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Structure function of pion and its compositeness

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Abstract. Using recent data on pion structure function and a rigorous inequality obtained recently using unitarity analyticity and Bjorken scaling a numerical upper bound on the wave-function-renormalisation constant of pion is computed. By the (somewhat drastic) act of neglecting the sea, it is shown that the bare part of the pion can be no more than 38%.

Keywords. Pion structure function; renormalisation constant; compositeness; unitarity; Bjorken scaling; analyticity.

1. Introduction

Recently some attention has been focussed on estimating the degree of compositeness of a particle from a knowledge of the numerical value of its wave-function-renormalisation constant. Fractional value of the renormalisation constant has been interpreted as the manifestation of the degree of compositeness of the particle. Using unitarity, analyticity, Bjorken scaling and the available data on the structure functions of the proton, Broadhurst (1972) was able to obtain a numerical upper bound on the wave-function-renormalization constant, Z_2, for the proton. Later Baluni and Broadhurst (1973) carried out a more rigorous analysis using unitarity, analyticity and the experimental data on πN scattering. A more sophisticated computation by Baluni and Broadhurst (1977) reveals that $Z_2 \leqslant 0{\cdot}25$ which implies that proton is at least 75% composite.

Since pion is the least massive of hadrons taking part in strong interaction, estimation of its compositeness may also furnish information on the lower limit of the degree of compositeness of other hadrons. To our knowledge there does not exist any numerical upper bound on the wave-function-renormalisation constant Z_3 of the pion. However, recently, theoretically rigorous upper bounds on, Z_3, have been derived (Parida and Giri 1977, 1978) in terms of integrals involving structure functions of the pion. But since the experimental data on the pion structure functions were not available at that time it was not possible to evaluate the upper bound numerically. In this paper we evaluate the upper bound on Z_3 numerically, using the recently published data on the pion structure function (Newman *et al* 1979). Several aspects of this result are also discussed.

2. Pion-structure function data and computation of numerical upper bound

We follow notations adopted in earlier papers (Parida and Giri 1977, 1978). Considering the pion-photon vertex occurring in inelastic electron pion scattering and using unitarity, analyticity and the hypothesis of Bjorken scaling, in addition to establishing a theorem on the compositeness of pion, the following inequalities on the wave-function-renormalization constant have been obtained (Parida and Giri 1978)*.

$$\frac{Z_3}{1-Z_3} \leqslant \int_1^\infty \frac{F_2(w)}{w} \frac{R(w)}{R(w)+1} dw \tag{1}$$

$$\frac{Z_3}{1-Z_3} \leqslant \int_1^\infty \frac{F_2(w)}{w} dw. \tag{2}$$

In (1) and (2), $F_2(w)$ is the Bjorken limit of the structure function

$$\lim_{\text{Bj}} \nu W_2 = F_2(w), \tag{3}$$

and $R(w)$ is the Bjorken limit of the longitudinal to the transverse photoabsorption cross section of the pion

$$R(w) = \lim_{\text{Bj}} \frac{\sigma_L}{\sigma_T} = \lim_{\substack{\nu, Q^2 \to \infty \\ w \text{ fixed}}} \left[(1+\nu^2/Q^2)\frac{W_2}{W_1} - 1\right]$$

$$= \frac{wF_2(w)}{F_1(w)} - 1, \tag{4}$$

where $w = 2m_\pi \nu / Q^2$ and $\lim_{\text{Bj}} 2m_\pi w_1 = F_1(w)$. (5)

Very recently the pion structure-function data have been extracted from the muon pair production data in pion-nucleon collision processes using the Drell-Yan formula (Newman *et al* 1979). From charge conjugation and isospin invariance it follows that the quark distribution functions for the two-valence quarks inside the pion are equal.

$$\bar{u}^\pi(x) = d^\pi(x), \tag{6}$$

where $x=1/w$. Newman *et al* (1979) have given several equivalent fits to the structure-function data on $x\,\bar{u}^\pi(x)$ for $0{\cdot}25 \leqslant x \leqslant 1$, one of which can be written as

$$f^\pi(x) = x\,\bar{u}^\pi(x) = ax^{1/2}(1-x)^b \tag{7}$$

where $a = 0{\cdot}9 \pm 0{\cdot}06,\ b = 1{\cdot}27 \pm 0{\cdot}06.$ (8)

*Note that there are some errors in writing down the inequalities (1) and (2) in the final form in the earlier paper (Parida and Giri 1978). The correct factors at the left hand side should be $Z_3/(1-Z_3)$ instead of $Z_3/(Z_3-1)$.

