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Magnetic moments of *b*-quark baryons in broken SU(5) symmetry

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Abstract. The magnetic moments of *b*-quark baryons within the framework of five quark models are derived. Also the transition magnetic moments of various *b*-quark baryons are calculated.

Keywords. Magnetic moments; transition moments; *b*-quark baryons; broken SU (5) symmetry.

1. Introduction

Among the most successful description of the new particles and their spectroscopy is the one based on a new quark flavour with charge $-\frac{1}{3}$, conventionally described as the beauty in broken SU(5)—symmetry (Herb *et al* 1977; Innes *et al* 1977). The discovery of upsilon resonance $\Upsilon(9{\cdot}4)$ (Herb *et al* 1977; Innes *et al* 1977), has made the study of the physical aspects and properties of $\Upsilon(9{\cdot}4)$ family interesting. In this paper we present the magnetic moments of beautiful baryons and the relations between them and also calculate the transition magnetic moments. This will enable us to write down wave functions in terms of quark wave functions. The baryon wave functions of three quarks and meson wave functions of a quark-antiquark pair include a spatial part, a spin part, a flavour unitary-symmetry part and a colour unitary-symmetry part (Johnson and Shah-Jahan 1977; Lipkin and Tavkhelidze 1965). The spatial part of a baryon wave function cannot be written explicitly as its form depends on the unknown details of quark dynamics. However, if we assume that the forces between quarks are basically attractive, the lowest energy states will have symmetric spatial wave functions.

So far as spin part of the wave function is concerned, we assume that the lowest lying states have zero-orbital angular momentum. Then the spin wave functions (for $m=3/2$ and $1/2$) of the 35-plet symmetric baryons of SU(5) are

$$\chi^{3/2}_{3/2} = \alpha\alpha\alpha; \quad \chi^{3/2}_{1/2} = \frac{1}{\sqrt{3}}(\alpha\alpha\beta+\alpha\beta\alpha+\beta\alpha\alpha), \tag{1}$$

where α and β denote quark spin functions with the third component $\frac{1}{2}$ and $-\frac{1}{2}$ respectively. The members of the 40-plet mixed baryons of SU(5) have spin $\frac{1}{2}$. The wave functions χ_m and χ'_m which are symmetric and antisymmetric respectively under the interchange of the spin coordinates of the first two quarks are written (for $m=\frac{1}{2}$)

$$\chi_{1/2} = \frac{1}{\sqrt{6}}(2\alpha\alpha\beta - \alpha\beta\alpha - \beta\alpha\alpha);$$

$$\chi'_{1/2} = \frac{1}{\sqrt{2}}(\alpha\beta\alpha - \beta\alpha\alpha). \tag{2}$$

2. The magnetic moments

The baryon magnetic moments can be expressed as a vector sum of the quark moments plus a contribution from any orbital angular momentum of the quarks (Franklin 1975). We write the magnetic moment operator of a quark as (Lipkin and Tavkhelidze 1965)

$$\vec{\mu}_q = \mu_q \vec{\sigma} \tag{3}$$

where $\vec{\sigma}$ is the Pauli spin operator and μ_q is the constant which depends on the flavour of quark. If it is assumed that the orbital angular moment of the quarks is zero, then the magnetic moment operator of a baryon is given by

$$\vec{\mu} = \sum_{i=1}^{3} \mu_q(i) \vec{\sigma}(i), \tag{4}$$

where the sum is over the three quarks of a baryon.

The value of the magnetic moment μ_B of any baryon is the expectation value of μ_3 (the third or z-component of $\vec{\mu}$) with respect to a baryon wave function B which is maximally polarised along the z-axis (Lipkin 1978), that is

$$\mu_B = (B, \sum_i \mu_q(i)\, \sigma(i)\, B). \tag{5}$$

It is possible to evaluate μ_B in terms of μ_q for any baryon once the flavour and spin wave functions of the baryon are specified. We shall now obtain the magnetic moments of the spin $(\frac{1}{2})^+$ and $(\frac{3}{2})^+$ baryons of the 40-plet and 35-plet in terms of the quark moments using the simplified baryon wave functions and derive the sum rules of the magnetic moments and their transition moments.

