Pramāṇa, Vol. 14, No 6, June 1980, pp. 425-431. © Printed in India

Electromagnetic form factors of ^{3}He and ^{3}H

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MS received 28 February 1980

Abstract. The electric and magnetic form factors of ^{3}He and ^{3}H are calculated with 3-nucleon wave functions obtained from the solution of Schrödinger equation with separable potentials of two different shapes which have already been employed in the coulomb energy calculation. The effect of important meson exchange corrections is evaluated and their dependence on the wave function studied. The form factors can depend rather sensitively on the nucleon form factors as well, and this dependence is studied by using two different parametrisations for the latter.

Keywords. Electromagnetic form factors; meson exchange currents; separable potentials; nucleon form factors.

1. Introduction

In the last few years interest in the electromagnetic form factors of ^{3}H and ^{3}He has sustained mainly because of the minimum in the ^{3}He charge form factor at about 11·6 fm^{-2} (McCarthy *et al* 1970; 1977). The occurrence of the minimum suggests the existence of strong correlations in the three-body wave function at short distances. In the last decade a large number of 'exact' calculations of the binding energy and the wave function of a three-nucleon system have been carried out, thanks to the availability of large computers. The electromagnetic form factors have been calculated in the impulse approximation (IA) with the 'exact' wave functions as well as with variational wave functions. While these calculations yielded a minimum in the ^{3}He charge form factor, a precise agreement between theory and experiment could not be obtained (Haftel and Kloet 1977). They made an exhaustive study of various corrections to the IA charge form factors. The various meson exchange current corrections turned out to be the most important ones. Calculation of the exchange current contribution (ECC) to the magnetic form factors have also been done, the most recent being by Hadjimichael (1978).

An alternative often employed in many three-body and other few-body calculations because of its simplifying character is the use of separable potentials with excellent results for the binding energy and doublet *n-d* scattering length (Mitra 1969; Schrenk *et al* 1970). An earlier investigation of the electromagnetic radii and form factors at low momentum transfers also gave encouraging results (Gupta *et al* 1967). However, when taken to higher momentum transfers the results (Mehdi and Gupta 1976) completely disagreed with the experimental findings—no minimum in the ^{3}He charge form factor was found even after the inclusion of a repulsive term in the singlet N-N potential. Calculations with a different potential shape (shape-II of Mehdi and Gupta (1976), which is one of a class of potentials proposed by Kharchenko *et al* (1968), though gave a vast improvement in the overall agreement with experiment in binding energy, various radii and form factors, as well as in the ^{3}He coulomb energy (Mehdi and Gupta 1979), failed to produce the elusive minimum. The inability to produce the required minimum appears to be a basic drawback of the separable potential model with soft core repulsion. This observation is further supported by the calculation of Haftel and Kloet (1977) in which the only potential that does not yield a minimum, even after the inclusion of repulsion in the singlet and tensor force in the triplet state, is separable potential.

Because of their extreme simplicity, separable potentials still serve as a useful model for many complex few-body systems. For a simple model calculation involving these complex systems, one would like to have a simple potential without the formidable complications of a tensor force term, and if possible, even a repulsive term. The shape-II potential seems to be a good candidate for this purpose—much better than the commonly used shape-I potential. One would however like to see if a reasonable agreement for the electro-magnetic form factors can be obtained with shape-II potentials, once the ECC, the most dominant of the various corrections, is included in the study.

The form factors, and especially the exchange contribution to these, depend rather significantly on the nucleon form factors as well. This comes about because the expressions for the form factors involve terms with opposite signs. Thus, in particular the precise position of the minimum could depend sensitively on the nucleon form factors for which quite a few sets of data exist. It will be of interst to study this sensitivity.

In this paper we report our results on the 3-nucleon form factors obtained with the inclusion of ECC. There are many elementary diagrams that contribute to exchange currents. Of these we, in our calculation, have included I(a) and I(b) of Kloet and Tjon (1974), which make the dominant contributions. The sensitivity of the results to the nucleon form factors is studied by doing the calculation for two different parametrisations of the nucleon form factors (de Vries *et al* 1964; Janssens *et al* 1966) which give equally good fit to the electron scattering data.

