeneracy (EXD) of ρ and A_2. Bolotov *et al* (1974b) using the data in the range $6 \leqslant p_{\mathrm{lab}} \leqslant 50$ GeV/c for the reaction $\pi^- p \to \pi^\circ n$ obtained the intercept for the ρ trajectory as

$$\alpha_\rho(0) = 0{\cdot}56 \pm 0{\cdot}02,$$

while the data for the reaction $\pi^- p \to \eta n$ measured in the same momentum range yielded (Bolotov *et al* 1974c)

$$\alpha_{A_2}(0) = 0{\cdot}52 \pm 0{\cdot}04,$$

for the A_2 intercept. These results favour EXD between the two trajectories. However, some years ago when data from FNAL became available over an extended energy range, it was found that the energy dependence for the reaction $\pi^- p \to \pi^\circ n$

($\pi^- p \to \eta n$) with ρ (A_2) as the exchange trajectory was different from the energy dependence observed at Serpukhov. In fact precise determination of the intercepts with this data showed that the concept of EXD which has often been taken as an article of faith must be considered a poor approximation to reality. The intercepts were found to be

$$\alpha_\rho (0) = 0{\cdot}48 \pm 0{\cdot}01,$$

$$\alpha_{A_2} (0) = 0{\cdot}37 \pm 0{\cdot}008,$$

so that they differ by about 0·11. Bouquet and Diu (1975) using measurements upto $p_{\rm lab} = 240$ GeV/c found that $\alpha_{A_2}(0) = 0{\cdot}37 \pm 0{\cdot}01$. Kamran (1980) has carried out an investigation of the A_2 intercept using data above 6 GeV/c up to the highest available energies. He has determined the A_2 intercept from KN total cross section and forward $d\sigma/dt$ ($\pi^- p \to \eta n$) data. Fits yield two possible answers for the A_2 intercept $\alpha_{A_2}(0) = 0{\cdot}3618 \pm 0{\cdot}0068$ and $0{\cdot}3475 \pm 0{\cdot}0026$. The intercept for A_2 trajectory used here is in close agreement with these results. The equation of the A_2 trajectory has been obtained from a fit to $\pi^- p \to \eta n$ scattering data by Saleem *et al* (1978). However, we must be rather careful in drawing a conclusion about the A_2 intercept. Large errors in experimental measurements of $d\sigma/dt$ do not allow a unique determination for the A_2 intercept. For instance, if we take the equation of the trajectory as

$$\alpha_{A_2} (t) = 0{\cdot}46 + 0{\cdot}9\, t,$$

the χ^2/pt value is again reasonably acceptable. It is only an accurate measurement of $d\sigma/dt$ for this and other reactions to which A_2 contributes which will help us in finding out the correct equation for the A_2 trajectory.

It would be interesting to find out whether the process $\pi^- p \to \eta n$ possesses the scaling property or not. It is well-known that virtually all elastic reactions have this property at high energies; the differential cross-section normalised by its forward value, $\sigma(s, t)/\sigma(s, o) \equiv \Phi(\tau)$, is a function depending on one dimensionless variable $\tau = tb(s)$ rather than on s and t separately, where $\sigma(s, t)$ stands for $\frac{d\sigma}{dt}(s, t)$. Empirically, the best choice for $b(s)$ corresponds to $\sigma(s, 0)/\sigma_{\rm el}(s)$. Recently, Divakaran (1978) has extended the domain of scaling to inelastic differential cross-sections for $2 \to 2$ reactions. The general scaling variable is

$$\tau = t\, \sigma_{\rm max}(s)/\sigma_{\rm int}(s),$$

and the scaling function

$$\Phi(s, \tau) = \sigma(s, t)/\sigma_{\rm max}(s),$$

where $\sigma_{\rm max}(s)$ is the highest value of $d\sigma/dt$ at the given s and $\sigma_{\rm int}(s)$ is the corresponding integrated value over t. This scaling has been tested in the inelastic reaction $\pi^- p \to \eta' n$ over $p_{\rm lab} = 15$ to 40 GeV/c and for $-t \leqslant 1{\cdot}6\,({\rm GeV/c})^2$. Figure 2 shows $-\tau$

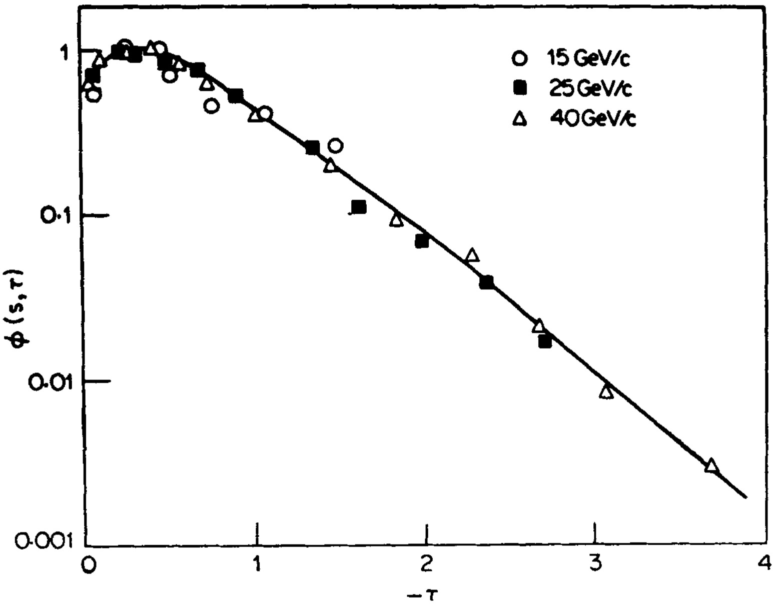

Figure 2. $\phi(\tau) \equiv \sigma(s, t)/\sigma_{max}(s)$ plotted against the scaling variable $\tau = t\, b(s)$, where $b(s) = \sigma_{max}(s)/\sigma_{int}(s)$ for $\pi^- p \to \eta' n$. The numbers are the beam momenta in GeV/c.

plotted against $\Phi(s, \tau)$. The errors on both abscissa and ordinate are omitted since they only clutter up the figure. The scaling is found to be good for this inelastic reaction.

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