2.1. $J^P = (\frac{1}{2})^+$ *baryons*

To evaluate the magnetic moments of $\sum_b^+$ we substitute the $\sum_b^+$ wave function of table 1 into equation (5) and get

$$\mu_{\Sigma_b^+} = \tfrac{1}{3}(4\mu_u - \mu_b). \tag{6}$$

The other results are

$$\mu(\Sigma_b^-) = \tfrac{1}{3}(4\mu_d - \mu_b);\ \mu(\Omega_b^-) = \tfrac{1}{3}(4\mu_s - \mu_b);$$

$$\mu\,(\lambda_b^+) = \tfrac{1}{3}\,(4\mu_c - \mu_b);$$

$$\mu(\Sigma_b^0) = \tfrac{2}{3}\,\mu_u + \tfrac{2}{3}\,\mu_d - \tfrac{1}{3}\,\mu_b;$$

$$\mu\,(\Xi_b^0) = \tfrac{1}{3}\,(2\mu_u + 2\mu_s - \mu_b);$$

$$\mu\,(A_b^+) = \tfrac{1}{3}\,(2\mu_u + 2\mu_c - \mu_b);$$

$$\mu\,(\Xi_b^-) = \tfrac{1}{3}\,(2\mu_d + 2\mu_s - \mu_b);$$

$$\mu\,(\lambda_b^0) = \tfrac{1}{3}\,(2\mu_d + 2\mu_c - \mu_b);$$

$$\mu\,(A_b^0) = \tfrac{1}{3}\,(2\mu_s + 2\mu_c - \mu_b);$$

$$\mu\,(\Xi_{bb}^-) = \tfrac{1}{3}\,(4\mu_b - \mu_d);\;\; \mu\,(\Xi_{bb}^0) = \tfrac{1}{3}\,(4\mu_b - \mu_u);$$

$$\mu\,(\Omega_{bb}^-) = \tfrac{1}{3}\,(4\mu_b - \mu_s);\;\; \mu\,(\Omega_{bb}^0) = \tfrac{1}{3}\,(4\mu_b - \mu_c);\text{ and}$$

$$\mu\,(\Lambda_b^0) = \mu_b. \tag{7}$$

Table 1. Simplified wave functions of the spin-(1/2)+ baryons

Particle	Wave function	Particle	Wave function
Σ_b^+	$uub\,\chi_m$	λ_b^+	$ccb\,\chi_m$
Σ_b^0	$udb\,\chi_m$	$\lambda_b'^0$	$dcb\,\chi'_m$
Λ_b^0	$udb\,\chi'_m$	λ_b^0	$dcb\,\chi_m$
Σ_b^-	$ddb\,\chi_m$	$A_b'^+$	$ucb\,\chi'_m$
Ξ_b^0	$usb\,\chi_m$	A_b^+	$ucb\,\chi_m$
$\Xi_b'^0$	$usb\,\chi'_m$	Ξ_{bb}^-	$bbd\,\chi_m$
Ω_b^-	$ssb\,\chi_m$	Ξ_{bb}^0	$bbu\,\chi_m$
Ξ_b^-	$dsb\,\chi_m$	Ω_{bb}^-	$bbs\,\chi_m$
$\Xi_b'^-$	$dsb\,\chi'_m$	Ω_{bb}^0	$bbc\,\chi_m$
A_b^0	$scb\,\chi_m$		
$A_b'^0$	$scb\,\chi'_m$		

The spin wave functions χ_m and χ'_m (for $m=1/2$) are given in equations (1) and (2).