2. Summary of the formalism

The general form of a rank two separable potential in the singlet state is

$$-M\langle \mathbf{p} \,|\, V_s \,|\, \mathbf{q}\rangle = \lambda_s\,[s(p)\,s(q) - h(p)\,h(q)],$$

where $h(p)$ provides the necessary repulsion. The traditional forms of $s(p)$ and $h(p)$, which we call shape-I, are

$$s(p) = (p^2 + \beta_s^2)^{-1}; \quad h(p) = np^2\,(p^2 + \beta_h^2)^{-2}.$$

For the triplet state we have, in the absence of tensor force, the single term potential

$$-M\langle \mathbf{p} \,|\, V_T \,|\, \mathbf{q}\rangle = \lambda_T\, c(p)\, c(q).$$

The traditional form of $c(p)$, like that of $s(p)$, is

$$c(p) = (p^2 + \beta_c^2)^{-1}.$$

Shape-II on the other hand corresponds to

$$s(p) = (p^2 + \bar{\beta}_s^2)^{-2};\ h(p) = \bar{n}\, p^2 (p^2 + \bar{\beta}_h^2)^{-2};\ c(p) = (p^2 + \bar{\beta}_c^2)^{-2},$$

with strength parameters changed to new values $\bar{\lambda}_s$ and $\bar{\lambda}_T$.

The parameters of the various sets of singlet potentials used in our previous study (Mehdi and Gupta 1976), as well as in the present one, and the two-body data to which they are fitted are listed in tables 1 and 2. The triplet parameters (β_c, λ_T or $\bar{\beta}_c$, $\bar{\lambda}_T$, as the case may be) are adjusted to give the correct deuteron binding energy (2.22 MeV) and scattering length (5·40 fm).

The impulse approximation expression for the various form factors are well-known and have been derived earlier (Gupta *et al* 1967). The extension to the case when the potential includes a repulsive term is straightforward. The calculation reduces to the evaluation of certain multiple integrals involving the 3-body wave function which is known from the solution of energy eigen-value problem.

Table 1. The potential parameters β_s, λ_s, β_h and n for various potential sets of shape-I and the two-body parameters a_s and r_{0s} (in Fermis) to which the former are fitted. α is the deuteron binding energy parameter. S and H represent, respectively, the attractive and repulsive parts of the $^1S_0$ potential and C^{eff} the 'effective' central part of $^3S_1$ potential. For meaning of suffixes, Y, N, G_1' see Mitra (1969) and Schrenk *et al* (1970). The triplet parameters, which correspond to $a_t = 5{\cdot}378$ fm and $r_{0t} = 1{\cdot}716$ fm, are the same for sets I, III and IV ($\beta_c = 6{\cdot}255\alpha$, $\lambda_T = 33{\cdot}36\alpha^3$). For set II these parameters are $\beta_c = 5{\cdot}8\alpha$, $\lambda_T = 22{\cdot}9\alpha^3$.

Set	Potential	β_s/α	λ_s/α^3	β_h/α	n	$-a_s$	r_{0s}
I	$C_Y^{\text{eff}} + S_Y$	6·255	23·4	—	—	23·7	2·151
II	$C_N + S_N$	5·8	18·6	—	—	23·7	2·33
III	$C_Y^{\text{eff}} + (S+H)_N$	8·0	62·4	8·0	2·333	23·7	2·355
IV	$C_Y^{\text{eff}} + (S+H)_{G_1'}$	5·7	18·95	7·8	2·74	18·0	2·70

Table 2. The potential parameters $\bar{\beta}_s$, $\bar{\lambda}_s$, $\bar{\beta}_h$ and $\bar{n}$ for various potential sets of shape-II and the two-body parameters a_s and r_{0s} (in Fermis) to which the former are fitted. C, S, H and α have the same meaning as in table 1. The triplet parameters ($\bar{\beta}_c = 9{\cdot}0\alpha$, $\bar{\lambda}_T = 1{\cdot}083 \times 10^{+6}\, \alpha^7$) are the same for all the potential sets and correspond to $a_t = 5{\cdot}405$ fm, $r_{0t} = 1{\cdot}76$ fm.

