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Coherent information processing by a pair of lenses in spherical wave illumination

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Abstract. A coherent optical information system has been analysed using the Fresnel diffraction theory. Considering the spherical wave illumination, the same system is used for spatial filtering and subsequent reimaging. The conditions for locating the spatial frequency plane and the image plane have been pointed out. The scale of the Fourier transform can be controlled by three degrees of freedom. The final image formed is inverted and magnified with respect to the input signal. The present analysis has been compared with those of Pernick and Moharir. Aberrations involved have also been discussed.

Keywords. Information processing; Fourier transforming elements; spatial filtering; aberrations; imaging system.

1. Introduction

1.1 *General*

The Fourier transformation can describe the many optical operations in optical information processing. The focusing elements like lenses and mirrors have been used for producing the Fourier transform of input function. Three separate configurations for performing the Fourier transform operations are well-known (Goodman 1968; Cathey 1974). Many workers have reported the various forms of Fourier transforming devices. Haskell (1970) studied the Fourier transforming properties of holograms. Husain-Abidi and Krile (1971) supported experimentally as well as theoretically the superiority of mirrors over the lenses as Fourier transforming element. Kasana *et al* (1976, 1978) reported the Fourier transforming properties of parabolic mirrors in spherical wave illumination in which exact Fourier transformation occurs provided the Newton's formula is valid. In fact, Newton's formula is a focusing condition and must be valid for every focusing optical element. Blandford (1969) described a new four-component system in which the overall length of the Fourier processor is reduced for use in optical information processing. von-Bieren (1971) designed a Fourier transforming element which consisted of two identical triplets. Wynne (1974) also gave the design data and optical performance of the Fourier transforming elements. He showed that the comparable level of aberration correction could be achieved. Pernick (1971) predicted that a pair of lenses could be used for spatial filtering and subsequent reimaging. Moharir (1974, 1975) also dealt with a pair of lenses for exact Fourier transformation using the

Fresnel diffraction theory and the spherical wave illumination. In single lens configuration, considering the spherical wave illumination, he indicates that the image plane corresponding to the point source will be the plane of spatial frequency spectrum.

In the present paper, we study a coherent optical information processing system using a pair of lenses and spherical wave illumination.

1.2 *Glossary*

$$\psi(x, y; D) \,\Delta\, \exp\,[ikD(x^2 + y^2)/2)],$$

$$D_i \,\Delta\, 1/d_i \text{ (where } i = 1, 2, 3, 4 \text{ and } 5),$$

$$F_j \,\Delta\, 1/f_j \text{ (where } j = 1 \text{ and } 2),$$

$$k \,\Delta\, 2\pi/\lambda.$$

The symbol Δ indicates a definition where d and f denote the axial distances and focal lengths, respectively. $\bar{\psi}$ denotes the complex conjugate of the function ψ. The properties of ψ-function have been described by Vander Lugt (1966).

2. Coherent optical processor

The optical system used for optical information processing is shown in figure 1. P_0 is the point source of coherent monochromatic light which propagates the spherical waves. The input transparency $s(x, y)$ has been placed in a plane (x, y) at a distance d_1 from the point source and at a distance d_2 from the lens L_1 of focal length f_1. The lens L_1 is followed by the second lens L_2 of focal length f_2 at a distance d_3 from L_1. The lens L_2 is at a distance d_4 from the spatial frequency plane (x_0, y_0). The dotted lines indicate the principal planes of the combination of lenses L_1 and L_2. The image plane exists at a distance d_5 from the spatial frequency plane.

3. Theory

The disturbance at any point (x, y) due to spherical waves of amplitude A originating from the point source P_0 can be calculated by using the paraxial approximation as:

$$U(x, y) = A\psi(x, y; D_1). \tag{1}$$

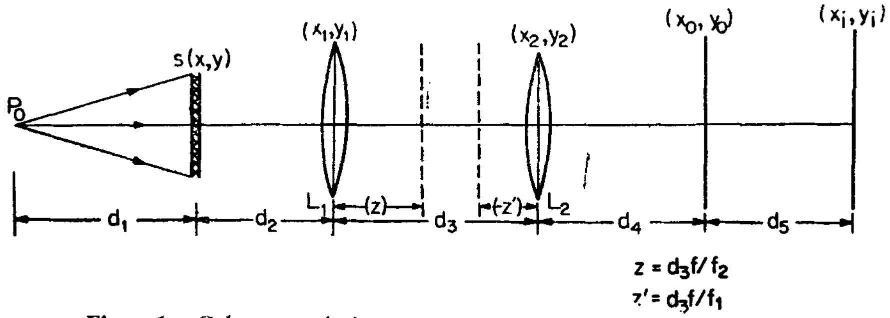

Figure 1. Coherent optical processor.

