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Quark model predictions on the masses of heavier vector mesons

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Abstract. We use the quark model considerations of Federman, Rubinstein and Talmi and the relation $(m_\psi/m_\phi) \simeq (m_\Upsilon/m_\psi) \simeq (m_{\Upsilon_t}/m_\Upsilon)$ to compute the masses of beautiful and tasty vector mesons.

Keywords. Vector mesons; quarkmodel; hadron masses.

1. Introduction

The three u, d and s quarks proposed by Gell-Mann (1964) and Zweig (1964) as the basic constituents of matter, have dominated the hadron spectroscopy until 1974. The indirect evidence for the fourth quark c, carrying the charm quantum number, was provided by the discovery of the J/ψ vector particle (Aubert *et al* 1974; Augustin *et al* 1974; Abrams *et al* 1974 and Brauns *et al* 1975) which can be understood as the $c\,\bar{c}$ bound state. The discovery of a new class of resonance states called the upsilon family (Herb *et al* 1977; Innes *et al* 1977 and Kephart *et al* 1977) and a new heavy lepton τ (Perl *et al* 1975) accompanied by its own neutrino ν_τ (Perl *et al* 1977) compels us to introduce new quark b and possibly t carrying new quantum numbers beauty and taste, respectively. With these heavier quarks, also demanded by lepton-quark symmetry (Harari 1975) and the unified gauge theories (Kobayashi and Maskawa 1973), heavier mesons and baryons are expected. Several authors (Boal 1978; Aubrecht *et al* 1979; Camiz *et al* 1979; Bose 1979; Singh *et al* 1979, Misra and Sastry 1979 and Martin 1979) have already studied some of the properties of the heavy hadrons.

In this paper we consider the masses of the beautiful and tasty mesons in a quark model including b and t quarks. We follow the considerations of Federman and his co-workers (Federman *et al* 1966 and Rubinstein 1966) that the mass of the hadron arises from a contribution from the sum of the masses of quarks it contains plus a sum of the two-body interaction energies. We have treated the problem in the framework of SU(12) $\supset$ SU(6) $\otimes$ SU(2) by including the spin dependent interactions. Taking the universality of the relation

$$\frac{m_\psi}{m_\phi} \simeq \frac{m_\Upsilon}{m_\psi} \simeq \frac{m_{\Upsilon_t}}{m_\Upsilon},$$

we compute the masses of beautiful and tasty vector mesons.

2. Preliminaries

Mesons are made up of a quark and an antiquark. We have therefore 36 mesons, generated in the direct product

$$6 \otimes \bar{6} = 1 \oplus 35. \tag{1}$$

These are represented in table 1. In the case of SU(12) the meson will belong to the representations present in the direct product

$$12 \otimes \overline{12} = 1 \oplus 143. \tag{2}$$

The 143-dimensional representation of SU(12) has the following SU(6) and spin contents

$$143 \supset {}^{3}35 \oplus {}^{1}35 \oplus {}^{3}1, \tag{3}$$

where the superscripts denote the spin multiplicity.

The mass of the meson is assumed to be the sum of the masses of the quarks contained in it plus a contribution from the sum of two-body interactions. By the mass of the quark we mean its effective mass when bound in a meson. It might arise from a scalar interaction which modifies the mass term in the Hamiltonian describing the system. We take the mass of any bound quark as being independent of the meson in which it is bound. For two-body interactions, it is assumed that

Table 1. Vector mesons

SU(6)	SU(5)	SU(4)	SU(3)	States
1	1	1	1	(Υ_t)
35	5	4	3	$(D_t^{*+}, D_t^{*0}, F_t^{*+})$
			1	(G_t^{*0})
		1	1	(H^{*+})
	$\bar{5}$	1	1	(H^{*+})
		4	1	$(\bar{G}_t^{*0})$
			3	$(D_t^{*-}, \bar{D}_t^{*0}, F_t^{*-})$
	1	1	1	(Υ)
	24	4	3	$(D_b^{*+}, D_b^{*0}, F_b^{*0})$
			1	(G^{*+})
		$\bar{4}$	1	(G^{*-})
			3	$(D_b^{*-}, \bar{D}_b^{*0}, \bar{F}_b^{*0})$
		1	1	(ψ)
		15	3	(D^{*+}, D^{*0}, F^{*0})
			$\bar{3}$	$(D^{*-}, \bar{D}^{*0}, \bar{F}^{*0})$
			1	(ϕ)
			8	$2\,(K^{*0}, K^{*0})$
				$\bar{2}\,(K^{*-}, \bar{K}^{*0})$
				$1\,(\omega)$
				$3\,(\rho^{+}, \rho^{0}, \rho^{-})$

it depends on the flavour and spin configuration of the quarks, as well as on whether the interaction is between two quarks or between quark and antiquark. We write the interaction energy between the q_1 and q_2 as V_{12}^S where 1 and 2 are the flavour indices and S is the spin index. So the mass operator for the meson will be

$$M = m_0 + m_1 + m_2 + V_{12}^S, \tag{4}$$

where m_0 will remain the same for all the mesons. We take $m_u = m_d = m$ and m_s, m_c, m_b and m_t as the masses of the respective quarks.