Set	Potential	$\bar{\beta}_s/\alpha$	$\bar{\lambda}_s \times 10^{-5}/\alpha^7$	$\bar{\beta}_h/\alpha$	n	$-a_s$	r_{0s}
I′	$C_{MG} + S_{MG_1}$	7·411	1·803	—	—	18·0	2·7
II′	$C_{MG} + S_{MG_2}$	7·16	1·412	—	—	18·0	2·8
III′	$C_{MG} + (S+H)_{MG_1}$	7·70	2·464	12·0	0·0953	18·0	2·7
IV′	$C_{MG} + (S+H)_{MG_2}$	7·40	1·839	10·0	0·0455	18·0	2·8
V′	$C_{MG} + (S+H)_{MG_3}$	7.60	2·260	8·5	0·0320	18·0	2·8

The 3-body wave function, in the absence of D-state, consists of a totally space-symmetric S-state, with a small admixture of S'-state ($\sim 1\,\%$). Thus for the purpose of evaluating the correction due to exchange effects one can safely approximate it by a pure S-state. In that case, following Kloet and Tjon (1971; 1974), the evaluation of meson exchange corrections to the various form factors also becomes fairly straightforward and reduces to the evaluation of just two multiple integrals.

The 3-nucleon electromagnetic form factors are the products of the ' body form factors ' with the electromagnetic form factors of the nucleons. The nucleon form factors are generally parametrised in terms of important pole contributions and many such parametrisations exist (de Vries *et al* 1964; Janssens *et al* 1966), which give equally good fit to the electron scattering data, especially at low momentum transfers. To study the dependence of the 3-nucleon form factors on the nucleon form factors the calculation has been done for both these parametrisations of the latter.

3. Results and discussion

The form factors in the impulse approximation both for shape-I and shape-II are shown in figures 1(a)–(d). The effect of the ECC as well as the nucleon form factors are depicted in figures 2 and 3. The IA results for the form factors (and for the radii as well) were discussed in Mehdi and Gupta (1976) and are included here for the sake of completeness. The main features to note are a marked overall improvement with shape-II potential sets (primed) compared to shape-I potential sets (unprimed) and a reduction in the effect of repulsive core with shape-II compared to its effect with shape-I. This wide difference between the two shapes can be understood from the effect of the shape on the wave function. To illustrate the point we show in figure 4 the deuteron wave function as obtained with the two shapes. The wave function near the origin is much smaller with shape-II than with the other. Thus with shape-II potential any two nucleons in ^{3}H or ^{3}He as well, tend to stay away from each other, thereby increasing the three-body radii and depressing the form factors. Since the wave function with shape-II is already quite hollowed out, the effect of introducing a repulsive core is much smaller than with shape-I, thus explaining both features mentioned above.

As for the meson exchange currents, first of all we observe that their effect on magnetic form factors is almost negligible both for ^{3}He and ^{3}H. The oscillation in the sign of the ECC has no dynamical significance but is due to the definition; the quantity μF_{mag}, where μ is the static magnetic moment (which is also affected by exchange currents), decreases in a uniform manner on the addition of these effects.

The effect of exchange currents on the charge form factors is quite different for ^{3}H and ^{3}He. Whereas for the former it remains small (starting of course with zero at zero momentum transfer), for the latter it is much more appreciable and is maximum at $\sim 4\ \text{fm}^{-2}$ after which it decreases slowly. Since the form factor itself falls quite rapidly, the relative effect of exchange currents keeps on increasing with increasing momentum transfer.

In figure 2 is also shown the effect of the nucleon charge from factor on the charge form factor of ^{3}He. As has already been mentioned, all the 3-nucleon form factors depend upon the values of the nucleon form factors. Of these, however, the ^{3}He