Here the constant phase factor has been dropped. The amplitude distribution just behind the input signal can be written as

$$U'(x, y) = A\, s(x, y)\, \psi(x, y;\, D_1), \tag{2}$$

where $s(x, y)$ represents the amplitude distribution of the input signal.

This disturbance advances towards the lens L_1 and it is modified by the phase transformation function $\bar{\psi}(x_1, y_1;\, F_1)$ of the lens during its passage through the lens. Hence, using the Fresnel diffraction formula, the field distribution just behind the lens L_1 can be written as follows:

$$U_1(x_1, y_1) = A\psi(x_1, y_1;\, D_2 - F_1) \int\int_{-\infty}^{\infty} \psi(x, y;\, D_1 + D_2)\, s(x, y)$$
$$\times \exp\left[-\, ikD_2(xx_1 + yy_1)\right] dx\, dy, \tag{3}$$

where it has been assumed that the linear dimensions of the signal in the input plane and that of the region in the next plane where the amplitude distribution is determined should be much smaller than their separation apart (Goodman 1968).

The propagation of this disturbance over a distance d_3 and after travelling through the lens L_2, it involves the phase transformation function $\bar{\psi}(x_2, y_2;\, F_2)$ of the lens L_2. So, the disturbance just behind the lens L_2 would be of the form:

$$U_2(x_2, y_2) = A\psi(x_2, y_2;\, D_3 - F_2) \int\int\int_{-\infty}^{\infty}\int \psi(x, y;\, a)\, s(x, y)\, \psi(x_1, y_1;\, b)$$
$$\times \exp\left[-\, i2\pi(p_1x_1 + q_1y_1)\right] dx\, dy\, dx_1\, dy_1, \tag{4}$$

where,
$$a = D_1 + D_2, \tag{5}$$
$$b = D_2 + D_3 - F_1, \tag{6}$$
$$p_1 = (xD_2/\lambda) + (x_2D_3/\lambda), \tag{7}$$
$$q_1 = (yD_2/\lambda) + (y_2D_3/\lambda).$$

Considering the integration over the variables x_1 and y_1

$$\int\int_{-\infty}^{\infty} \psi(x_1, y_1;\, b) \exp\left[-\, i2\pi(p_1x_1 + q_1y_1)\right] dx_1\, dy_1$$

$$= \text{Fourier transform of } [\psi(x_1, y_1;\, b)] \text{ at } p_1, q_1$$

$$= \bar{\psi}(x, y;\, D_2^2/b)\, \dot{\psi}(x_2, y_2;\, D_3^2/b)$$

$$\times \exp\left[-\, ikD_2D_3\,(xx_2 + yy_2)/b\right]. \tag{8}$$

Using equations (4) and (8), we have:

$$U_2(x_2, y_2) = A\psi(x_2, y_2;\, D_3 - [D_3^2/b] - F_2) \int\int_{-\infty}^{\infty} s(x, y)$$

$$\times\, \psi[x, y;\, a - (D_2^2/b)] \exp [-ikD_2D_3(xx_2 + yy_2)/b]\; dx\, dy. \qquad (9)$$

The field distribution in (x_0, y_0) plane would be:

$$U_0(x_0, y_0) = A\, \psi\, (x_0, y_0;\, D_4) \int\int\int\int_{-\infty}^{\infty} \psi\, (x, y;\, a - [D_2^2/b])\, s(x, y)$$

$$\times\, \psi\, (x_2, y_2;\, c) \exp [-i2\pi\, (p_2x_2 + q_2y_2)]\; dx\, dy\, dx_2\, dy_2, \qquad (10)$$

where $$c = D_4 + D_3 - (D_3^2/b) - F_2, \qquad (11)$$

$$p_2 = (xD_2D_3/\lambda b) + (x_0D_4/\lambda),$$

$$q_2 = (yD_2D_3/\lambda b) + (y_0D_4/\lambda). \qquad (12)$$

Now, dealing with the integration over dx_2, dy_2:

$$\int\int_{-\infty}^{\infty} \psi\, (x_2, y_2;\, c) \exp [-i2\pi\, (p_2x_2 + q_2y_2)]\; dx_2\, dy_2$$

$$= \text{Fourier transform of } [\psi(x_2, y_2;\, c)] \text{ at } p_2, q_2$$

$$= \bar{\psi}\, (x, y;\, D_2^2D_3^2/b^2c)\, \bar{\psi}\, (x_0, y_0;\, D_4^2/c)$$
$$\times \exp [-ikD_2D_3D_4\, (xx_0 + yy_0)/bc]. \qquad (13)$$