Other fits differ from (7) almost by a negligible amount for $x \geqslant 0{\cdot}25$ for which valence quark contribution dominates and the sea quark contribution is negligible. The formula (7) which can be extrapolated upto the region $x = 0$ represents the valence quark contribution in the region $0 \leqslant x \leqslant 1$. Also the fit (7) is preferred to some other fits proposed by Newman *et al* because of the following attractive features: The fit leads to a finite value of the integral of the valence quark function over the range $0 \leqslant x \leqslant 1$ and possesses the desirable behaviour of the type $x^{1/2}$ near $x = 0$ suggested by other theoretical analyses (Landshoff and Polkinghorne 1970; Kuti and Weisskopf 1971). Further, the value of b in (8) is consistent with the theoretical predictions of Altarelli *et al* (1975) and Donachie and Landshoff (1976). The pion structure function in the scaling limit due to the valence quark contributions can be written as

$$\nu W_2 = \frac{4}{9} \times \bar{u}^{\pi}(x) + \frac{1}{9} x d^{\pi}(x),$$

$$= \frac{5}{9} f^{\pi}(x), \tag{9}$$

where we have used (6) and (7). It may be remarked that experimental data on R is not available yet for the pion. In view of this we use inequality (2) instead of (1) to calculate the numerical upper bound. Using inequality (2) and equations (7) to (9) we obtain

$$\frac{Z_3}{1-Z_3} \leqslant \frac{5}{9} \int_0^1 \bar{u}^{\pi}(x)\, dx = 0{\cdot}616 \pm 0{\cdot}015, \tag{10}$$

which yields

$$Z_3 \leqslant 0{\cdot}38 \pm 0{\cdot}01. \tag{11}$$

A unity value of the wave-function-renormalisation constant implies that the particle is purely elementary, while a vanishing wave-function-renormalisation constant implies that it is composite, although the sufficiency condition for compositeness has been questioned (Hayashi *et al* 1967; Tirapegui 1972). An intermediate value of the renormalisation constant between zero and unity has been interpreted as the degree of compositeness (Broadhurst 1972; Baluni and Broadhurst 1973, 1977). In (11) the value of Z_3 is less than unity by at least 0·62. This result implies that, if the sea is neglected, the bare part of the pion can be no more than 38% or the pion is at least 62% composite.

3. Discussion

In obtaining the numerical upper bound we have not included possible contribution to the structure function due to sea quarks. The sea quark contribution is expected to be important near $x = 0$ and has been suggested to be (Farrar 1974; Feynman and Field 1977)

$$f_s^{\pi}(x) = 0{\cdot}1\ (1-x)^5, \tag{12}$$

which yields $F_2(x=0)=0{\cdot}1$. This will make the inequality (2) diverge at the upper limit without leading to any meaningful result. But there is the possibility that the inequality (1) may yield useful results if $R(x)$ approaches zero as x^{ϵ} with $\epsilon > 0$. However, because of lack of any numerical data on R in the whole range of x for the pion at present we have not been able to utilise the inequality (1) with sea quark contribution. In spite of this limitation our result is the first one in the literature revealing the degree of compositeness of the pion from unitarity, analyticity, Bjorken scaling and the experimental data on structure function.

As remarked earlier (Parida and Giri 1978), if it is found that $R(x)=0$ for all x, it means that the pion is 100% composite. According to the sophisticated analysis of Baluni and Broadhurst (1977) proton is at least 75% composite.

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References

Altarelli G, Cabibbo N, Maiani L and Petronzio R 1975 *Nucl. Phys.* **B92** **413**
Baluni V and Broadhurst D J 1973 *Phys. Rev.* **D7** 3738
Baluni V and Broadhurst D J 1977 *Phys. Rev.* **D15** 230
Broadhurst D J 1972 *Phys. Rev.* **D5** 2615
Donnachie A and Landshoff P V 1976 *Nucl. Phys.* **B112** 223
Farrar G R 1974 *Nucl. Phys.* **B77** 429
Feynman R P and Field R D 1977 *Phys. Rev.* **D15** 2590
Hayashi K *et al* 1967 *Fortschr. Phys.* **15** 625
Kuti J and Weisskopf V F 1971 *Phys. Rev.* **D4** 3418
Landshoff P V and Polkinghorne J C 1970 *Nucl. Phys.* **B19** 432
Newman C B *et al* 1979 *Phys. Rev. Lett.* **42** 951
Parida M K and Giri N 1977 *Pramāṇa* **9** 501
Parida M K and Giri N 1978 *Pramāṇa* **10** 551
Tirapegui E 1971 *Phys. Rev.* **D4** 3807