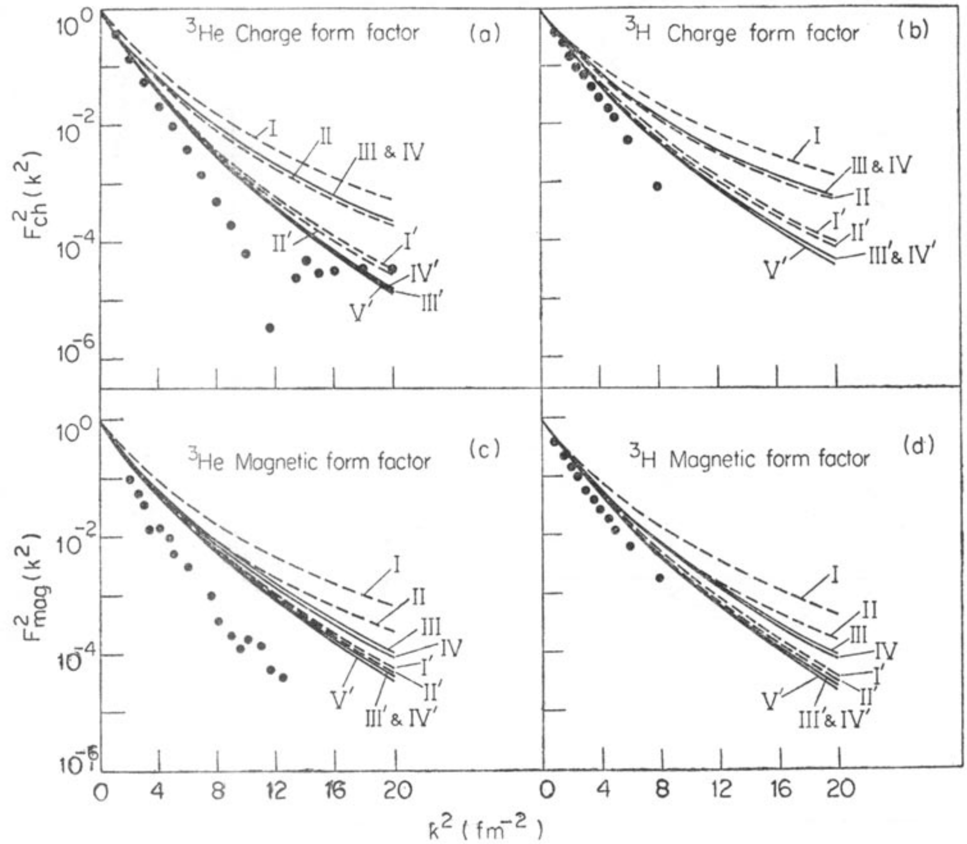

Figure 1. Charge and magnetic form factors of ^{3}H and ^{3}He in the impulse approximation for the various potential sets of tables 1 and 2. Solid and dashed curves represent the form factors for potentials with and without the repulsive term, respectively. Data points are from McCarthy *et al* 1970.

charge form factor which is the difference of IA and ECC terms, both being of the same order, is the most sensitive one. Indeed, whereas the Janssens *et al* parametrisation is able to produce a change in the sign of the form factor, the de Vries *et al* parametrisation is not (the thick and thin lines respectively in figure 2). The effect on the other form factors (not shown in the figure) is comparatively small.

The dependence of ECC on the wave function can be seen from curves I-III in figure 2. The higher the form factor in the IA, the more is the depression brought about by the inclusion of ECC. Thus the ECC tends to reduce the differences in the form factor due to different 3-body wave functions. In particular, the effect of a soft core term which is already much less for shape-II than for shape-I potentials, is further reduced when exchange currents are included, so much so that for IA+ECC form factors are nearly the same with or without the repulsive core. The effect of the tensor force however may not follow the same pattern because of the introduction of *D*-state.

In conclusion, wherever the tensor force effects can be neglected the shape-II potential gives a better fit to the three-body data including the form factors and provides a more approximate wave function for use in other calculations than shape-I.

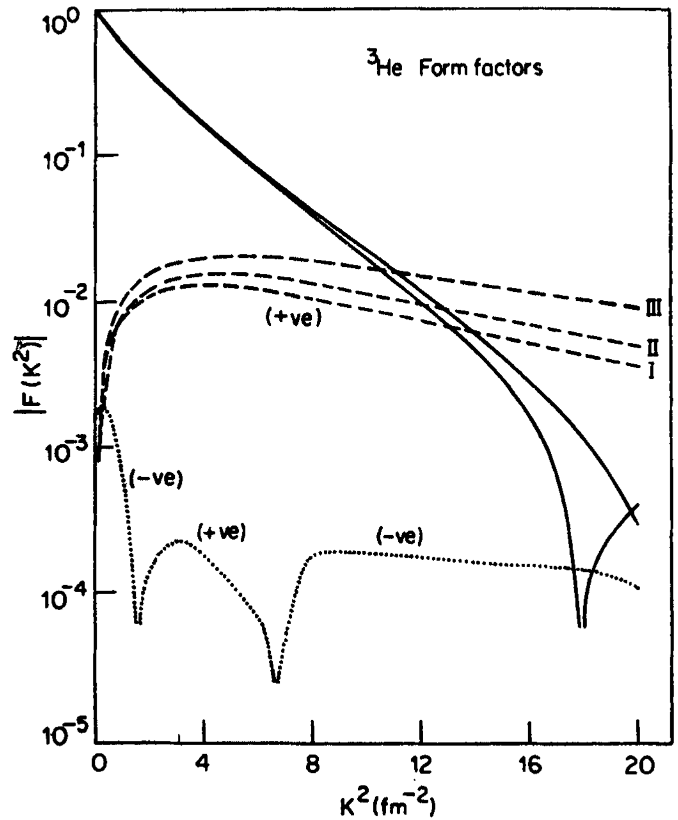

Figure 2. The exchange current contribution for ^{3}Hef orm factors. The three dashde lines (I-III) represent the ECC to charge form factor for potential sets V′, II′ and I, respectively. The dotted line represents the ECC for magnetic form factor. The solid lines represent the IA+ECC to charge form factor for the de Vries *et al* (1964) and Janssens *et al* (1966) parametrisation to the nucleon form factors.