Substituting equation (13) in equation (10), we get

$$U_0\, (x_0, y_0) = A\, \psi\, (x_0, y_0;\, D_4 - [D_4^2/c]) \int\int_{-\infty}^{\infty} s\, (x, y)$$

$$\times\, \psi\, (x, y;\, a - [D_2^2/b] - [D_2^2D_3^2/b^2c])$$

$$\times \exp [-ikD_2D_3D_4(xx_0 + yy_0)/bc]\; dx\, dy. \qquad (14)$$

This is the resultant expression showing the amplitude distribution across the spatial filtering plane. The right hand side of equation (14) indicates the Fourier transform relationship, however, distorted due to the two quadratic phase factors.

The validity and correctness of the expression can be checked and verified by establishing the various existing Fourier transforming conditions. In fact, the curvature terms of this expression are responsible for giving these various conditions which are as follows:

3.1 *Input function placed in front of the lens*

When the input signal is placed in front of the combined lens system at a distance d_2, the expression (14) will give the Fourier transform of signal $s(x, y)$ only if the ψ-function which depends on variables x and y is unity. Hence

$$D_1 = D_2\,[F_1\,(D_3 + D_4) - D_3 D_4 + F_2 D_3 - F_1 F_2]/bc. \tag{15}$$

With the help of (6) and (11), we get

$$bc = (D_2 D_3 + D_3 D_4 + D_4 D_2) - F_1\,(D_3 + D_4) - F_2\,(D_2 + D_3) + F_1 F_2. \tag{16}$$

Substituting equation (16) in (15) and solving, we get

$$d_1 + d_2 = f\left[d_3 + d_4\left(1 - \frac{d_3}{f_2}\right)\right] \Big/ [d_4 + (d_3 f/f_1) - f].$$

On adding $(d_3 f/f_2)$ both sides and simplifying further, we obtain

$$U(V - f)/f = d_4 + d_3[1 + f(d_3 - f_1)/f_1 f_2],$$

or $$1/f = (1/U) + (1/V), \tag{17}$$

where $$U = d_1 + d_2 + (d_3 f/f_2),$$

$$V = d_4 + (d_3 f/f_1), \tag{18}$$

and $$1/f = (1/f_1) + (1/f_2) - (d_3/f_1 f_2).$$

Again, using equations (6) and (11), we have

$$R = D_4\left(1 - \frac{D_4}{c}\right) = \frac{D_4[D_3(D_2 - F_1 - F_2) - F_2(D_2 - F_1)]}{D_3(D_2 - F_1 - F_2) - F_2(D_2 - F_1) + D_4(D_2 + D_3 - F_1)},$$

$$= \frac{1 - [d_2 + (d_3 f/f_2)]/f}{d_2[1 - (V/f)] + d_3 + d_4\,[1 - (d_3/f_2)]}.$$

Eliminating V and d_4 from this equation with the help of (17) and (18), we obtain:

$$R = \{1 - [d_2 + (d_3 f/f_2)]/f\}\,(U - f)/f d_1,$$

$$= [1 - (d_2 + d_3 f/f_2)/f]\,[1 + (d_2 + d_3 f/f_2 - f)/d_1]/f \tag{19}$$

Also, we can show using (16),

$$Q = bc/D_2 D_3 D_4,$$

$$= \left[d_2 + d_3 + d_4 - d_4\left(d_2 + \frac{d_3 f}{f_2}\right) \Big/ f - (d_2 d_3/f_1)\right].$$

Substituting the value of (d_4/f) from (18), we have:

$$Q = [d_2 + (d_3 f/f_2)] + [d_4 + (d_3 f/f_1)] \left[1 - \left(d_2 + \frac{d_3 f}{f_2}\right) \Big/ f\right]. \quad (20)$$