3. Masses of the mesons

From the quark contents and with the help of equation (4), the masses of the mesons can easily be calculated. They depend linearly on m, m_s, m_c, m_b, m_t and V's. It has been noted (Federman *et al* 1966; Hendry 1967; Hendry and Lichtenberg 1975) that the assumption (the quark-quark (antiquark) interaction is a two-body interaction) is not strong enough to obtain decuplet equal spacing rule and the well-known Gell Mann-Okubo octet mass formula. However, to obtain these, one has to assume that there is a relation among the quark-quark (antiquark) interaction energies, namely,

$$2V_{12}^1 = V_{11}^1 + V_{22}^1. \tag{5}$$

Then the following sum rules are obtained.

3.1. *Vector mesons*

Charm sector

$$\underset{(\text{1·80 GeV})}{2K^*} = \underset{(\text{0·778 GeV})}{\rho} + \underset{(\text{1·02 GeV})}{\phi} \qquad [\text{1·79 GeV}], \tag{6}$$

$$\underset{(\text{4·0 GeV})}{2D^*} = \underset{(\text{0·778 GeV})}{\rho} + \underset{(\text{3·095 GeV})}{\psi} \qquad [\text{3·88 GeV}], \tag{7}$$

$$\underset{(\text{4·2 GeV})}{2F^*} = \underset{(\text{1·02 GeV})}{\phi} + \underset{(\text{3·095 GeV})}{\psi} \qquad [\text{4·12 GeV}], \tag{8}$$

Beauty sector

$$2D_b^* = \rho + \Upsilon \qquad [\text{10·238 GeV}], \tag{9}$$

$$2F_b^* = \phi + \Upsilon \qquad [\text{10·48 GeV}], \tag{10}$$

$$2G^* = \psi + \Upsilon \qquad [\text{12·555 GeV}]. \tag{11}$$

Taste sector

$$2D_t^* = \rho + \Upsilon_t \qquad [\text{29·668 GeV}], \tag{12}$$

$$2F_t^* = \phi + \Upsilon_t \qquad [\text{29·93 GeV}], \tag{13}$$

$$2G_t^* = \psi + \Upsilon_t \qquad [\text{32·005 GeV}], \tag{14}$$

$$2H^* = \Upsilon + \Upsilon_t \qquad [\text{38·37 GeV}]. \tag{15}$$

The round brackets contain the known mass values. We can calculate the masses of the beautiful vector mesons as the experimental values of the particles on the right side of equations (9) to (11) are known. The predicted masses are given in square brackets along with the respective equation and compared with other predictions (table 2).

A remarkable relationship between the masses of the lowest mesonic states of the $s\,\bar{s}$, $c\,\bar{c}$ and $b\,\bar{b}$ families has been noticed (Khare 1979) namely

$$m_{\psi}/m_{\phi} \simeq (m_{\Upsilon}/m_{\psi}) \simeq 3{\cdot}02. \tag{16}$$

The universality of this relation,

$$\frac{m_{\psi}}{m_{\phi}} \simeq \frac{m_{\Upsilon}}{m_{\psi}} \simeq \frac{m_{\Upsilon_t}}{m_{\Upsilon}} \simeq 3{\cdot}02, \tag{17}$$

gives

$$m_{\Upsilon_t} \simeq \frac{m\Upsilon^2}{m_{\psi}} \simeq 28{\cdot}9 \text{ GeV}. \tag{18}$$

Jenkovszky (1978) has also considered the constant ratio between the masses of successively heavier quark of different flavours

i.e.

$$\frac{m_c}{m_s} \simeq \frac{m_b}{m_c} \simeq \frac{m_t}{m_b} \simeq 3. \tag{19}$$

With this value of m_{Υ_t}, we can then predict the masses of the other vector mesons in the taste sector from (12) to (15). The calculated masses are given in square brackets along with the equations.

3.2 *Pseudoscalar mesons*

Taking spin-dependent interactions, no relation can be obtained for pseudoscalar mesons. However, if we take the interaction to be spin-independent, and use the same two-body interaction relation (5), we obtain the relations similar to the ones for the vector mesons. In the case of pseudoscalar mesons the estimation of the beautiful and tasty meson masses is not possible, as the experimental values of η_b and η_t are not known.

Table 2. Masses of the beautiful mesons (in GeV)

Particles	Predicted masses			
	Present analysis	Misra and Sastry (1979)	Aubrecht (1979)	Martin (1979)
D_b^*	5·12	5·08	5·09	5·27–5·37
F_b^*	5·24	5·20	5·21	5·47–5·8
G^*	6·28	6·32	6·24	6·24–6·62

4. Conclusions

With the available values of the masses of charmed mesons we see that the vector meson linear mass relations ((6)-(8)) are satisfied and so we may expect the same thing for the corresponding relations in the beauty and taste sectors. By taking the interaction to be spin-dependent, seven relations are found for the (beautiful and tasty) vector mesons, but no independent relation is obtained for pseudoscalar mesons. If the interaction is taken to be spin-independent, using relation (5), analogous relations are found for pseudoscalar mesons. Assuming the mass relation

$$\frac{m_{\not{\phi}}}{m_{\phi}} \simeq \frac{m_{\Upsilon}}{m_{\not{\phi}}} \simeq \frac{m_{\Upsilon_t}}{m_{\Upsilon}}$$

we predict the mass, $m_{\Upsilon_t} \simeq 28{\cdot}9$ GeV. With this value we compute the masses of tasty vector mesons. However, the masses of the pseudoscalar mesons cannot be predicted because of the non-availability of the mass η_b. The validity of the quark model for the internal symmetry SU(6) and the assumption of the quark interactions, which have been quite successful at SU(3) and SU(4) levels, will be tested only in future when the expected particles are seen.

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