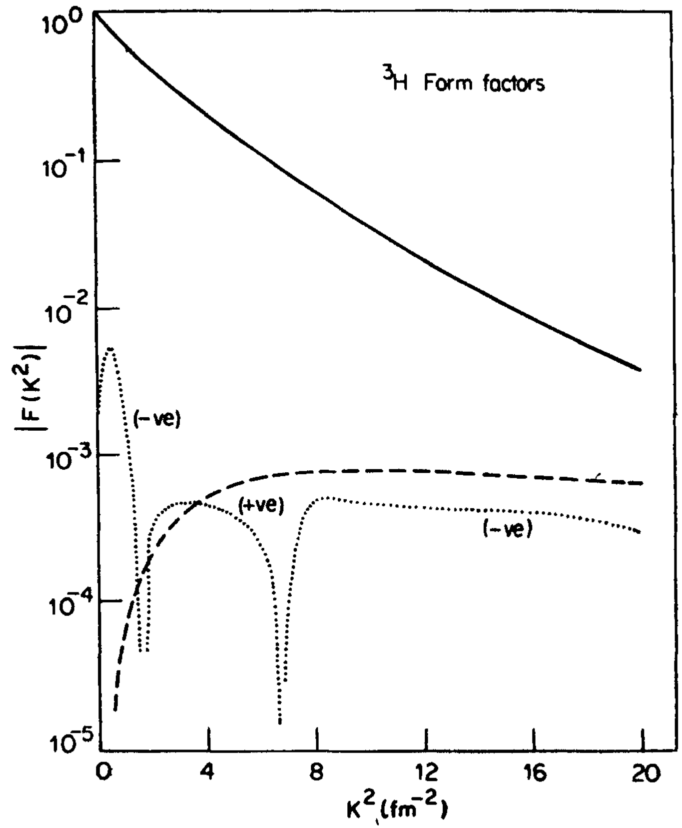

Figure 3. The ECC to the magnetic form factor (dotted line), charge form factor (dashed line) and IA+ECC to the charge form factor (solid line) for ^{3}H.

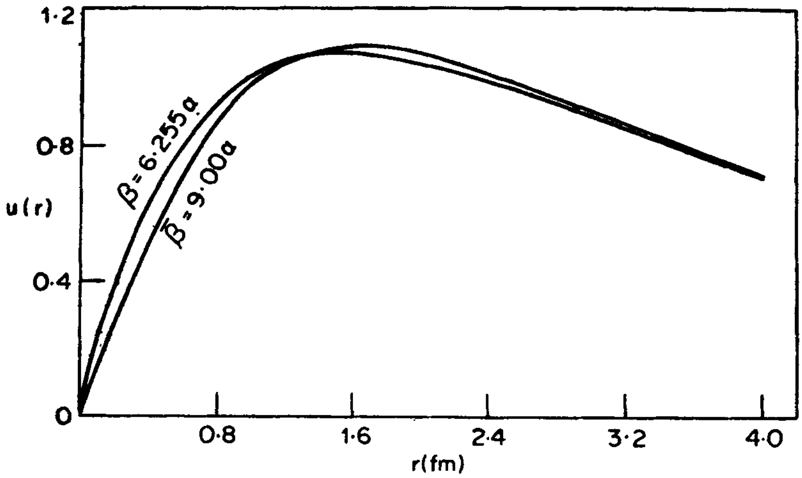

Figure 4. The deuteron wave function for the two potential shapes, I and II, with $\beta=6.255\alpha$ for shape-I and $\beta=9{\cdot}0\alpha$ for shape-II.

Also, the repulsive core has a much less crucial role to play and the wave function obtained without its inclusion is nearly as accurate as the one with it. For finer agreement on the position of the minimum and the secondary maximum, apart from the effects of exchange currents and three-body forces, etc., the nucleon form factors also play an important role.

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