Taking into account (17), (19) and (20), the amplitude distribution in the Fourier transform plane would be

$$U_0 (x_0, y_0) = A\, \psi (x_0, y_0;\, R)\, S (x_0/\lambda Q,\, y_0/\lambda Q), \quad (21)$$

$$\text{where } S\left(\frac{x_0}{\lambda Q}, \frac{y_0}{\lambda Q}\right) = \int\int_{-\infty}^{\infty} s (x, y) \exp [- i2\pi (xx_0 + yy_0) / \lambda Q]\, dx\, dy.$$

Hence, when the input function is placed at any distance from the combined lens configuration, an extra quadratic phase factor precedes the Fourier transform. Indeed, this quadratic phase factor affects the area of the Fourier transform i.e. it lies on a sphere of radius $(1/R)$. The speculation of (18) implies that U and V represent the position of the point source and its image from the planes which are at the distances $(d_3 f/f_2)$ and $(- d_3 f/f_1)$ from the centres of lenses L_1 and L_2, respectively, as shown in figure 1 by dotted lines. Equation (17) defines the lens law. Hence, the spatial frequency plane should be recognised from (17) as the plane where the image of the point source appears.

Applying again the Fresnel diffraction formula and using (21), the amplitude distribution across the image plane at a distance d_5 from the filtering plane may be written as

$$\begin{aligned} U_i (x_i, y_i) = {} & A\, \psi (x_i, y_i;\, D_5) \int\int_{-\infty}^{\infty} \psi (x_0, y_0;\, D_5 + R) \\ & \times S (x_0/\lambda Q,\, y_0/\lambda Q) \\ & \times \exp [- ikD_5 (x_0 x_i + y_0 y_i)]\, dx_0\, dy_0. \end{aligned} \quad (22)$$

This expression will lead to the image of the original signal if the quadratic phase factor with (x_0, y_0) is eliminated. This can be accomplished by requiring that

$$\begin{aligned} d_5 &= - 1/R, \\ &= f^2 / [d_2 + (d_3 f/f_2) - f] \left[1 + \left(d_2 + \frac{d_3 f}{f_2} - f\right) \Big/ d_1\right]. \end{aligned} \quad (23)$$

Let $U' = d_2 + (d_3 f/f_2)$,

$$V' = d_4 + (d_3 f/f_1) + d_5.$$

Using (17), we can show that the parameters U' and V' satisfy the relation

$$1/f = (1/U') + (1/V'). \quad (24)$$

This expression is again the lens law between the signal and its image. Thus, if the lens law holds good, we obtain

$$U_i(x_i, y_i) = A\,\psi(x_i, y_i;\, D_5)\, s(-Qx_i/d_5, -Qy_i/d_5). \tag{25}$$

This is the field distribution in the image plane which gives information about the original signal, of course, with a quadratic phase term. This phase factor is of no importance because the intensity across the image plane is of real interest. Hence, the intensity in the image plane can be expressed as

$$I(x_i, y_i) = |A\, s(-Qx_i / d_5, -Qy_i / d_5)|^2. \tag{26}$$

This implies that the final image formed is the inverted and the magnified replica of the original signal. The image plane should be recognised from (24) as a plane where the image of the input signal appears. This condition is different than that of (17). Hence, an extra imaging assembly is not required to have the image of the signal. The same system performs the spatial filtering and subsequent reimaging. Therefore, if a filter is inserted in the filtering plane, the resulting image would be the convolution of the signal with the impulse response of the filter.

The distances d_i are considered to be positive and assumed that the combination of lenses acts as a converging lens. It is evident from (17) and (23), that the spatial frequency plane and the image plane will be real if

$$U > f < U'.$$

3.2 *Scaling parameter*

In optical information processing, especially in matched filtering operation, the scale of the Fourier transform should be under the control of the experimenter. The scaling factor of the Fourier transform, for combined lens configuration, is given by

$$\begin{aligned} a &= \lambda Q, \\ &= \lambda \{[d_2 + (d_3 f / f_2)] + [d_4 + (d_3 f / f_1)] \\ &\quad \times \left[1 - \left(d_2 + \frac{d_3 f}{f_2}\right) \Big/ f\right]\}. \end{aligned} \tag{27}$$

The effective focal length (f) of the combined lens can be varied either by changing the individual focal lengths of the lenses or by changing their separation apart. Therefore, the factors affecting the scale of the Fourier transform are:

(i) individual focal lengths of lenses,

(ii) separation between the two lenses (i.e. zoom effect),

(iii) position of input signal between the point source and the lens L_1 (d_2).