From the above results we get the following sum rules:

$$\mu\,(\Sigma_b^-) + \mu\,(\Xi_{bb}^-) = \mu_d + \mu_b, \tag{8}$$

$$\mu\,(\Sigma_b^+) + \mu\,(\Xi_{bb}^0) = \mu_u + \mu_b, \tag{9}$$

$$\mu\,(\Omega_b^-) + \mu\,(\Omega_{bb}^-) = \mu_s + \mu_b, \tag{10}$$

$$\mu\,(\lambda_b^+) + \mu\,(\Omega_{bb}^0) = \mu_c + \mu_b, \tag{11}$$

$$\begin{aligned}\mu\,(\Xi_b^0) + \mu\,(\lambda_b^0) &= \mu\,(A_b^+) - \mu\,(\Xi_b^-),\\ &= \tfrac{2}{3}\,(\mu_u - \mu_d).\end{aligned} \tag{12}$$

2.2. $J^P = (\frac{3}{2})^+$ baryons

Magnetic moments for J^P=(3/2)$^+$ baryons are obtained using the simplified baryon wave functions of table 2 and equation (1). We get the following relations:

$$\mu(\Sigma_b^{*0}) = \tfrac{1}{3}\,(\mu_u + \mu_d + \mu_b);\ \mu\,(\Xi_b^{*-}) = \tfrac{1}{3}\,(\mu_d + \mu_s + \mu_b);$$

Table 2. Simplified wave functions of the spin-(3/2)$^+$ baryons

Particle	Wave function
Σ_b^{*+}	$uub\ \chi_m^{3/2}$
Σ_b^{*-}	$ddb\ \chi_m^{3/2}$
Σ_b^{*0}	$udb\ \chi_m^{3/2}$
Ξ_b^{*0}	$usb\ \chi_m^{3/2}$
Ξ_b^{*-}	$dsb\ \chi_m^{3/2}$
Ω_b^{*-}	$ssb\ \chi_m^{3/2}$
λ_b^{*0}	$dcb\ \chi_m^{3/2}$
A_b^{*+}	$ucb\ \chi_m^{3/2}$
A_b^{*0}	$scb\ \chi_m^{3/2}$
λ_b^{*++}	$ccb\ \chi_m^{3/2}$
Ξ_{bb}^{*-}	$bbd\ \chi_m^{3/2}$
Ξ_{bb}^{*0}	$bbu\ \chi_m^{3/2}$
Ω_{bb}^{*-}	$bbs\ \chi_m^{3/2}$
Ω_{bb}^{*0}	$bbc\ \chi_m^{3/2}$
Ω_{bbb}^{-}	$bbb\ \chi_m^{3/2}$

$\mu(\Xi_b^{*0}) = \frac{1}{3}(\mu_u + \mu_s + \mu_b)$; $\mu(A_b^{*+}) = \frac{1}{3}(\mu_u + \mu_s + \mu_b)$;

$\mu(A_b^{*0}) = \frac{1}{3}(\mu_s + \mu_c + \mu_b)$; $\mu(\Sigma_b^{*+}) = \frac{1}{3}(2\mu_u + \mu_b)$;

$\mu(\Sigma_b^{*-}) = \frac{1}{3}(2\mu_d + \mu_b)$; $\mu(\Omega_b^{*-}) = \frac{1}{3}(2\mu_s + \mu_b)$;

$\mu(\lambda_b^{*+}) = \frac{1}{3}(2\mu_c + \mu_b)$;

$\mu(\lambda_b^{*0}) = \frac{1}{3}(\mu_d + \mu_c + \mu_b)$;

$\mu(\Xi_{bb}^{*-}) = \frac{1}{3}(2\mu_b + \mu_d)$;

$\mu(\Xi_{bb}^{*0}) = \frac{1}{3}(2\mu_b + \mu_u)$;

$\mu(\Omega_{bb}^{*-}) = \frac{1}{3}(2\mu_b + \mu_s)$;

$\mu(\Omega_{bb}^{*0}) = \frac{1}{3}(2\mu_b + \mu_c)$; and

$$\mu(\Omega_{bbb}^{*-}) = \mu_b. \tag{13}$$

From the above results we get the following sum rules;

$$\mu(\Xi_b^{*0}) - \mu(\Xi_b^{*-}) = \mu(A_b{}^{*+}) - \mu(\lambda_b^{*0}),$$

$$= \tfrac{1}{2}[\mu(\Sigma_b^{*+}) - \mu(\Sigma_b^{*-})],$$

$$\mu(\Xi_{bb}^{*0}) - \mu(\Xi_{bb}^{*-}) = \tfrac{1}{3}(\mu_u - \mu_d). \tag{14}$$