In either case of achieving the variable scale of Fourier transform, the various distances d_i are retained such that the condition (17) and (24) are simultaneously satisfied. The latter two would be preferred, especially, the third one because in this case only the image plane is to be located and the spatial frequency plane remains invariable as evident from the conditions of (17) and (24). Thus, comparatively the last one would be more flexible. This property of controllable scaling factor would be important in information processing where the scale of the feature of interest (in the input plane) is either unknown or of different sizes.

3.3 *Magnification parameter*

The magnification of the final image can be expressed as

$$M = d_5/Q = f(U' - f) = V'/U'. \tag{28}$$

So the image magnification depends on the effective focal length as well as on the position of input signal from the first principal plane. Thus, when the scale of the Fourier transform is changed by varying the position of input signal (d_2), the magnification of the image is also affected.

3.4 *Exact Fourier transform condition*

When the condition of exact Fourier transform is imposed across the spatial frequency plane, an extra phase factor preceding the Fourier transform of (21) should also disappear, which leads to the condition

$$f = d_2 + (d_3 f / f_2). \tag{29}$$

So for the exact Fourier transformation, the input signal must be placed in the front focal plane of the system and also the condition of (17) is simultaneously satisfied. Therefore, the Fourier transform is given by

$$S(x_0 / \lambda f, y_0 / \lambda f) = \int_{-\infty}^{\infty}\int s(x, y) \exp[-ik(xx_0 + yy_0)/f]\, dx\, dy.$$

The scale of Fourier transform has now lost one degree of freedom (i.e. d_2) and only depends on the effective focal length. The system is still flexible. Equation (23) shows that the final image is formed at infinity i.e. $V' = \infty$. An extra imaging assembly is required to bring the image in the observation plane. The distance d_2 will be positive if

$$f_2 > d_3.$$

4. Results and discussions

Since a pair of lenses acts as a single lens of effective focal length f, the Fourier transforming properties of a single lens should be valid even for a pair of lenses.

Pernick's analysis in plane wave illumination makes use of geometrical optics which is inadequate to explain certain information processing conditions. According to his analysis, the separation between two lenses is such that the combination acts as a single diverging lens as evident from the erect and unmagnified image of the original signal which is impractical for the spatial filtering.

Moharir's treatment of the same configuration for exact Fourier transformation in a spherical wave illumination by using the diffraction theory does not give explicit results. His analysis does not specify exactly the position of the input plane and spatial frequency plane while we have indicated (§ 3.4) that the input function should be located in the exact front focal plane of the system and filtering plane should be recognised from equation (17) as the plane where the image of the point source appears. This is the exact explanation of the Fourier transforming properties. He has expressed that the scale of the Fourier transform can be controlled by changing the focal length of the second lens which will introduce the design, economic and practical problems also. But in our case, we have suggested the use of zoom effect for achieving the desired scaling factor.

Also, despite the more legitimate word scaling factor which determines the size of the Fourier transform spectrum, both the authors (Pernick and Moharir) have used the word magnification in the spatial frequency plane which does not reflect its actual meaning.

We have used the same assembly for the spatial filtering and reimaging. This occurs when the input function is at any distance from the system. The final image formed is inverted and a magnified replica of the original signal. Thus, if a suitable spatial filter is inserted in spatial frequency plane, the resulting amplitude distribution in the image plane would be the convolution of the input signal with the impulse response of the filter. The scale of the Fourier transform spectrum can be controlled by three degrees of freedom which is not available with a single lens. The complex spatial filter may be recorded holographically by using the condition of exact Fourier transform with desired scaling factor and the processing can be performed by the condition discussed under § 3.1. The conditions predicted here for coherent information processing favours the exact expected results.

The study of the Seidel aberrations has not been made. However, these aberrations would always be present in this configuration. The full control of aberrations requires minimum six lens elements because it has five degrees of freedom which are greater than the aberrations as von-Bieren has pointed out. His symmetrical system consists of a pair of identical triplets. If one of the triplets has a focal length f_1 and the other f_2, the present analysis can be applied to that symmetrical system which would give better and identical results for optical information processing. Wynne has also thrown light on this point. He has pointed out that the comparable level of aberration correction can be achieved by system of one or two thin lens elements. Hence, an aberration free system is available for optical information processing.

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