3. Transition moments

The expression for nonvanishing transition magnetic moment of two different baryons (Singh 1977) is written as

$$\mu_{BB'} = \Big(B, \sum_i \mu_q(i)\, \sigma_3(i)\, B'\Big), \tag{15}$$

where B and B' are two different baryons. If B and B' have different spins, the baryon with the smaller spin is maximally polarised along the Z-axis and both baryons have the same value of Jz. The transition moments for $(1/2)^+$ baryons are obtained as,

$$\langle \Sigma_b^0 | \mu | \Lambda_b^0 \rangle = \frac{1}{\sqrt{3}}(\mu_d - \mu_u); \tag{16}$$

$$\langle \Xi_b^0 | \mu | \Xi'^0_b \rangle = \frac{1}{\sqrt{3}}(\mu_S - \mu_u); \tag{17}$$

$$\langle \lambda_b^0 | \mu | \lambda'^0_b \rangle = \frac{1}{\sqrt{3}}(\mu_c - \mu_d). \tag{18}$$

3.1. *Transition moments* $\langle (3/2)^+ \mid \mu \mid (1/2)^+ \rangle$

We now calculate the transition moments between two baryons—one belonging to $(3/2)^+$ and the other $(1/2)^+$ — and we get the following results:

$$\langle \Sigma_b^{*0} \mid \mu \mid \Sigma_b^0 \rangle = \frac{2}{3\sqrt{2}}(\mu_u + \mu_d - 2\mu_b); \tag{19}$$

$$\langle \Xi_b^{*0} \mid \mu \mid \Xi_b^0 \rangle = \frac{2}{3\sqrt{2}}(\mu_u + \mu_s - 2\mu_b); \tag{20}$$

$$\langle A_b^{*+} \mid \mu \mid A_b^+ \rangle = \frac{2}{3\sqrt{2}}(\mu_u + \mu_c - 2\mu_b); \tag{21}$$

$$\langle \Xi_b^{*-} \mid \mu \mid \Xi_b^- \rangle = \frac{2}{3\sqrt{2}}(\mu_d + \mu_S - 2\mu_b); \tag{22}$$

$$\langle \lambda_b^{*0} \mid \mu \mid \lambda_b^0 \rangle = \frac{2}{3\sqrt{2}}(\mu_d + \mu_c - 2\mu_b); \tag{23}$$

$$\langle A_b^{*0} \mid \mu \mid A_b^0 \rangle = \frac{2}{3\sqrt{2}}(\mu_S + \mu_c - 2\mu_b); \tag{24}$$

$$\langle \Sigma_b^{*-} \mid \mu \mid \Sigma_b^- \rangle = \frac{4}{3\sqrt{2}}(\mu_d - \mu_b); \tag{25}$$

$$\langle \Sigma_b^{*+} \mid \mu \mid \Sigma_b^+ \rangle = \frac{4}{3\sqrt{2}}(\mu_u - \mu_b); \tag{26}$$

$$\langle \Omega_b^{*-} \mid \mu \mid \Omega_b^- \rangle = \frac{4}{3\sqrt{2}}(\mu_S - \mu_b), \quad \text{and} \tag{27}$$

$$\langle \lambda_b^{*+} \mid \mu \mid \lambda_b^+ \rangle = \frac{4}{3\sqrt{2}}(\mu_c - \mu_b). \tag{28}$$

4. Conclusions

Using the quark model we haves hown how to obtain the magnetic moment of 35-plet symmetric and 40-plet mixed baryons of SU(5) as well as a number of transition magnetic moments in terms of only five parameters, the magnetic moments of the five quarks. Several relations among the magnetic moments of $J^P = (1/2)^+$ and $(3/2)^+$ baryons and transition moments $\langle 3/2 \mid \mu \mid 1/2 \rangle$ have been obtained which are valid for the total as well as anomalous magnetic moment.

The few non-vanishing transition magnetic moments calculated here are important for calculating the decay from one baryon to another *via* photon emission (Quigg and Rosner 1977; Schachinger *et al* 1978).

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